Trade Policy under Monopolistic Competition with Firm Selection*

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Abstract

We analyze unilateral, efficient and Nash trade policies in a symmetric, two-country version of the Melitz-Ottaviano (2008) model. Starting at global free trade, we show that a country gains from the introduction of (1) a small import tariff; (2) a small export subsidy, if trade costs are low and the dispersion of productivities is high; and (3) an appropriately combined small increase in its import and export tariffs. The welfare of its trading partner, however, falls in each of these three cases. The market may provide too little or too much entry, depending on a simple relationship among model parameters. Correspondingly, global free trade is generally not efficient, even within the class of symmetric trade policies. We also provide conditions under which, starting at the symmetric Nash equilibrium, countries can mutually gain by exchanging small reductions in import tariffs, export tariffs or combinations thereof. More generally, we show that Nash equilibria are inefficient while "politically optimal" policies are efficient, indicating a central role for the terms-of-trade externality. We also discuss why the model's implications for the treatment of export subsidies in trade agreements differ from those that obtain in a model with CES preferences for the differentiated-goods sector.

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1 Introduction

A large literature now exists that analyzes the economics of trade agreements.[1] As Bagwell and Staiger (1999, 2002) argue, in the standard competitive framework, the purpose of a trade agreement is to facilitate an escape from a terms-of-trade driven Prisoners' Dilemma problem. This framework offers interpretations for GATT/WTO negotiations that lower tariff caps as well as for other key GATT/WTO design features.[2] It does not, however, provide an easy interpretation for the WTO's strong restrictions on export subsidies. In addition, despite the explosion of research on gains from trade in heterogeneous-firms models, only a small literature as yet has analyzed trade policies in such models.[3] Motivated by these and other considerations, we analyze trade policies in the heterogeneous-firms model of Melitz and Ottaviano (2008). Among other findings, we identify conditions under which countries have unilateral incentives to introduce beggar-thy-neighbor export subsidies.

While export subsidies were treated in a fairly permissive manner in the early years of GATT, they are banned (with certain exceptions) in the WTO.[4] By contrast, WTO member countries are free to impose positive (non-discriminatory) import tariffs that do not exceed their respective negotiated tariff caps. From the perspective of the standard terms-of-trade model, the relatively severe treatment of export subsidies in the WTO is puzzling. A higher import tariff typically generates a negative terms-of-trade externality for a country's trading partner, whereas a higher export subsidy normally provides the partner with a positive terms-of-trade externality. Indeed, the standard terms-of-trade model suggests that governments with political-economic objectives "under-supply" export subsidies in comparison to the level that would be efficient from their joint perspective. This implication contrasts sharply with the WTO's prohibition of export subsidies, indicating either that the rules on export subsidies are too severe or that the standard theory is missing something important. In this context, it is of particular interest to explore any new implications that heterogeneous-firms models may provide as regards the use and treatment of export subsidies.

We consider a symmetric, two-country version of the Melitz-Ottaviano model, which we modify slightly to include ad valorem import and export tariffs. In this model, firms observe trade policies, decide whether or not to incur the fixed cost associated with entry, observe their productivity realizations, and engage in monopolistic competition.

[1] See the contributions in Bagwell and Staiger (2016a), Bagwell et al (2016) and Maggi (2014).

[2] For broadly related analyses with imperfectly competitive markets and homogeneous firms, see Bagwell and Staiger (2012a,b, 2015) and Ossa (2011).

[3] See Melitz and Redding (2014) for a survey of the literature on heterogeneous-firms models of trade. Research that analyzes trade policies from this perspective is discussed below.

[4] GATT restrictions on export subsidies tightened over time. See Sykes (2005) for discussion of the evolution of subsidy rules in the GATT/WTO.

Consumer preferences are described by a quadratic utility function that is defined over a continuum of varieties and that exhibits "love of variety." The two markets are segmented, and a firm that locates in one market incurs a trade cost when exporting to the other market. As Melitz and Ottaviano show, a reduction in trade costs impacts the selection of firms into the domestic and export markets. The least productive firms are forced to exit, average mark-ups fall, and product variety increases. We allow that trade also may be subject to import and export tariffs. Unlike changes in "wasteful" trade costs, changes in tariffs have tariff-revenue implications and also impact welfare through this channel. We assume that tariff revenue is re-distributed to consumers in a lump-sum fashion.

The model has two sectors. In addition to the differentiated-goods sector, the model includes an outside-good sector. The outside good is freely traded with no trade costs and serves as the numeraire good. It is supplied at constant unit cost by a competitive industry. Markups thus vary not just among firms in the differentiated-goods sector but also across sectors. In the absence of domestic policy instruments, the cross-sectoral dispersion in markups suggests a possible role for trade policy in directing resources toward the differentiated-goods sector. This intuition, however, is incomplete, since as we show excessive entry into the differentiated-goods sector can occur without policy interventions.

To develop our findings, we highlight three driving forces in the model. The first effect is a *selection effect*: an increase in country l's import tariff or in country h's export tariff results in a lower cut-off cost level for domestic sales in country l and an increase in the cut-off cost level for domestic sales in country h. The second and related effect is a *firm-delocation effect*: an increase in country l's import tariff or in country h's export tariff likewise leads to an increase in the number of entrants into country l, a decrease in the number of entrants into country h, an increase in the number of varieties sold in country l, and a decrease in the number of varieties sold in country h. An important implication of these findings is that the model generates a *Metzler Paradox*: an increase in country l's import tariff or in country h's export tariff results in a decrease in the average price in country l and an increase in the average price in country h.[5] Finally, for the closed-economy version of the model, we decompose the externalities that would be generated were a social planner to raise entry beyond the level provided by the market. The *entry-externality effect* from additional entry derives in expectation from the direct consumer surplus gain from a new variety, the consumer surplus loss on pre-existing varieties, the benefit of an increase in the number of varieties, and a business-stealing effect. We sign each of these components and find that the sign of the net externality is determined by a simple relationship among model parameters, with a negative (positive) externality

[5] We highlight the selection and firm-delocation effects in order to derive and interpret our trade-policy findings. These effects are originally derived by Melitz and Ottaviano in the context of their analysis of the consequences of unilateral reductions in trade costs. See Section 4 of their paper.

existing if and only if a demand parameter α is above (below) a critical level, where α impacts the substitution level between the differentiated varieties and the outside good. A negative (positive) externality indicates that the market provides excessive (insufficient) entry into the differentiated-goods sector.

After highlighting these forces, we turn to the paper's primary focus and derive several trade-policy results. We assume that each country evaluates trade policies from the perspective of its national welfare, which in this model is summarized by the consumer surplus enjoyed on the differentiated-goods sector plus income, where trade policy influences income by generating tariff revenue or subsidy expenses. The model entails free entry, and so profits do not enter into the country's welfare function.

For our first set of results, we assume that countries start at global free trade and consider the implications of small, unilateral trade-policy changes. We first show that a country can gain by imposing a small import tariff, since it thereby generates a lower average domestic price, greater domestic variety and tariff revenue. Second, we also identify conditions under which a country can gain by introducing a small export subsidy. A country contemplating the introduction of a small export subsidy faces a tradeoff: an export subsidy generates entry and lowers the average price while expanding variety in the intervening country, but it also gives rise to a subsidy expense. We find that the intervening country gains from the introduction of a small export subsidy if selection effects are strong in that trade costs are small and the dispersion of productivity is high. A small export subsidy also can be attractive when selection effects are weak if in addition the demand parameter α that describes the relative appeal of the differentiated-goods sector is not too high. Third, we show that a country can gain by introducing a small import tariff and export tariff, where the tariffs are calibrated to keep the domestic cut-off cost level, and thus the average price and level of variety in the domestic market, constant. This intervention maintains a constant domestic consumer surplus for the differentiated-goods sector while also generating tariff revenue (on both imports and exports).

All three of the described interventions are beggar-thy-neighbor interventions: starting at global free trade, when a country introduces a small import tariff, a small export subsidy, or combined small import and export tariffs of the described kind, its trading partner suffers a reduction in welfare. While the unilateral appeal of a small export subsidy is dependent upon model parameters, the negative international externality that is associated with such a policy is not. The key point is that all of the described policy interventions raise the cut-off cost level in the foreign country and thus increase the average price and reduce the level of variety in this country.

These findings support a relationship between key trade cost and dispersion parameters in the heterogeneous-firms literature and the nature of optimal trade-policy interventions. Our export-subsidy findings are perhaps of greatest interest. We find that a country has incentive to introduce such a policy when trade costs are low and productivity dispersion is great, a setting which may be more likely in the current era and perhaps for some sectors more than others. Our findings also offer a partial perspective on the WTO's prohibition of export subsidies. To the extent that governments use trade agreements to limit the scope in the long run for beggar-thy-neighbor policies, our findings suggest that restrictions on export subsidies could be attractive once governments have achieved through preceding negotiations an outcome that is sufficiently close to global free trade.

We next consider efficient trade policies. We show that countries can use tariffs to effect lump-sum transfers, which implies that efficiency is measured by the sum of the two countries' welfare functions. We then show that the logic of the entry-externality effect extends to the two-country model of trade. Starting at global free trade, the impact on efficiency of a small increase in any tariff, or of a small and symmetric increase in the overall trade barrier between countries, is determined by a simple relationship among model parameters, with an efficiency gain (loss) occurring if and only if the demand parameter α is above (below) a critical level, indicating that the market provides excessive (insufficient) entry under global free trade. Thus, if entry is excessive under free trade, then a restriction on the introduction of small export subsidies would be efficiency enhancing, at least once countries get sufficiently close to free trade. For this setting, the model therefore provides an efficiency-based rationale for a prohibition on the use of export subsidies. Furthermore, such a restriction could be effective: even when entry is excessive at free trade, the introduction of a small export subsidy could be unilaterally attractive to the intervening country. When the market provides excessive entry at free trade, however, the model does not provide a similar efficiency-based rationale for caps on import tariffs.

We offer as well characterizations of Nash policies. Such characterization are relatively complicated, since an evaluation of a small tariff change starting at non-zero tariffs requires consideration of the tariff-revenue changes attributable to changes in entry levels. We show that, if the overall trade barrier is symmetric across the two directions of trade and the joint-welfare function is quasi-concave in this symmetric barrier, then the symmetric Nash equilibrium entails a higher-than-efficient overall trade barrier. Hence, starting at the symmetric Nash equilibrium, countries can mutually gain by exchanging small reductions in import tariffs, export tariffs or combinations thereof. Our findings here provide a possible interpretation for why early GATT rounds emphasized negotiated reductions in import tariffs while taking a more permissive stance toward export subsidies.

We also provide results about the levels of Nash import and export tariffs, respectively. If the joint-welfare function is quasi-concave and model parameters are such that the market does not provide insufficient entry under free trade, then, in a symmetric Nash equilibrium, the import tariff must be higher than the export tariff. We also establish a limiting result: as the trade cost goes to infinity, the Nash import tariff converges to a positive number while the limiting sign of the Nash export tariff is positive (negative) if the market provides excessive (insufficient) entry under free trade. Thus, a Nash export subsidy is implied when the trade cost is sufficiently high and the market provides insufficient entry.

We also show that welfare functions can be expressed as functions of local and world prices. Representing welfare functions in this way, we show that Nash tariffs are inefficient but that "politically optimal" tariffs are efficient. Our analysis thus indicates a sense in which the terms-of-trade externality represents the fundamental reason for a trade agreement in the Melitz-Ottaviano model. This finding complements previous findings as surveyed by Bagwell and Staiger (2016b) for settings with perfect competition and for settings with imperfect competition and homogeneous firms. We thus offer here a first demonstration that politically optimal policies are efficient for a heterogeneous-firms model. From this perspective, we conclude that firm heterogeneity does not provide a new rationale for trade agreements; at the same time, we emphasize that the design of the trade agreement (e.g., whether restrictions on export subsidies are desirable) may be impacted by parameters that describe the dispersion of heterogeneity.

Finally, we relate our findings to those developed in our sequel paper, Bagwell and Lee (2018). There, we conduct a similar analysis of trade policies in a two-sector model with quasi-linear utility but with two main differences: consumer utility in the differentiated-goods sector is captured by a CES preference function, and, following Melitz (2003), selection is achieved as a consequence of fixed production costs.[6] Many trade-policy characterizations hold in common across the two models; however, a key difference is that the entry-externality effect is always positive in the CES model. Hence, if countries start at global free trade, then for the CES model the introduction of an export subsidy always benefits the intervening country and *raises* joint welfare. Starting at global free trade, the CES model thus differs in that it does not provide an efficiency-based rationale for a restriction on export subsidies. We provide here an intuitive but partial perspective as to the underlying source of the different entry-externality effects across the two models. We note that the expected profit of a firm conditional on its survival falls with greater entry in the Melitz-Ottaviano model but is constant with respect to the level of entry in the CES model. The CES model thus shuts down one channel for the business-stealing effect that would otherwise work against a positive entry-externality effect.

Related Work Our work builds on a large literature that studies trade policy for markets with free entry, imperfect competition and homogeneous firms.[7] Under Cournot

[6]For tractability and to facilitate comparison, Bagwell and Lee (2008) follow Chaney (2008) and assume that firm-level productivities are drawn from a Pareto distribution.

[7]A large literature also exists that analyzes strategic roles for trade policies when profits exist and can be shifted across firms. See Brander (1995) for a survey and Amador and Bagwell (2013), Bagwell and

competition, Venables (1985) establishes firm-delocation effects and a Metzler paradox, and he further shows that a country gains from the introduction of a small import tariff or a small export subsidy, where the optimality of the latter intervention is more qualified but holds for a linear-demand setting. Bagwell and Staiger (2012b) further analyze this model. Under the assumption of linear demand, they show that total tariffs that deliver free trade are efficient in the symmetric class, and they develop for this model unilateral trade-policy results that parallel the three results mentioned above.[8] They also develop related findings regarding liberalization paths from the symmetric Nash equilibrium.

Relative to these papers, our contribution is to characterize unilateral, efficient and Nash policies in a model with monopolistic competition and heterogeneous firms. Our work thereby forges a link between the incentives for strategic and beggar-thy-neighbor export subsidy policies on the one hand and parameters related to product differentiation, trade costs and productivity dispersion on the other hand. Further, since global free trade is not generally efficient in the model that we consider, a simultaneous ban on export subsidies and import tariffs receives less support in this model than in the linear Cournot delocation model that Bagwell and Staiger (2012b) analyze.

In a homogeneous-firms model with monopolistic competition and CES preferences, Venables (1987) establishes that the introduction of a small import tariff can increase welfare in the intervening country, by expanding varieties and thereby generating a fall in the domestic price index.[9] Campolmi et al (2014) build on Venables' model and establish several additional findings. They show that too few varieties are delivered under free trade. When only trade policies are available, they remark (see their footnote 13) that such policies can be used in a second-best way to enhance efficiency. They further show that, starting at global free trade, a country can gain from the unilateral introduction of a small import tariff or small export subsidy. For this setting, they also characterize Nash tariffs when each country is restricted to use only import tariffs or only export tariffs.

Our analysis differs in structure and findings. Structurally, we analyze a heterogeneous-firms model in which selection effects exist and markups are variable. With respect to findings, while Campolmi et al consider policy scenarios (e.g., regarding the role of domestic policies) that we do not address, we also consider trade-policy scenarios that Campolmi et al do not address. In our analysis of unilateral policies, for example, we show that a country can gain from the simultaneous introduction of small import and export tariffs.

Staiger (2012a), DeRemer (2013), Etro (2011), Mrazova (2011) and Ossa (2012) for recent contributions.

[8] With respect to findings for unilateral trade policies, one difference is that Bagwell and Staiger (2012b) do not establish for the linear Cournot delocation model that the introduction of a small import and export tariff results in a decrease in the trading partner's welfare.

[9] Venables (1987) assumes a Cobb-Douglas utility function, ensuring that expenditure shares in the differentiated-goods and outside-good sectors are fixed. See also Bagwell and Staiger (2015), Helpman and Krugman (1989), and Ossa (2011) for extended analyses. The former two papers assume as here that the utility function is quasi-linear whereas Ossa uses a Cobb-Douglas utility function.

Regarding efficiency, we formally characterize the entry-externality effect that underlies the entry distortion in the Melitz-Ottaviano model. We also show that politically optimal tariffs are efficient.[10] Finally, our analysis of symmetric Nash trade policies assumes that import and export tariffs are simultaneously selected, and we characterize the efficiency of these Nash policies and identify corresponding efficiency-enhancing liberalization paths.[11]

Our work is also related to a small literature that explores trade policy under monopolistic competition when firms are heterogeneous and preferences take a CES form, as in Melitz (2003). Nash trade policies are characterized by Felbermayr et al (2013) for the one-sector Melitz model. Demidova and Rodriguez-Clare (2009) analyze a small-country version of this model, finding that the optimal unilateral export policy is then an export tariff. Haaland and Venables (2016) consider a two-sector model with an outside good, and analyze optimal unilateral trade and domestic taxes for a family of small-country models. Costinot et al (2016) offer a general treatment of the one-sector model, assume that each country has available a full set of domestic and trade policy instruments, and characterize optimal unilateral tariffs both when tariffs are firm-specific and when they are uniform. They identify a central role for the terms-of-trade externality in their analysis of unilateral trade-policy intervention. Finally, Caliendo et al (2017) examine import tariffs in a multi-sector model that features entry distortions and trade in intermediates.

Relative to this work, we consider a large-country, two-sector model with a linear outside good when consumers have quadratic preferences, leading to variable markups, and domestic policies are unavailable. In this context, we study unilateral, efficient and Nash trade policies. While we share with Costinot et al an emphasis on a central role for the terms-of-trade externality, we focus on a different model and base our argument on the efficiency properties of the politically optimal and Nash trade policies. Finally, our sequel paper, Bagwell and Lee (2018), considers a model similar to that analyzed here but with CES preferences for the differentiated-goods sector. A comparison with the results in that paper clarifies the role of quadratic preferences for our findings. As mentioned above, the two models have different predictions relating to the entry-externality effect and the corresponding treatment of export subsidies in trade agreements.

Our analysis of the entry-externality effect is related to recent work by Dhingra and Morrow (forthcoming) and Nocco et al (2014). For a family of one-sector monopolistic competition models with heterogeneous firms and additively separable preferences, Dhin-

[10] Campolmi et al argue that politically optimal policies are inefficient for their model when domestic and trade policies are available. As Bagwell and Staiger (2016b, p. 465) argue, however, Campolmi et al impose a different definition of politically optimal policies than is imposed in previous work (see Bagwell and Staiger 1999, 2001, 2012a, 2015, 2016b).

[11] Campolmi et al consider Nash policies when import and export tariffs are simultaneously determined only when domestic policies are also available to target the monopolistic distortion. In our model, domestic policies are not available, and so Nash trade policies must be selected with markups in mind.

gra and Morrow show that the market outcome with CES preferences is first best. By comparison, we conduct our analysis in the Melitz-Ottaviano model, wherein preferences are not additively separable and an outside good exists. In our analysis of the entry-externality effect, we also consider a second-best scenario, in which the social planner directly controls only the number of entrants.

Nocco et al offer an extensive analysis of the efficiency properties of the market outcome in the Melitz-Ottaviano model. They characterize the equilibrium and first-best outcomes and show that the market equilibrium level of entry is above (below) that in the first-best allocation if the demand parameter α is higher (lower) than a critical level. The first-best outcome can be decentralized through firm-specific per-unit production subsidies accompanied by a lump-sum entry tax per entrant and a lump-sum tax on consumers. They also consider a second-best scenario that arises when any per-unit (i.e., specific) production subsidy must be offered to all firms and financed by a lump-sum tax on consumers. The second-best level of entry exceeds that provided by the market. Our analysis of the entry-externality effect is related but can be understood as a different second-best scenario in which production subsidies are unavailable and lump-sum transfers between consumers and firms can be used to subsidize or tax the fixed cost of entry.

Other recent work examines trade policy while building on the Melitz-Ottaviano model. Spearot (2014) enriches that model to allow for heterogeneous dispersion parameters across countries. Among other results, he provides conditions under which a higher import tariff increases competition in the domestic market.[12] Spearot (2016) further develops this analysis by considering a multi-sector, multi-country model with heterogeneous dispersion parameters, in which the outside good is removed. He estimates shape parameters and provides counterfactual analyses of several trade-policy shocks. Demidova (2017) characterizes optimal unilateral import tariffs for small and large countries when the outside good is removed from the Melitz-Ottaviano model. She finds that the Metzler paradox then no longer holds, and she shows that the resulting optimal tariffs are positive for both small and large countries.[13] Our work is complementary. Like a large body of existing trade-policy research based on partial-equilibrium models, we use an outside good to eliminate the general-equilibrium wage effects associated with trade-policy changes. This approach seems reasonable, for example, when analyzing import or export

[12] See Spearot (2013) for an empirical analysis of the implications of the Melitz-Ottaviano model in response to tariff liberalizations. Chen et al (2009) also offer empirical support for predictions of the Melitz-Ottaviano model.

[13] The optimal unilateral tariff is higher for a large country, due to the terms-of-trade externality. In our (large-country) model, an import tariff also generates a terms-of-trade gain for the intervening country, and indeed the Metzler paradox acts to reinforce this gain. The terms-of-trade implications of export policies in our model are more novel, since an export subsidy results in a terms-of-trade *gain* for the exporting country. We discuss this point further following Proposition 10. See also Bagwell and Staiger (2012b, 2015).

policies for specific sectors, such as is often the case in WTO disputes, since sector-based trade policies are less likely to generate economy-wide wage effects. Using the additional tractability that this approach offers, we characterize unilateral, efficient and Nash import and export policies, as well as liberalization paths.

Ultimately, the relevance of the Metzler paradox is an empirical issue. While a large literature finds imperfect pass-through for various industries, evidence of a negative pass-through rate (i.e., a Metzler paradox) is less common. The study by Ludema and Yu (2016) is of particular interest. They examine the response of US export prices to foreign tariff cuts at the firm level over a time period when the Uruguay Round was implemented. The pass-through of the average firm is typically negative, in line with the Metzler paradox. They argue, however, that this is partly due to quality upgrading, whereby U.S. firms respond to foreign tariff cuts by upgrading quality and increasing export prices. They thus conclude that they find evidence in support of a "quasi-Metzler paradox."

The paper is organized as follows. Section 2 presents a symmetric, two-country version of the Melitz-Ottaviano model, modified slightly to include tariffs. Section 3 highlights the driving forces that emerge from this model and inform our subsequent trade-policy analysis. Our three findings for unilateral trade-policy interventions are presented in Section 4, while our characterizations of efficient and Nash trade policies are contained in Section 5. Section 6 contains our results on politically optimal tariffs and the terms-of-trade rationale for trade agreements. Section 7 provides a partial perspective concerning the underlying reason for differences that emerge when preferences instead take a CES form. Section 8 concludes. The Appendix contains omitted proofs.

2 Model

We develop our tariff analysis in the context of a symmetric, two-country version of the Melitz and Ottaviano (2008) model. There are two symmetric countries, home (H) and foreign (F). The markets are segmented, and international trade entails trade costs as well as ad valorem export and import tariffs. The key difference between our setup and that of Melitz and Ottaviano is that we include import and export tariffs. We present the model with this modest adjustment in order to provide expressions that facilitate our analysis of tariff policies in subsequent sections.

Consumer Behavior

Each country has a unit mass of consumers. Consumer preferences are defined over a continuum of differentiated varieties and a numeraire (outside) good. All consumers in country $l \in \{H, F\}$ share the same quasi-linear quadratic preferences given by

$$U^l \equiv \max_{\left\{q_0^l, \left\{q_i^l\right\}_{i \in \Omega^l}\right\}} \left[q_0^l + \alpha \int_{i \in \Omega^l} q_i^l di - \frac{1}{2}\gamma \int_{i \in \Omega^l} \left(q_i^l\right)^2 di - \frac{1}{2}\eta \left(\int_{i \in \Omega^l} q_i^l di \right)^2 \right] \tag{1}$$

s.t.

$$q_0^l + \int_{i \in \Omega^l} p_i^l q_i^l di \leq w^l + TR^l + \Pi^l \equiv I^l \tag{2}$$

where q_0^l, q_i^l, and p_i^l represent the consumption of the numeraire good in country l, the consumption of differentiated good $i \in \Omega^l$ in country l, and the price of differentiated good i in country l. The set Ω^l represents a continuum of varieties that are potentially available for consumption in country l.[14] Consumer income consists of a numeraire-good holding w^l, aggregate profit Π^l, and government transfers TR^l. We discuss the determinants of consumer income, I^l, in greater detail below.

The preference parameters α, γ, and η are all positive. The parameters α and η capture the substitution level between the differentiated varieties and the numeraire, while the parameter γ measures the degree of product differentiation within the set of differentiated varieties. For example, in the limiting case where $\gamma = 0$, a consumer's preferences regarding the differentiated varieties are completely summarized by the aggregate consumption of these varieties: $Q^l \equiv \left(\int_{i \in \Omega^l} q_i^l di\right)$.

Following Melitz and Ottaviano, we assume that the numeraire good is consumed ($q_0^l > 0$) and proceed to derive the inverse demand for variety i as $p_i^l = \alpha - \eta Q^l - \gamma q_i^l$ for $i \in \Omega^{*l}$ where $\Omega^{*l} \subset \Omega^l$ denotes the set of varieties for which $q_i^l > 0$. The intercept for the demand for variety i is thus $\alpha - \eta Q^l$. We may now integrate over the corresponding demand functions $q_i^l = (\alpha - \eta Q^l - p_i^l)/\gamma$ to express Q^l in terms of the average price and the measure N^l of consumed varieties in Ω^{*l}. Proceeding in this way yields

$$q_i^l = (p_{\max}^l - p_i^l)\frac{1}{\gamma} \text{ for } i \in \Omega^{*l} \tag{3}$$

where

$$p_{\max}^l \equiv \frac{\alpha\gamma + \eta N^l \overline{p}^l}{\gamma + \eta N^l} \tag{4}$$

defines the key demand intercept term and where

$$\overline{p}^l \equiv (\int_{i \in \Omega^{*l}} p_i^l di)(\frac{1}{N^l}) \tag{5}$$

is the average price of a consumed variety in country l. It is now evident that the set Ω^{*l}

[14] Such varieties may be produced domestically or imported.

is defined as the largest subset of Ω^l for which $p_i^l \leq p_{\max}^l$. Notice that $\alpha \geq p_{\max}^l$ if and only if $\alpha \geq \overline{p}^l$.

To evaluate welfare, we require a representation of indirect utility. As Melitz and Ottaviano show, the indirect utility function takes the following convenient form:

$$U^l = I^l + \frac{1}{2}(\eta + \frac{\gamma}{N^l})^{-1}(\alpha - \overline{p}^l)^2 + \frac{1}{2}\frac{N^l}{\gamma}\sigma_{p^l}^2, \tag{6}$$

where

$$\sigma_{p^l}^2 \equiv \frac{1}{N^l}\int_{i\in\Omega^{*l}}(p_i^l - \overline{p}^l)^2 di \tag{7}$$

and where we recall also the assumption that the numeraire good is consumed: $q_0^l > 0$. This assumption in turn holds if and only if

$$I^l > \int_{i\in\Omega^{*l}} p_i^l q_i^l di = \overline{p}^l Q^l - N^l \sigma_{p^l}^2 \frac{1}{\gamma}, \tag{8}$$

where Q^l is calculated using (3). We also define consumer surplus in this setting as follows:

$$CS^l \equiv U^l - I^l, \tag{9}$$

and we note that consumer surplus is higher when the average price is lower, the variance of prices is higher, and the level of product variety is higher, where it is understood that we hold other terms constant when increasing any one term.

Firm Behavior in Domestic Market

Production in this economy utilizes labor, which is the only factor. Labor is supplied in an inelastic fashion in a competitive labor market. As is standard, labor can be used to produce the numeraire good under constant returns to scale in a one-to-one manner, where the numeraire good is sold in a competitive market. We thus set the wage in each country equal to one: $w^l = 1$. The supply of labor to the differentiated-goods sector is thus perfectly elastic at the unitary wage.

In the differentiated-goods sector, each variety $i \in \Omega^l$ is produced by a monopolistically competitive firm. To enter the market, a firm pays a fixed cost $f_e > 0$ and draws its marginal production cost c_i, which indicates the unit labor requirement. The cost c_i is drawn from a Pareto distribution with c.d.f. $G(c_i) = (c_i/c_M)^k$ for $c_i \in [0, c_M]$ where $k > 1$ represents a shape parameter and $c_M > 0$ represents the upper bound of c_i. The parameter k is important and determines the dispersion of productivity. Higher dispersion corresponds to a lower value for k. For example, in the limit where $k = \infty$, every firm has the same marginal cost c_M. Likewise, in the limiting case where $k = 1$, the level of dispersion is maximized and c_i follows a uniform distribution.

Depending on its productivity draw, a firm that enters country l may exit, produce only in country l, or produce in country l and also export to country h, where $h \in \{H, F\}$ and $h \neq l$.[15] Following Melitz and Ottaviano, we assume that markets are segmented and that firms engage in monopolistic competition in each market. Thus, a firm makes separate decisions about its domestic and export prices, and each firm takes as given the number of firms and the average price in a market when selecting its price for that market.

Consider first the domestic market. A firm located in country l with cost level c selects its price in the domestic market, p_D^l, to maximize domestic-market profit, $(p_D^l - c)(p_{\max}^l - p_D^l)\frac{1}{\gamma}$, where $p_{\max}^l$ is defined above in (4). Let the resulting profit-maximizing price for domestic sales be denoted as $p_D^l(c)$. Defining $q_D^l(c) \equiv (p_{\max}^l - p_D^l(c))\frac{1}{\gamma}$ and $\pi_D^l(c) \equiv (p_D^l(c) - c)q_D^l(c)$, it follows that $p_D^l(c) = \frac{p_{\max}^l + c}{2}$, $q_D^l(c) = \frac{p_{\max}^l - c}{2\gamma}$ and $\pi_D^l(c) = \frac{1}{4\gamma}(p_{\max}^l - c)^2$. Notice that $p_D^l(c) \geq c$ if and only if $p_{\max}^l \geq c$. We may define the cut-off cost level for sales in the domestic market, c_D^l, as

$$c_D^l \equiv p_{\max}^l, \tag{10}$$

With this definition, and following Melitz and Ottaviano, we may represent profit-maximizing domestic variables for $c \leq c_D^l$ as

$$\begin{aligned} p_D^l(c) &= \frac{c_D^l + c}{2}, \\ q_D^l(c) &= \frac{c_D^l - c}{2\gamma}, \\ \pi_D^l(c) &= \frac{1}{4\gamma}(c_D^l - c)^2. \end{aligned} \tag{11}$$

A firm with cost level c in country l sells in the domestic market if and only if $c \leq c_D^l$.

Firm Behavior in Export Market

Consider next the export market. A firm located in country l with cost level c selects its delivered export price for consumers in country h, which we denote as p_X^l, while taking as given the number of varieties sold and the average price in country h. A firm with cost level c incurs the cost $\tau \cdot c$ when delivering a unit to the foreign market, where we assume $\tau > 1$. The trade cost τ thus ensures that a firm incurs a greater cost when delivering a unit of its variety to the export market. Notice that we assume the trade cost is independent of the designation of the export market.

Exported varieties are also subject to ad valorem export tariffs and import tariffs. We

[15] Throughout, the use of index h is understood to mean the country other than country $l : h \neq l$.

assume that the ad valorem export tariff, $\tilde{t}^l$, is levied on the exporting firm, with the factory gate price used for valuation. The ad valorem import tariff, t^h, is paid by the importing consumer, with the factory gate price again used for valuation. The factory gate price is thus $\frac{p_X^l}{1+t^h}$, and the exporting firm thus receives $\frac{1-\tilde{t}^l}{1+t^h} p_X^l$ for every unit that is exported. We assume that $t^h > -1$ and $\tilde{t}^l < 1$, where the former assumption ensures that the after-tax consumption price of an imported good is positive and the latter assumption ensures that the firm's after-tax price received on a unit exported is positive.

We are now prepared to analyze profit-maximizing choices in the export market. A firm located in country l with cost level c selects its delivered export price, p_X^l, to maximize its export-market profit,

$$\left(\frac{p_X^l}{\chi^h} - \tau \cdot c \right) \frac{p_{\max}^h - p_X^l}{\gamma} \tag{12}$$

where

$$\chi^h \left(t^h, \tilde{t}^l \right) \equiv \frac{1+t^h}{1-\tilde{t}^l}$$

and $p_{\max}^h$ is defined above in (4), once l is replaced with h. χ^h is a convenient measure of the overall trade barrier due to trade policy in exporting from country l to country h. Under our assumptions, $\chi^h > 0$ and χ^h is increasing in t^h and $\tilde{t}^l$. We note further that $\chi^h > 1$ if and only if $t^h + \tilde{t}^l > 0$ so that the total tariff along this channel is positive.

Let the resulting profit-maximizing price for export sales be denoted as $p_X^l(c)$, and define $q_X^l(c) \equiv (p_{\max}^h - p_X^l(c))\frac{1}{\gamma}$ and $\pi_X^l(c) \equiv (\frac{p_X^l(c)}{\chi^h} - \tau \cdot c) q_X^l(c)$. The cut-off cost level for sales in the export market, c_X^l, now may be defined as

$$c_X^l = \frac{p_{\max}^h}{\tau \cdot \chi^h} = \frac{c_D^h}{\tau \cdot \chi^h}. \tag{13}$$

With these definitions in place, we may represent profit-maximizing export variables for for $c \leq c_X^l$ as

$$\begin{aligned} p_X^l(c) &= \frac{\tau \cdot \chi^h}{2} \left(c_X^l + c \right) \\ q_X^l(c) &= \frac{\tau \cdot \chi^h}{2\gamma} (c_X^l - c) \\ \pi_X^l(c) &= \frac{\tau^2 \cdot \chi^h}{4\gamma} (c_X^l - c)^2. \end{aligned} \tag{14}$$

A firm with cost level c in country l sells in the export market if and only if $c \leq c_X^l$.

Free Entry Conditions

In the long run, each entrant expects zero profit. The expected profit for a firm located in country l is given as

$$\overline{\pi}^l \equiv \int_0^{c_D^l} \pi_D^l(c) dG(c) + \int_0^{c_X^l} \pi_X^l(c) dG(c). \tag{15}$$

We may thus express the free-entry conditions as two equations

$$\overline{\pi}^l = f_e \text{ for } l = H, F \tag{16}$$

which with (13) may be used to determine the cut-off levels, c_D^l and c_X^l for $l = H, F$. The domestic cut-off levels, c_D^l and c_D^h, in turn determine $p_{\max}^l$ and $p_{\max}^h$ and thus, by (4) and (5), the number of varieties sold in the domestic and export markets, N^l and N^h. From here, the number of entrants, N_E^l and N_E^h, may be determined, as confirmed below.

Following Melitz and Ottaviano, we now proceed in the described fashion and complete the solution of the model. Solving for the cut-off levels yields:

$$\begin{aligned} c_D^l &= [\frac{\phi\gamma(1-\rho^h)}{1-\rho^l\rho^h}]^{\frac{1}{k+2}} \\ c_X^l &= c_D^h(\frac{\rho^h}{\tau})^{\frac{1}{k+1}}, \end{aligned} \tag{17}$$

where

$$\phi \equiv 2(k+1)(k+2)(c_M)^k f_e > 0$$

and

$$\rho^l \equiv (\tau)^{-k}\left(\chi^l\right)^{-(k+1)} \tag{18}$$

We assume henceforth that $\rho^l \in (0,1)$.[16] Our assumptions above imply $\rho^l > 0$; thus, the new assumption is that $\rho^l < 1$.[17]

The next step is to determine the number of varieties sold in each market. To this end, we first compute $\overline{p}^l$. The expected price in country l is determined by prices from domestic firms as well as from exporters in country h. As Melitz and Ottaviano show, under the Pareto distribution, the expected price in country l from domestic producers is the same as that from foreign exporters, and takes the form

$$\overline{p}^l = c_D^l \cdot \frac{2k+1}{2k+2}, \tag{19}$$

[16] Given $\tau > 1$, if $\tau^{1/2} \cdot \chi^l > 1$, then $\rho^l < 1$ for all $k > 1$. We also assume throughout that $c_M > c_D^l$ for the tariffs under consideration. At global free trade, $c_M > c_D^l$ holds if and only if $c_M > [\frac{2(k+1)(k+2)f_e\gamma(1-(\tau^{-k}))}{1-(\tau)^{-2k}}]^{\frac{1}{2}}$.

[17] In a model without firm heterogeneity, Bagwell and Staiger (2012b) impose a related assumption.

which indicates that a higher domestic cut-off level leads to a higher average price.[18] Substitution of (10) and (19) into (4) now yields a solution for N^l in terms of c_D^l :

$$N^l = \frac{2\gamma(\alpha - c_D^l)(k+1)}{\eta c_D^l}. \tag{20}$$

As reported in (17), the free-entry conditions yield a specific value for c_D^l, which may be plugged into (20) to determine the free-entry solution for N^l in terms of model parameters.[19]

The numbers of entrants, N_E^l, in the two countries can now be determined as the solutions to the following two equations

$$N^l = G(c_D^l)N_E^l + G(c_X^h)N_E^h, \tag{21}$$

assuming a positive mass of entrants in both countries. The solution to this system is

$$N_E^l = \frac{(c_M)^k}{1-\xi^l\xi^h}[\frac{N^l}{(c_D^l)^k} - \frac{\xi^l N^h}{(c_D^h)^k}] \tag{22}$$

where $\xi^l \equiv \rho^l \cdot \chi^l < 1$ follows from our assumptions.[20] Substituting (20) into (22) yields

$$N_E^l = \frac{2(k+1)(c_M)^k\gamma}{\eta[1-\xi^l\xi^h]}[\frac{\alpha - c_D^l}{(c_D^l)^{k+1}} - \frac{\xi^l(\alpha - c_D^h)}{(c_D^h)^{k+1}}] \tag{23}$$

A maintained assumption is that the trade policies under consideration are such that $N_E^l > 0$ for $l = H, F$. Given $\xi^l \in (0,1)$, we see from (23) that this assumption implies $\alpha > c_D^l$ for $l = H, F$.[21] In later sections, we give particular consideration to trade policies

[18] The expected price on domestic and imported varieties can be respectively computed as

$$\frac{1}{G(c_D^l)}\int_0^{c_D^l} p_D^l(c)dG\,(c) = c_D^l[\frac{2k+1}{2(k+1)}] = \frac{1}{G(c_X^h)}\int_0^{c_X^h} p_X^h(c)dG\,(c)\,.$$

[19] Given $\alpha > c_D^l$, the set Ω^{*l} is non-empty.

[20] Our assumption that $\rho^l < 1$ can be restated as $\tau^{-\frac{k}{1+k}} < \chi^l$. For $k > 1$ and $\tau > 1$, $\tau^{-1} < \tau^{-\frac{k}{1+k}} < \chi^l$ holds. Therefore, $\tau \cdot \chi^l > 1$ and $\xi^l = \left(\tau \cdot \chi^l\right)^{-k} < 1$ are implied. Using (13), we note that $\tau \cdot \chi^l > 1$ is equivalent to $c_X^h < c_D^l$, which is to say that the cut-off cost level for foreign sales in the home market is below that for domestic sales in the home market.

[21] We are now in position to represent our assumptions that $q_0^l > 0$, $N^l > 0$ and $N_E^l > 0$ in terms of model parameters. For example, if tariffs are symmetric with $t^l = t^h$ and $\tilde{t}^l = \tilde{t}^h$, so that $\rho^l = \rho^h$ and $c_D^l = c_D^h$, then by (20) and (23) we know for $l = H, F$ that N^l and N_E^l are positive if and only if $\alpha > c_D^l$, which by (17) and the definition of ϕ holds if and only if $f_e < \alpha^{k+2}(1+\rho^l)/[2\gamma(k+1)(k+2)(c_M)^k]$. Furthermore, as f_e approaches this upper bound from below, we can show that $q_0^l \to 1$. Thus, under symmetric tariffs and for f_e sufficiently close to its upper bound, we can be sure for each country $l \in \{H, F\}$ that the numeraire good is consumed in positive quantity ($q_0^l > 0$) and that the differentiated

that constitute global free trade (i.e., trade policies for which all import and export tariffs are set equal to zero). The content of our maintained assumption for this case is thus that $\alpha > c_D^{FT}$, where c_D^{FT} is the value taken by $c_D^H = c_D^F$ under global free trade.

Finally, we return to the expression for consumer welfare U^l given in (6). As indicated in (19), the Pareto distribution delivers a simple expression for $\overline{p}^l$. It is likewise true that the price variance confronted by domestic consumers is the same for varieties produced domestically as for varieties imported from abroad. The corresponding expression is

$$\sigma_{p^l}^2 = \frac{(c_D^l)^2}{4} \frac{k}{(k+1)^2(k+2)} \tag{24}$$

Using (6), (19), (20) and (24), and following Melitz and Ottaviano, it is now possible to derive a simple expression for consumer welfare:

$$U^l = I^l + \frac{(\alpha - c_D^l)}{2\eta}[\alpha - c_D^l \frac{k+1}{k+2}]. \tag{25}$$

An immediate corollary is that consumer surplus takes the form

$$CS^l = \frac{(\alpha - c_D^l)}{2\eta}[\alpha - c_D^l \frac{k+1}{k+2}]. \tag{26}$$

Tariff Revenue

In the model described above, consumer income is comprised of a unit of labor income, profits and tariff revenue. We have already discussed labor income; furthermore, in a free-entry equilibrium, expected profits are zero. The remaining income source to consider is thus tariff revenue.

To define import tariff revenue, we first define the value of country l's imports prior to the imposition of its import tariff:

$$IMP^l = \frac{N_E^h}{1+t^l} \int_0^{c_X^h} p_X^h(c) q_X^h(c) dG\left(c\right).$$

Using (13) and (14), we may substitute and derive that

$$IMP^l = \frac{N_E^h}{1+t^l} \frac{(\tau \cdot \chi^l)^{-k} \cdot (c_D^l)^{k+2}}{2\gamma(k+2)(c_M)^k} \tag{27}$$

Import tariff revenue for country l is then $t^l \cdot IMP^l$.

In analogous fashion, we may define the value of country l's exports prior to the

sector has a positive mass of entry ($N_E^l > 0$) and positive consumption ($N^l > 0$).

imposition of country h's import tariff as

$$EXP^l = \frac{N_E^l}{1+t^h} \int_0^{c_X^l} p_X^l(c)\, q_X^l(c)\, dG(c)$$

Using (13) and (14), we may substitute and derive that

$$EXP^l = \frac{N_E^l}{1+t^h} \frac{(\tau \cdot \chi^h)^{-k} \cdot (c_D^h)^{k+2}}{2\gamma(k+2)(c_M)^k} \tag{28}$$

Export tariff revenue for country l is then $\widetilde{t}^l \cdot EXP^l$.

Welfare

We are now prepared to define the welfare function that a national-income maximizing government would seek to maximize. This is the welfare function against which we will evaluate trade-policy interventions, and it is defined as

$$U^l = 1 + t^l \cdot IMP^l + \widetilde{t}^l \cdot EXP^l + \frac{(\alpha - c_D^l)}{2\eta}[\alpha - c_D^l \frac{k+1}{k+2}] \tag{29}$$

where we recall from (26) that the last term equals consumer surplus, CS^l. The indirect utility function in (29) takes the same form as that derived by Melitz and Ottaviano, except that we include tariff revenue as a source of consumer income. We notice that tariffs affect tariff revenue and thereby consumer income both directly and also indirectly through the induced long-run impact on trade values, IMP^l and EXP^l.

With the model now defined, we are prepared to consider the welfare impacts of trade policy. We perform this analysis in the following sections. Throughout, we maintain the assumption that any trade policies under consideration are such that the model assumptions presented above are satisfied.

3 Driving Forces

For our purposes, the model has three main driving forces: the selection effect, the firm-delocation effect, and the entry-externality effect. In this section, we briefly highlight some key features of the model that are associated with these forces. We note that Melitz and Ottaviano also derive the selection and firm-delocation effects as part of their analysis of the consequences of unilateral reductions in trade costs. We briefly highlight these effects here in order to derive and interpret our trade-policy findings in subsequent sections.

3.1 Selection Effect

In the heterogeneous-firms model considered here, trade policy affects both the number and the efficiency of entering firms. A higher import tariff increases the number of entrants and generates a higher level of competition, so that firms must be more efficient to survive. In particular, a higher home import tariff (or a higher foreign export tariff) lowers the cut-off cost level for domestic sales in the home market and raises the cut-off cost level for domestic sales in the foreign market. We now summarize this discussion in a proposition.

Proposition 1 *(Selection effect)*

$$\frac{dc_D^l}{dt^l}, \frac{dc_D^l}{d\tilde{t}^h} < 0 < \frac{dc_D^h}{dt^l}, \frac{dc_D^h}{d\tilde{t}^h}$$

Proof. *Proofs are in the Appendix.* ■

We note that t^l and $\tilde{t}^h$ affect c_D^l and c_D^h only through χ^l by (17) and (18).[22]

3.2 Firm-delocation Effect and the Metzler Paradox

We now consider in more detail the impact of trade policy on the number of firms. Intuitively, an increase in the home import tariff makes it harder for foreign firms to export. As a result, the expected profit for home firms increases, and the expected profit for foreign firms decreases. To satisfy the free-entry conditions, the number of home entrants increases and the number of foreign entrants decreases, and the number of surviving varieties similarly increases in the home country and decreases in the foreign country. In this sense, a home import tariff "delocates" firms from the foreign country to the home country. A similar logic holds when the home country lowers its export tariff or equivalently increases its export subsidy. The following proposition summarizes this discussion:

Proposition 2 *(Firm-delocation effect) For countries l and h with $l, h \in \{H, F\}$ and $l \neq h$, an increase in country l's import tariff or in country h's export tariff results in an increase in the number of entrants in country l, a decrease in the number of entrants in*

[22] As Proposition 1 confirms, for each cut-off cost level, higher home import and foreign export tariffs along a given trade channel push that cost level in the same direction. The magnitude of the shift, however, may differ across the trade-policy instruments. The asymmetry arises, since, for a given delivered export price p_X^h to consumers in country l, a higher import tariff t^l lowers the corresponding factory gate price, $p_X^h/(1+t^l)$ whereas a higher export tariff $\tilde{t}^h$ leaves the factory gate price unaltered. For each of the following three inequalities, $\frac{\partial \chi^l}{\partial t^l} < \frac{\partial \chi^l}{\partial \tilde{t}^h}$, $\frac{dc_D^l}{d\tilde{t}^h} < \frac{dc_D^l}{dt^l}$ and $\frac{dc_D^h}{dt^l} < \frac{dc_D^h}{d\tilde{t}^h}$, we can show that the inequality holds if and only if $t^l + \tilde{t}^h > 0$. For the findings of this paper, however, we require only that increases in these respective tariffs shift the cut-off cost levels in the same direction, a requirement that Proposition 1 provides.

country h, an increase in the number of varieties sold in country l, and a decrease in the number of varieties sold in country h:

$$\frac{dN_E^l}{dt^l}, \frac{dN_E^l}{d\tilde{t}^h} > 0 > \frac{dN_E^h}{dt^l}, \frac{dN_E^h}{d\tilde{t}^h}$$

$$\frac{dN^l}{dt^l}, \frac{dN^l}{d\tilde{t}^h} > 0 > \frac{dN^h}{dt^l}, \frac{dN^h}{d\tilde{t}^h}.$$

Using (17), (18), (20) and (23), we note that t^l and $\tilde{t}^h$ affect the numbers of entrants and varieties sold in countries l and h only through χ^l.[23]

We consider next the implications of trade policy for average prices. In fact, the firm-delocation effect is strong enough in this model to generate a Metzler paradox. As Proposition 2 establishes, a higher home import tariff (or a higher foreign export tariff) increases the number of entering firms in the domestic market and ultimately results in a higher number of surviving firms selling in this market. A higher home import tariff thus results in a reduction in the cut-off cost level for sales in the domestic market, which by (19) implies that the average price in the home market falls. A similar logic indicates that a higher home import tariff causes the average price in the foreign market to rise.

Proposition 3 *(Metzler paradox) For countries l and h with $l, h \in \{H, F\}$ and $l \neq h$, an increase in country l's import tariff or in country h's export tariff results in a decrease in the average price in country l and an increase in the average price in country h:*

$$\frac{d\bar{p}^l}{dt^l}, \frac{d\bar{p}^l}{d\tilde{t}^h} < 0 < \frac{d\bar{p}^h}{dt^l}, \frac{d\bar{p}^h}{d\tilde{t}^h}$$

Using (17), (18) and (19), we note that t^l and $\tilde{t}^h$ affect c_D^l and c_D^h and thereby $\bar{p}^l$ and $\bar{p}^h$, respectively, only through χ^l.[24] As noted in the Introduction, the Metzler paradox is a driving force in other models of trade policy, too, including the homogeneous-firms Cournot model used by Venables (1985) and Bagwell and Staiger (2012b).

3.3 Entry-externality Effect

An important consideration in characterizing efficient trade policies is whether the market is distorted in the absence of trade-policy interventions. We thus now consider the exter-

[23] In particular, N^l depends on t^l and $\tilde{t}^h$ due only to the role of χ^l in determining c_D^l, while N_E^l depends on t^l and $\tilde{t}^h$ due to the role of χ^l in determining c_D^l, c_D^h and ξ^l. As in footnote 22, it is also possible to report relative magnitudes. For each of the following four inequalities, $\frac{dN_E^l}{d\tilde{t}^h} > \frac{dN_E^l}{dt^l}$, $\frac{dN_E^h}{dt^l} > \frac{dN_E^h}{d\tilde{t}^h}$, $\frac{dN^l}{d\tilde{t}^h} > \frac{dN^l}{dt^l}$ and $\frac{dN^h}{dt^l} > \frac{dN^h}{d\tilde{t}^h}$, we can show that the inequality holds if and only if $t^l + \tilde{t}^h > 0$.

[24] As in footnote 22, it is also possible to report relative magnitudes. For each of the following two inequalities, $\frac{d\bar{p}^l}{d\tilde{t}^h} < \frac{d\bar{p}^l}{dt^l}$ and $\frac{d\bar{p}^h}{dt^l} < \frac{d\bar{p}^h}{d\tilde{t}^h}$, we can show that the inequality holds if and only if $t^l + \tilde{t}^h > 0$.

nalities associated with entry in the Melitz-Ottaviano model. In particular, our approach is to decompose the difference between the market and socially optimal entry levels so that we can intuitively explain the source of any market failure. This work provides a context in which to interpret subsequent results in our trade-policy analysis.

Before we characterize the externalities associated with entry, we recall from (25) that consumer welfare is determined as the sum of income and consumer surplus:

$$U^l = I^l + \frac{(\alpha - c_D^l)}{2\eta}[\alpha - c_D^l \frac{k+1}{k+2}],$$

where this expression holds for any N^l and not just the value determined in the market equilibrium. In the market equilibrium, the free-entry conditions ($\bar{\pi}^l = f_e$ for $l \in \{H, F\}$) determine the entry level. The marginal entrant, however, does not consider the external effect of its entry decision on consumer welfare, and so the entry level in the market equilibrium need not coincide with the socially optimal entry level.

In order to decompose and clarify the externalities associated with entry, we consider a simple closed-economy setting. As Melitz and Ottaviano show, consumer surplus takes the same form in the closed-economy setting:

$$CS \equiv \frac{(\alpha - c_D)}{2\eta}[\alpha - c_D \frac{k+1}{k+2}], \tag{30}$$

where c_D denotes the cut-off cost level for the closed-economy model. We may now define

$$\bar{\pi} \equiv \int_0^{c_D} \pi_D(c) dG(c),$$

where $\pi_D(c) = \frac{1}{4\gamma}(c_D - c)^2$, and we likewise require that the number of entrants, N_E, and the number of surviving varieties, N, for the closed-economy setting satisfy $N = G(c_D)N_E$ and $N = 2\gamma(\alpha - c_D)(k+1)/(\eta c_D)$.

We now consider the problem of a social planner who selects the level of entry N_E in a closed economy with the objective:

$$\max_{N_E} CS + N_E(\bar{\pi} - f_e). \tag{31}$$

In this exercise, the social planner chooses the number of entrants N_E to maximize consumers' welfare, which is the sum of consumer surplus and aggregate profit $\Pi \equiv N_E(\bar{\pi} - f_e)$.[25] Given the relationships just described, when the planner selects N_E, choices for c_D and N are implied and values for CS and $\bar{\pi}$ thus follow. We note that a change in the number of entrants could be implemented in a decentralized setting by

[25] We may ignore wage income, which is a constant in this model.

using lump-sum transfers between consumers and firms so as to subsidize or tax the fixed costs of entry.[26]

For the utility function (1) used by Melitz and Ottaviano, we can rewrite consumer surplus as follows:

$$CS = \int_{i \in \Omega^*} \frac{\gamma}{2} (q_i^*)^2 \, di + \frac{\eta}{2} \left(\int_{i \in \Omega^*} q_i^* di \right)^2 \tag{32}$$

where $q_i^* = (p_{\max} - p_i)/\gamma$ is the optimized consumption level for variety i at price p_i, given the number of entrants.[27] An interesting feature is that $\frac{\gamma}{2} (q_i^*)^2$ corresponds to the triangular region under the demand curve for variety i and thus represents consumer surplus at variety i. The first term in (32) is thus the sum of consumer surplus at each variety; hence, the second term in (32) should be explained by variety effects.

Based on this understanding, and after allowing for profit-maximizing pricing by firms, we may represent consumer surplus as follows:

$$CS = N_E \cdot \overline{CS} + VE$$

where

$$\overline{CS} = \int_0^{c_D} \frac{\gamma}{2} \left(q_D \left(c \right) \right)^2 dG \left(c \right) \tag{33}$$

represents expected consumer surplus at single varieties and $q_D(c) = (c_D - c)/(2\gamma)$. For a given value of c_D, the variety effect, VE, is then defined as the difference between CS as given in (32) and $N_E \cdot \overline{CS}$ with $\overline{CS}$ given by (33). As noted above, by choosing N_E, the planner effectively chooses c_D and N, and so values for CS, $\overline{CS}$, VE, $\bar{\pi}$ and Π follow.

The socially optimal N_E^* maximizes utility as defined in (31). The first-order condition takes the following form:

$$\overline{CS} + N_E \frac{d\overline{CS}}{dN_E} + \frac{dVE}{dN_E} + N_E \frac{d\bar{\pi}}{dN_E} + \bar{\pi} - f_e = 0$$

By contrast, the market determines the entry level to satisfy $\bar{\pi} = f_e$. We thus define the externalities that a market economy does not consider as follows:

$$EXT = \left(\overline{CS} + \frac{dVE}{dN_E} + N_E \frac{d\overline{CS}}{dN_E} + N_E \frac{d\bar{\pi}}{dN_E} \right) \tag{34}$$

[26] If the planner sought to decrease N_E below the market equilibrium level, then a per-entrant and lump-sum entry tax $T_E > 0$ could be imposed, so that entry would occur until $\bar{\pi} - f_e - T_E = 0$. Consumers would enjoy additional income of $N_E(\bar{\pi} - f_e)$ once the tax is redistributed. A lump sum subsidy to entry is similar except that $T_E < 0$ and so consumers provide a lump-sum redistribution to entrants.

[27] For the closed-economy model, $p_{\max} \equiv (\alpha\gamma + \eta N \overline{p})/(\gamma + \eta N)$, where $\overline{p} \equiv (\int_{i \in \Omega^*} p_i di)/N$ and Ω^* is the set of varieties for which $p_{\max} > p_i$.

where in expectation $\overline{CS} > 0$ represents the direct consumer surplus gain from a new variety, $\frac{dVE}{dN_E} > 0$ represents the beneficial variety effect from a new entrant, $N_E \frac{d\overline{CS}}{dN_E} < 0$ represents a substitution effect (i.e., the consumer surplus losses on pre-existing varieties when additional entry occurs), and $N_E \frac{d\bar{\pi}}{dN_E} < 0$ represents a business-stealing effect.

In the Appendix, we derive and sign all of the terms in (34) and establish the following:

Proposition 4 *(Entry-externality effect) Starting at the market equilibrium, additional entry generates a negative externality if and only if $\alpha > 2 \cdot c_D^m$; that is,*

$$EXT < 0 \text{ if and only if } \alpha > 2 \cdot c_D^m, \tag{35}$$

where c_D^m is the cutoff cost level in the market equilibrium under free entry.

To interpret this proposition, we can image starting with a market equilibrium, where the level of entry is determined by the free-entry conditions, and then considering the impact of a marginal change in the level of entry on welfare. In this scenario, $c_D^m = (\phi\gamma)^{\frac{1}{k+2}}$ corresponds to the cost level in the market equilibrium under free entry.[28] According to Proposition 4, if α is at least twice as large as this cutoff level, then the last entrant resulted in a drop in welfare, and so the social planner could generate a gain in welfare with a small reduction in the level of entry. Intuitively, the number of entrants is increasing in α in this model, and negative externalities such as the substitution effect and business-stealing effect are weighted by the number of entrants.

As noted in the Introduction, Nocco et al (2014) also report a critical value for α such that the market supplies too much entry relative to the first-best level when α exceeds this value. Our finding is related and complementary. Our result is derived in a second-best context, however, where the planner does not have direct control over firm-specific output levels.[29]

4 Unilateral Trade Policies

In the previous sections, we presented the solution to the model and highlighted three driving forces. Now we can discuss trade policies and welfare implications of this model. We focus in this section on unilateral trade-policy incentives when countries start at global free trade.

[28] See Melitz and Ottaviano (2008) for the derivation of c_D^m.

[29] The critical value that Nocco et al (2014) derive is closely related but has a different coefficient on c_D^m. Nocco et al also consider a second-best setting in which the planner can use a per-unit production subsidy financed by a lump-sum tax on consumers. In this case, however, and as they show, c_D^m is unaffected by policy. In terms of our decomposition above, such a policy would eliminate all effects in EXT except $\overline{CS}$. As they show, the market thus under-supplies variety in this case.

4.1 Introduction of a Small Import Tariff

We suppose that both countries initially adopt free trade with import and export tariffs. From this starting point, the introduction of a small home import tariff generates a welfare gain for the home country and a welfare loss for the foreign country.[30]

Proposition 5 *(Small import tariff) For countries l and h with $l, h \in \{H, F\}$ and $l \neq h$, if both countries initially adopt a policy of free trade, then the introduction of a small import tariff by country l generates a welfare gain for country l and a welfare loss for country h:*

$$\begin{aligned} \frac{dU^l}{dt^l}\Big|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^l}{dt^l} + IMP^l \Big|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} > 0 \\ \frac{dU^h}{dt^l}\Big|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^h}{dt^l}\Big|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} < 0. \end{aligned}$$

When the home country introduces a small import tariff, the firm-delocation effect implies that the number of home entrants increases and the number of foreign entrants decreases. Under the Metzler paradox, the average price falls in the home country and rises in the foreign country. As well, consumers in the home country enjoy greater variety whereas foreign consumers experience a decrease in variety. Since the average price decreases and the variety effect increases in the home country, consumer surplus increases in the home country while the opposite occurs in the foreign country. Finally, the introduction of a small home import tariff also generates a positive tariff-revenue gain for the home country, where this gain corresponds to the import value. Due to these price, variety and revenue effects, the introduction of a small import tariff by the home country results in a home-country welfare increase and a foreign-country welfare decrease.

4.2 Introduction of a Small Export Subsidy

We also consider the introduction of a small export subsidy under global free trade. The following proposition shows that the introduction of a small home export subsidy always decreases foreign welfare while it increases home welfare when the selection effect is strong (i.e., when the trade cost is low and the dispersion of firms' productivities is high) or when the selection effect is weak and the demand parameter α is small.

Proposition 6 *(Small export subsidy) For countries l and h with $l, h \in \{H, F\}$ and $l \neq h$, if both countries initially adopt a policy of free trade, then the introduction of a*

[30] In Proposition 5, we assume that both countries initially adopt free trade for simplicity. More generally, the key requirement is that the country about which the welfare statement is made adopts a policy of free trade, while the other country adopts any initial policy consistent with positive entry.

small export subsidy by country l has the following effects: 1). It generates a welfare gain for country l,

$$\frac{dU^l}{d\tilde{t}^l}\,|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \;=\; \frac{dCS^l}{d\tilde{t}^l}+EXP^l\,|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \;<\; 0, \tag{36}$$

when (a) the selection effect is strong in that $\tau < (4+2k)^{1/k}$ *or (b) the selection effect is weak in that* $\tau \geq (4+2k)^{1/k}$ *and*

$$\alpha < \left(1+\frac{\tau^k}{\tau^k-2(k+2)}\right)c_D^{FT} \tag{37}$$

2). It generates a welfare loss for country h,

$$\frac{dU^h}{d\tilde{t}^l}\,|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \;=\; \frac{dCS^h}{d\tilde{t}^l}>0. \tag{38}$$

Proposition 6 indicates that the introduction of a small export subsidy (i.e., the introduction of a small negative export tariff) by the home country always hurts foreign welfare but can raise home welfare. The foreign welfare loss may be understood in terms of the firm-delocation effect and the logic identified in Proposition 5. The impact on home welfare, however, is more complicated. When a country introduces an export subsidy, it must balance any consumer-surplus gain against the tariff-revenue loss, where the tariff-revenue loss corresponds to the export value. Proposition 6 indicates that the net effect of a small export subsidy for the home country is positive if the selection effect is strong or if the selection effect is weak and the demand parameter α is small.

Intuitively, the introduction of a small export subsidy generates a lower average price and a greater level of variety in the home market, which leads to a gain in consumer surplus. This gain overwhelms the subsidy expense when selection effects are strong; however, when selection effects are weak, it is possible that the subsidy expense dominates in the home-country welfare calculation. We note, though, that the introduction of an export subsidy does generate a gain for the home country in the weak-selection case when α is below a threshold value as captured in (37).

To see this tradeoff more clearly, consider the extreme case in which the trade cost approaches infinity ($\tau \to \infty$). In this limiting case, the home country can be interpreted in terms of our closed-economy analysis, and (37) is reducible to the idea of Proposition 4 that additional entry is desirable if and only if $\alpha < 2 \cdot \lim_{\tau\to\infty} c_D^{FT}$. Thus, if there are too few entrants without government intervention, then additional entry generates a positive externality, and policies that encourage entry are attractive.[31]

Building from this insight, we now establish a simple corollary to Proposition 6:

Corollary 1 *If both countries initially adopt a policy of free trade, and if $\alpha \leq 2 \cdot c_D^{FT}$, then the introduction of a small export subsidy by country l generates a welfare gain for country l.*

The proof is immediate if selection effects are strong. For the situation where selection effects are weak, the proof follows since $\alpha \leq 2 \cdot c_D^{FT}$ then implies that (37) holds.

To provide some intuition for Corollary 1, we may refer to the logic underlying the entry-externality effect as captured in Proposition 4. Based on that reasoning, we may expect that the market provides the right level of entry or too little entry when $\alpha \leq 2 \cdot c_D^{FT}$.[32] Accordingly, the introduction of a small export subsidy then enables the intervening country to obtain a larger slice of a "global pie," where the pie itself is unaffected (to the first order) or expanded as a consequence of the subsidy.

We emphasize that $\alpha \leq 2 \cdot c_D^{FT}$ is sufficient but not necessary for a country to gain from the introduction of a small export subsidy. Indeed, Proposition 6 indicates that the introduction of a small export subsidy is attractive to the intervening country when the selection effect is strong, regardless of the value of α; furthermore, the introduction of a small export subsidy may also be beneficial to the intervening country when the selection effect is weak and $\alpha > 2 \cdot c_D^{FT}$, provided that α is not so large as to then violate (37).[33]

We may compare Proposition 6 with Venables' (1985) findings. For the homogeneous-firms Cournot model, he shows that the introduction of a small home export subsidy harms the foreign country but benefits the home country, at least when demand is linear. Our finding above likewise shows that the unilateral benefit of a small export subsidy is more qualified than that for a small import tariff, but in the heterogeneous-firms model that we analyze here the key considerations that determine the unilateral benefit of a small export subsidy are related to parameters that describe the significance of the selection effect and the externality associated with entry. Our findings thus suggest that strategic export subsidy policies may be more effective in some sectors than others. Sectors characterized by low trade costs and high productivity dispersion (i.e., strong-selection characteristics) would seem natural candidates for small strategic subsidies according to Proposition 6.

Proposition 6 offers a partial perspective on the WTO's prohibition of export subsidies. To the extent that governments use trade agreements to limit the scope in the long run for

[31] Under an infinite trade cost, trade policy doesn't affect welfare. But the sign is maintained as the trade cost goes to infinity, in that $\frac{dU^l}{dt^l}$ approaches zero from below when $\alpha < 2 \cdot \lim_{\tau \to \infty} c_D^l$, where $\lim_{\tau \to \infty} c_D^l = (\varphi\gamma)^{\frac{1}{k+2}} = c_D^m$ follows from (17).

[32] Formal support for this exepctation is provided below in Propositions 8 and 9.

[33] In Proposition 6, condition (b) can hold even if $\alpha > 2 \cdot c_D^{FT}$ since the parenthetical term in (37) exceeds 2 when the selection effects is weak.

beggar-thy-neighbor policies, Proposition 6 suggests that restrictions on export subsidies could be attractive once governments have achieved through preceding negotiations an outcome that is sufficiently close to global free trade. In this context, an interesting feature of the analysis provided here is that the appeal of restrictions is greater under conditions that may be descriptive of the current trading environment - namely, low trade costs and high productivity dispersion - since a country has unilateral incentive to introduce a small export subsidy when these conditions prevail. A more complete evaluation of the treatment of export subsidies in this model, however, requires a characterization of the efficiency frontier, a topic we consider below.

4.3 Introduction of a Small Import and Export Tariff

We now consider a different unilateral path from global free trade and allow that the home country simultaneously increases its import and export tariffs. The idea is to propose a simultaneous increase in home tariffs so as to maintain home consumer surplus while also generating tariff revenue. Bagwell and Staiger (2012b) explore such an intervention for the linear Cournot delocation model. The current setting is more complex, however, as consumer surplus is influenced by price and variety effects while firms are heterogeneous. Even so, we are able to utilize the structure of the Melitz-Ottaviano model and deliver related results. As (26) indicates, the price and variety influences on consumer surplus are all channeled through the cut-off cost level for domestic sales in the home market. Building from this insight, we show that the home country can gain by raising its import and export tariffs so as to maintain its cut-off cost level for domestic sales.

To formally explore this idea, let us consider the tariffs for any country l that serve to fix c_D^l. We suppose again that all tariffs are initially set at free trade. Using (17), we then find that the introduction of slight changes in country l's tariffs that preserve c_D^l satisfy

$$\frac{\partial \tilde{t}^l}{\partial t^l}\Big|_{t^h=\tilde{t}^h=t^l=\tilde{t}^l=0,\bar{c}_D^l} = -\frac{\frac{\partial c_D^l}{\partial t^l}}{\frac{\partial c_D^l}{\partial \tilde{t}^l}}\Big|_{t^h=\tilde{t}^h=t^l=\tilde{t}^l=0,\bar{c}_D^l} = \tau^{-k} > 0. \tag{39}$$

Intuitively, and as Proposition 1 confirms, a higher import tariff lowers c_D^l whereas a higher export tariff raises c_D^l. The particular positive relationship that maintains c_l^D then takes an especially simple form starting from global free trade. Since $TR^l = t^l \cdot IMP^l + \tilde{t}^l \cdot EXP^l$, we have that

$$\frac{dTR^l}{dt^l}\Big|_{t^l=\tilde{t}^l=0} = IMP^l > 0 \text{ and } \frac{dTR^l}{d\tilde{t}^l}\Big|_{t^l=\tilde{t}^l=0} = EXP^l > 0,$$

and so the proposed tariff changes are also sure to raise country l's tariff revenue. Referring to (26) and (29), we may now conclude that the proposed tariff changes leave country l's consumer surplus unaltered, raise country l's tariff revenue, and thus generate a gain in country l's welfare.[34]

Further utilizing the structure of the Melitz-Ottaviano model, we also find that the welfare of country h must fall when country l departs from global free trade and introduces this policy variation. To see why, we use (17) and show that the introduction of slight changes in country l's tariffs that preserve c_D^h must satisfy

$$\frac{\partial \tilde{t}^l}{\partial t^l}\Big|_{t^h=\tilde{t}^h=t^l=\tilde{t}^l=0,\bar{c}_D^h} = -\frac{\frac{\partial c_D^h}{\partial t^l}}{\frac{\partial c_D^h}{\partial \tilde{t}^l}}\Big|_{t^h=\tilde{t}^h=t^l=\tilde{t}^l=0,\bar{c}_D^h} = \tau^k > 0, \tag{40}$$

where $\tau^k > \tau^{-k}$ under our assumptions. Thus, for a given increase in t^l, the increase in $\tilde{t}^l$ that maintains c_D^l is not sufficiently great to maintain c_D^h. Since Proposition 1 implies that c_D^h is decreasing in $\tilde{t}^l$, we conclude that the proposed tariff variation for country l causes an increase in c_D^h. Using (26), it is straightforward to show that a country's consumer surplus is decreasing in its cut-off cost level for domestic sales.[35] Since country h has a policy of free trade, country l's policy change has no impact on country h's tariff revenue. We thus conclude from (29) that country h is harmed by the proposed tariff variation for country l.

The following proposition summarizes our findings:

Proposition 7 *(Small import and export tariffs) For countries l and h with $l, h \in \{H, F\}$ and $l \neq h$, if both countries initially adopt a policy of free trade, then the introduction of a small import tariff and a small export tariff by country l that satisfies (39) is sure to increase country l's welfare and lower country h's welfare.*

A notable feature of Proposition 7 is that the welfare implications hold for all trade costs, demand and dispersion parameters. Proposition 7 is related to Bagwell and Staiger's (2012b) finding for the linear Cournot delocation model; however, a novel feature of Proposition 7 is that it also addresses the externality associated with the described intervention. In particular, Proposition 7 indicates that this unilateral policy intervention, too, imposes a cost on the trading partner, provided that policies are initially placed at global free trade.

[34] It is now clear that the home country's gain from the proposed tariff changes does not require that the foreign country also adopt a policy of free trade. On the other hand, if the home country's initial tariffs were to differ from free trade, then the home country would gain from the proposed tariff change if and only if the change raises home-country tariff revenue. This tariff-revenue condition holds when the home country starts at free trade but need not hold otherwise.

[35] See Lemma 4 in the Appendix.

5 Efficient and Nash Trade Policies

In this section, we offer characterizations of efficient and Nash trade policies.

5.1 Efficient Symmetric Trade Policies

We begin by considering efficient trade policies. An initial point, confirmed in the following lemma, is that the sum of the two countries' welfare functions depends on tariffs only through the overall barriers to trade, χ^H and χ^F.

Lemma 1 *Joint welfare, $U \equiv U^H + U^F$, depends on individual tariffs, $\{t^H, \tilde{t}^H, t^F, \tilde{t}^F\}$, only through $\chi^H = \frac{1+t^H}{1-\tilde{t}^F}$ and $\chi^F = \frac{1+t^F}{1-\tilde{t}^H}$.*

As we explain next, this property implies that countries can effect lump-sum transfers through tariff policy variations that maintain $\chi^H = \frac{1+t^H}{1-\tilde{t}^F}$ and $\chi^F = \frac{1+t^F}{1-\tilde{t}^H}$.

To understand how countries may achieve lump-sum transfers, we consider an increase in t^l that is balanced against a reduction in $\tilde{t}^h$ so as to preserve $\chi^l = \frac{1+t^l}{1-\tilde{t}^h}$. Using (27), (28) and (29), we can rewrite U^l as

$$U^l = 1+\frac{t^l}{1+t^l}\cdot\frac{N_E^h(\tau\cdot\chi^l)^{-k}\cdot(c_D^l)^{k+2}}{2\gamma(k+2)(c_M)^k}+\frac{\widetilde{t}^l}{1+t^h}\cdot\frac{N_E^l(\tau\cdot\chi^h)^{-k}\cdot(c_D^h)^{k+2}}{2\gamma(k+2)(c_M)^k}+\frac{(\alpha-c_D^l)}{2\eta}[\alpha-c_D^l\frac{k+1}{k+2}],$$

where c_D^l, c_D^h, N_E^l and N_E^h are all fixed given that χ^l and χ^h are unaltered.[36] We now simply observe that an increase in t^l that is balanced against a decrease in $\tilde{t}^h$ so as to preserve χ^l acts to raise $\frac{t^l}{1+t^l}$. It follows that the change raises U^l. With the welfare sum for the two countries fixed by Lemma 1, the increase in U^l must be offset by an equal loss in U^h.

Lump-sum transfers achieved through tariff instruments operate by changing world prices at fixed trade volumes, and this mechanism is at work here as well. To develop this point, we define the (average) world price associated with an import good for country l:

$$\overline{p}^{wl} \equiv \frac{\overline{p}^l}{(1+t^l)}. \tag{41}$$

An increase in t^l that is balanced against a decrease in $\tilde{t}^h$ so as to preserve $\chi^l = \frac{1+t^l}{1-\tilde{t}^h}$ must also preserve c_D^l and thus by (19) $\overline{p}^l$ as well; hence, country l enjoys a terms-of-trade gain from such an adjustment, as the world price of its import good falls. Country h suffers a matching terms-of-trade loss, as the world price of its export good falls. The resulting

[36] This can be verified using expression (12) for export-market profit and our maintained assumption (8) that $q_0^l > 0$ which ensures that tariff revenue does not alter demand in the differentiated sector.

increase in U^l exactly offsets the loss in U^h, since with χ^l (and χ^h) fixed there is no change in the value of trade volumes or in consumer surplus in either country.

Since countries can effect lump-sum transfers, efficient trade policies are those which maximize the sum of the two countries' welfare functions, $U = U^H + U^F$. Formally, and using Lemma 1, we can define efficient tariffs using a two-step process. First, we solve the following program

$$\max_{\{\chi^H, \chi^F\}} U(\chi^H, \chi^F)$$

so as to determine the overall trade barriers, χ^H and χ^F, that maximize joint welfare, $U(\chi^H, \chi^F)$. Second, the set of *efficient tariffs* is then defined by the set of underlying tariffs, $\left\{t^H, \tilde{t}^H, t^F, \tilde{t}^F\right\}$, that induce the overall trade barriers as determined in the first step. Notice that there is a continuum of ways to combine t^l and $\tilde{t}^l$ so as to induce a given value for χ^l, and so efficient tariffs are not unique.

We focus primarily on tariffs that are efficient within the class of tariffs that generate a symmetric overall trade barrier along each channel of trade. Let χ^* maximize $U(\chi^H, \chi^F)$ over values of χ that satisfy $\chi = \chi^H = \chi^F$; that is, let χ^* maximize $U(\chi, \chi)$. For any value χ, we say that the tariffs $\left\{t^H, \tilde{t}^H, t^F, \tilde{t}^F\right\}$ are *χ-symmetric tariffs* if they induce $\chi^H = \chi^F = \chi$.[37] *Efficient χ-symmetric tariffs* are then χ-symmetric tariffs for which $\chi = \chi^*$. Efficient χ-symmetric tariffs are thus efficient within the class of χ-symmetric tariffs.[38] In line with our preceding discussion, we note that χ^* can be induced by a continuum of underlying tariffs.

We assume that a maximizer χ^* exists that is consistent with the assumptions in Section 2 and interior.[39] We thus assume that the associated value of ρ^l satisfies $\rho^l \in (0, 1)$ and also that a positive number of firms enters in each country when the overall barrier is $\chi^H = \chi^F = \chi^*$.[40] Interiority means that χ^* satisfies the associated first-order condition:

$$\frac{dU(\chi, \chi)}{d\chi}|_{\chi=\chi^*} = 0. \tag{42}$$

We do not maintain an assumption as to the quasi-concavity of $U(\chi, \chi)$ but rather introduce this assumption within the propositions below when it is utilized.

Our next proposition considers the specific question of whether global free trade (i.e., $t^H = \tilde{t}^H = t^F = \tilde{t}^F = 0$) constitutes an efficient trade policy.

[37] Among other tariffs, the set of χ-symmetric tariffs includes tariffs for which the home and foreign countries set the same import tariffs ($t^H = t^F$) and the same export tariffs ($\tilde{t}^H = \tilde{t}^F$).

[38] As our preceding discussion confirms, countries can effect lump-sum transfers using χ-symmetric tariffs, and so efficient χ-symmetric tariffs maximize joint welfare within the class of χ-symmetric tariffs.

[39] It is convenient to note these assumptions here as part of our discussion of efficient χ-symmetric tariffs. The assumptions play no role in our next two propositions (i.e., Propositions 8 and 9).

[40] As indicated in footnote 16, given $\tau > 1$, a sufficient condition for $\rho^l \in (0, 1)$ at χ^* for $k > 1$ is that $\chi^* > \tau^{-1/2}$. We also assume that c_M exceeds c_D^l as determined by (17).

Proposition 8 *(Free trade and efficiency) If both countries initially adopt a policy of free trade so that* $t^H = \tilde{t}^H = t^F = \tilde{t}^F = 0$*, then the introduction of a small increase in any tariff raises joint welfare if and only if* $\alpha > 2 \cdot c_D^{FT}$*, lowers joint welfare if and only if* $\alpha < 2 \cdot c_D^{FT}$*, and has no first-order effect on joint welfare if and only if* $\alpha = 2 \cdot c_D^{FT}$*.*

Proposition 8 thus indicates that global free trade is inefficient whenever $\alpha \neq 2 \cdot c_D^{FT}$.

Proposition 8 resonates with Proposition 4, which suggests that additional entry generates a negative externality for the economy when $\alpha > 2 \cdot c_D^{FT}$. According to Proposition 8, starting at global free trade, a trade agreement can generate higher welfare for its members if the agreement calls for a slight increase in any tariff when $\alpha > 2 \cdot c_D^{FT}$, and a symmetric decrease (i.e., a subsidy) in any tariff when $\alpha < 2 \cdot c_D^{FT}$. Finally, if $\alpha = 2 \cdot c_D^{FT}$, then a trade agreement cannot induce a first-order gain to its members with a small movement in any tariff.[41]

To explore the implications for trade-agreement design, let us consider a strong-selection environment so that $\tau < (4 + 2k)^{1/k}$. Starting at global free trade, Proposition 6 then indicates that a unilateral export subsidy is attractive to the intervening country. Therefore, if $\alpha = 2 \cdot c_D^{FT}$, then an efficiency-enhancing trade agreement would restrict small unilateral departures from global free trade in any form, even though each country has a unilateral incentive to depart from global free trade with a small import tariff or a small export subsidy. The case of $\alpha = 2 \cdot c_D^{FT}$ generates results analogous to those found by Bagwell and Staiger (2012b) under global free trade, since they show that any small movement from global free trade reduces efficiency in the linear Cournot delocation model. Other cases are possible here, however. Regarding the treatment of export subsidies, the unilateral incentive for a country to impose a small export subsidy would be beneficial (detrimental) for efficiency if $\alpha < 2 \cdot c_D^{FT}$ ($\alpha > 2 \cdot c_D^{FT}$). Thus, conditional on starting at global free trade and for the strong-selection environment, the model is consistent with effective and efficiency-enhancing restrictions on the use of export subsidies in a trade agreement if $\alpha \geq 2 \cdot c_D^{FT}$.[42] When $\alpha > 2 \cdot c_D^{FT}$, however, the model does not provide an efficiency-based rationale for restrictions on the introduction of small import tariffs.

Our second proposition is similar but starts with any tariffs that achieve free trade in the sense that the overall trade barrier along each channel is zero (i.e., $\chi^H = \chi^F = 1$).

[41] Our policy analysis here contrasts interestingly with the second-best analysis of Nocco et al (2014). As described previously, they consider per unit (i.e., specific) production subsidies that are provided on every produced unit within a closed-economy setting. By contrast, we consider ad valorem policies that are provided only on traded units within an open-economy setting. The cut-off cost level is impacted by a subsidy or tariff in the policy analysis that we examine.

[42] Similarly, in the weak-selection environment where $\tau \geq (4 + 2k)^{1/k}$, a unilateral export subsidy is attractive to the intervening country if $\alpha < (1 + \frac{\tau^k}{\tau^k - 2(k+2)})c_D^{FT}$, where the term in the parenthesis exceeds 2. Conditional on starting at global free trade and for the weak-selection environment, the model is consistent with effective and efficiency-enhancing restrictions on the use of export subsidies in a trade agreement if $(1 + \frac{\tau^k}{\tau^k - 2(k+2)})c_D^{FT} > \alpha \geq 2 \cdot c_D^{FT}$.

There is a continuum of such tariffs, including the tariffs that deliver global free trade.

Proposition 9 *(Free trade and efficiency under χ-symmetric policies) If the two countries initially adopt tariffs that achieve free trade so that $\chi^H = \chi^F = 1$, then the introduction of small tariff changes that induce a small and symmetric increase in $\chi = \chi^H = \chi^F$ raises joint welfare if and only if $\alpha > 2 \cdot c_D^{FT}$, lowers joint welfare if and only if $\alpha < 2 \cdot c_D^{FT}$, and has no first-order effect on joint welfare if and only if $\alpha = 2 \cdot c_D^{FT}$.*

The implications of Proposition 9 for trade-agreement design are similar to those developed above for Proposition 8, except that Proposition 9 allows for a larger set of initial tariffs and then considers χ-symmetric departures whereby the overall trade barrier adjusts symmetrically across the two trade channels. A specific implication of Proposition 9 is that global free trade is not in general an efficient trade policy, even within the restricted class of χ-symmetric tariffs.

An interesting finding in the proof of Proposition 9 is that, starting at χ-symmetric tariffs, a small and symmetric increase in $\chi = \chi^H = \chi^F$ raises the cut-off cost level in each country, $c_D^H = c_D^F$. Consumer surplus then falls in each country, due to the consequent increase in the average price and decrease in the number of varieties consumed in each country. Thus, for example, while an increase in a country's import tariff lowers its cut-off cost level and thereby raises its consumer surplus, symmetric increases in import tariffs that maintain symmetry between $\chi^H$and χ^F raise each country's cut-off cost level and thereby lower each country's consumer surplus. Melitz and Ottaviano (2008, p. 309) derive a similar finding when the (symmetric) trade cost τ is changed.

5.2 Nash Trade Policies and Liberalization Paths

Next, we consider Nash trade policies. A *Nash equilibrium* is a set of tariffs, $\{t^H, \tilde{t}^H, t^F, \tilde{t}^F\}$, that simultaneously solves

$$\max_{t^l, \tilde{t}^l} U^l \text{ for } l = H, F,$$

where we recall that $t^l > -1$ and $\tilde{t}^l < 1$. The tariffs so defined are *Nash tariffs*. A *symmetric Nash equilibrium* is then a Nash equilibrium in which $t^H = t^F$ and $\widetilde{t}^H = \widetilde{t}^F$. For a symmetric Nash equilibrium, we denote the *symmetric Nash tariffs* as a pair $(t^N, \widetilde{t}^N)$, where $t^N \equiv t^H = t^F$ is the symmetric Nash import tariff and $\widetilde{t}^N \equiv \widetilde{t}^H = \widetilde{t}^F$ is the symmetric Nash export tariff. Clearly, symmetric Nash tariffs must be χ-symmetric tariffs. We represent the associated symmetric Nash value for χ as $\chi^N \equiv (1 + t^N)/(1 - \widetilde{t}^N)$.

We assume that there exists a symmetric Nash equilibrium that is consistent with the assumptions in Section 2 and interior. We thus assume that $t^N > -1$, $\widetilde{t}^N < 1$, the associated value of ρ^l satisfies $\rho^l \in (0, 1)$ and also that a positive number of firms enters in each country when the overall barrier is given by $\chi^H = \chi^F = \chi^N$.[43] Our assumption of interiority means that t^N and $\widetilde{t}^N$ satisfy the associated first-order conditions:

$$\frac{dU^l}{dt^l}|_{t^l=t^h=t^N,\widetilde{t}^l=\widetilde{t}^h=\widetilde{t}^N} = \frac{dU^l}{d\widetilde{t}^l}|_{t^l=t^h=t^N,\widetilde{t}^l=\widetilde{t}^h=\widetilde{t}^N} = 0 \text{ for } l = H, F. \tag{43}$$

We next establish a condition under which the symmetric Nash equilibrium generates an overall trade barrier that is higher than efficient. With this result, we are also able to identify efficiency-enhancing liberalization paths.

Proposition 10 *(Nash, efficiency and liberalization paths) If $U(\chi, \chi)$ is quasi-concave in χ, then the symmetric Nash equilibrium is inefficient with a value for χ that is too high: $\chi^N > \chi^*$. Starting at the symmetric Nash equilibrium, countries thus mutually gain by symmetrically exchanging small reductions in import tariffs, export tariffs, or combinations thereof.*

As shown in detail in the Appendix, the approach of the proof of Proposition 10 is to add the Nash first-order conditions for t^l and $\widetilde{t}^l$ as derived from (43), impose symmetry, and write the resulting summed expression as a function of χ^N. Comparing this expression with the first-order condition for χ^* as derived from (42) and using the assumed quasi-concavity of the joint welfare function then delivers $\chi^N > \chi^*$. Proposition 10 thus indicates that countries enjoy mutual gains using χ-symmetric tariffs and starting at the symmetric Nash equilibrium only if they negotiate reductions in the overall trade barrier along each channel so that $\chi = \chi^H = \chi^F$ falls. Proposition 10 further indicates that starting at the symmetric Nash equilibrium countries are sure to enjoy mutual gains if they exchange small and symmetric reductions in their tariffs.[44]

An interesting finding in this regard is that a small and symmetric reduction in export tariffs generates mutual gains, even though each country's policy change then imposes a terms-of-trade *loss* on its partner. Bagwell and Staiger (2012b) make a similar observation for the linear Cournot delocation model. The key intuition is that country l's export tariff reduction facilitates a higher average price in country h with less of a terms-of-trade loss for country h than country h would have experienced had it tried to generate a higher average price in its own market by reducing its import tariff. Since at a Nash equilibrium country h is indifferent about small adjustments in its own policies, it therefore benefits from an export tariff reduction by country l.

[43] As indicated in footnote 16, given $\tau > 1$, a sufficient condition for $\rho^l \in (0, 1)$ to hold at χ^N for any $k > 1$ is that $\chi^N > \tau^{-1/2}$. We also assume that c_M exceeds c_D^l as determined by (17).

[44] Such tariff changes lead to small reductions in $\chi^H = \chi^F$. An exchange that is small and symmetric across countries suffices for mutual gains, but mutual gains may not be present under a signficicantly asymmetric liberalization path. For example, if the reduction in $\chi^H = \chi^F$ is achieved through reductions in the import and export tariffs of one country only, then mutual gains are not present since the liberalizing country moves away from its best-response tariffs and receives nothing in return.

If the symmetric Nash equilibrium entails the use of export subsidies (i.e., negative export tariffs), Proposition 10 thus provides that countries enjoy mutual gains by exchanging small and symmetric increases in their export subsidies. More generally, Proposition 10 indicates that trade policies are too restrictive in a symmetric Nash equilibrium. As such, it offers an interpretation for why early GATT rounds emphasized negotiated reductions in import tariffs but treated export subsidies in a more permissive way.

Proposition 10 is of special interest when viewed in combination with Propositions 6 and 8. Starting at global free trade, Proposition 6 establishes that the introduction of a small export subsidy by the home country always hurts foreign welfare but can raise home welfare. As suggested above, to the extent that governments use trade agreements to limit the scope in the long run for beggar-thy-neighbor policies, Proposition 6 suggests that restrictions on export subsidies could be attractive once governments have achieved through preceding negotiations an outcome that is sufficiently close to global free trade. At the same time, we note from Proposition 8 that global free trade is generally not efficient in the Melitz-Ottaviano model; thus, a simultaneous ban on export subsidies and import tariffs achieves less support in this model than in the linear Cournot delocation model considered by Bagwell and Staiger (2012b).

We next build on Proposition 10 and provide conditions under which we can sign the level of intervention in efficient χ-symmetric and symmetric Nash tariffs. Our next proposition establishes conditions under which efficient χ-symmetric trade policies entail a total tariff that is positive ($\chi^* > 1$) but an overall trade barrier that is below that in a symmetric Nash equilibrium.

Proposition 11 *(Nash and efficient tariffs) Assume $\alpha > 2 \cdot c_D^{FT}$ and that $U(\chi, \chi)$ is quasi-concave in χ. Then $\chi^N > \chi^* > 1$.*

The finding that $\chi^N > \chi^*$ is taken from the Proposition 10, while the finding that $\chi^* > 1$ follows from Proposition 9 and the fact that $\chi = 1$ under global free trade.

In models of trade agreements in which governments have political-economic preferences and efficiency is measured relative to those preferences, a common finding is that Nash tariffs exceed efficient tariffs in total, where the total efficient tariff is positive when governments attach a greater welfare weight to profit in import-competing sectors.[45] In the present model, we use ad valorem import and export tariffs in a segmented market setting, and the relevant measure for overall protection into country l is χ^l, where $\chi^l > 1$ if and only if the total tariff $t^l + \tilde{t}^h$ is positive. Proposition 11 establishes conditions under which a similar ranking occurs in the Melitz-Ottaviano model, even though the model has a zero-expected-profit condition. As Proposition 9 suggests, the assumption

[45] See, e.g., Bagwell and Staiger (2001) and Grossman and Helpman (1995).

that $\alpha > 2 \cdot c_D^{FT}$ is used in establishing that the efficient χ-symmetric tariff policy entails positive protection: $\chi^* > 1$. Under quasi-concavity of the joint welfare function in $\chi = \chi^H = \chi^F$, we may conclude from Proposition 10 that the protection measure χ is higher in the symmetric Nash equilibrium than under efficient χ-symmetric tariff policies. Bagwell and Staiger (2012b) provide a related result for the linear Cournot delocation model, but an important difference is that global free trade is efficient in that model.

We next provide conditions under which the levels of the symmetric Nash import and export tariffs can be ranked.

Proposition 12 *(Nash tariff ranking) Assume $\alpha \geq 2 \cdot c_D^{FT}$ and that $U(\chi, \chi)$ is quasi-concave. Then, in any symmetric Nash equilibrium, $t^N > \tilde{t}^N$.*

Proposition 12 shows that the symmetric Nash import tariff is higher than the symmetric Nash export tariff. For example, if both policies are positive, as Proposition 7 suggests could be the case, then the Nash import tariff is the higher of the two.

For the linear Cournot delocation model with specific tariffs, Bagwell and Staiger (2012b) establish this ranking as well. The key insight in both frameworks is that an import tariff both generates tariff revenue and lowers the domestic price whereas an export tariff provides additional tariff revenue but at the cost of raising the domestic price. Here, we derive the same ranking, but in a monopolistic competition model with heteregeneous firms and for ad valorem tariffs.

We now drop the symmetry restriction and consider the limiting behavior of Nash tariffs as $\tau \to \infty$.[46] To gain some intuition, we refer to Corollary 1 and observe that the home country has a unilateral incentive to introduce an export subsidy starting from global free trade if $\alpha < 2 \cdot c_D^m$ where $\lim_{\tau \to \infty} c_D^l = (\varphi\gamma)^{\frac{1}{k+2}} = c_D^m$ follows from (17). We can likewise show that, for τ sufficiently large and again starting from global free trade, the home country has a unilateral incentive to introduce an export tariff if $\alpha > 2 \cdot c_D^m$. Thus, when the trade cost τ is sufficiently large, whether the home country wishes to introduce an export subsidy or tariff hinges on the sign of the entry-externality effect as defined in (34) for the closed economy. Based on these findings, we may conjecture that the sign of the limiting value of the Nash export policy for a given country is determined by the sign of $\alpha - 2 \cdot c_D^m$.

The following proposition confirms this conjecture by characterizing Nash import and export tariffs as the trade cost τ goes to infinity.

Proposition 13 *(Nash in the limit) Nash tariffs in the limiting case of $\tau \to \infty$ can be characterized as:*

$$\lim_{\tau \to \infty} t^N = \frac{1}{k} > 0 \tag{44}$$

[46] For the following proposition, our assumption is thus that there exist Nash tariffs that are consistent with the assumptions in Section 2 and interior.

and

$$\lim_{\tau \to \infty} \frac{\tilde{t}^N}{1 - \tilde{t}^N} = \frac{1}{2(2+k)k} \frac{(\alpha - 2 \cdot c_D^m)}{(\alpha - c_D^m)} \tag{45}$$

where $c_D^m = \lim_{\tau \to \infty} c_D^l$.

Proposition 13 does not impose a symmetry restriction on the Nash tariffs; instead, when we derive (44) and (45), we confirm that the Nash tariffs must approach symmetric limits.

In line with our expectations, we see from (45) that $\lim_{\tau \to \infty} \tilde{t}^N$ is positive if and only if $\alpha > 2 \cdot c_D^m$. An interesting implication is thus that an export subsidy can be part of Nash equilibrium, at least for sufficiently high trade costs and when the market economy does not provide enough varieties. By contrast, we find that the import tariff is always positive as the trade cost goes to infinity, with a limit value that is inversely related to the dispersion parameter k. We note that the characterization $\lim_{\tau \to \infty} t^N = 1/k$ in (44) is also implied by Demidova's (2017) analysis of unilateral import tariffs in the Melitz-Ottaviano model when the assumption of an outside good is dropped.[47]

An interesting finding in the proof of Proposition 13 is that, as the trade cost τ goes to infinity, the import tariff t^l impacts c_D^h at a higher order than c_D^l while the export tariff $\tilde{t}^l$ impacts c_D^l at a higher order than c_D^h.[48] In effect, the dominant impact of a tariff in the limit is on the cost type for the country whose products directly incur (rather than are protected by) the tariff. This explains why the export tariff $\tilde{t}^l$ plays the role in (45) of adjusting c_D^l in response to the sign of $\alpha - 2 \cdot c_D^m$.

5.3 Numerical Example

We consider now a simple numerical illustration of our main model. Consider the following parameters: $\alpha = 2$, $c_M = 1$, $k = 1.1$, $f_e = 0.1$, $\tau = 1.1$ and $\gamma = 1 = \eta$. Under this specification, at global free trade, we find that $c_D^{FT} = 0.885 < 1 = \alpha/2$ and that the consumption of the numeraire good satisfies $q_0^* = 0.332 > 0$. For this specification, the efficient symmetric trade policies satisfy $\chi^* = 1.03$. Thus, consistent with Propositions 8 and 11, this example satisfies $\alpha > 2 \cdot c_D^{FT}$ and the efficient symmetric trade policies call for a positive total tariff, $\chi^* > 1$. The symmetric Nash equilibrium entails the following tariffs: $t^N = 0.78$ and $\tilde{t}^N = 0.26$ and thus $\chi^* \simeq 2.41$. Thus, as Proposition 12 indicates, the symmetric Nash import tariff exceeds the symmetric Nash export tariff. In line with Proposition 11, the overall trade barrier in the symmetric Nash equilibrium also exceeds

[47] We refer the reader to Demidova's (2017) equation (15) in her Proposition 1.

[48] As $\tau \to \infty$, the derivative of any cut-off cost level with respect to any tariff goes to zero. Our point here is that, as $\tau \to \infty$, certain derivatives go to zero at a lower order than do others. Notice also that the relative impacts here are consistent with the findings reported for the special case of global free trade in (39) and (40), since $\tau^k > 1 > \tau^{-k}$.

that under efficient symmetric tariffs. We can also verify for this specification that $U(\chi, \chi)$ is quasi-concave over tariffs that are consistent with the assumptions in Section 2.

An interesting feature of this example is that the Nash export tariff is positive. This feature may be surprising given the optimal export subsidy result in Proposition 6, but it can be readily interpreted in light of the complementary relationship between import and export tariffs identified in Proposition 7. Intuitively, when the home country has a positive import tariff in place, as it does here, it has an enhanced incentive to use an export tariff, since the resulting expansion in the foreign country's export value then generates import tariff revenue for the home country.[49] Bagwell and Staiger (2012b) show generally that the Nash import and export tariffs are both positive in the Cournot delocation model and interpret their finding in terms of this complementary relationship between import and export tariffs. Our numerical example confirms the possibility of a similar effect in the Melitz-Ottaviano model.

6 The Terms-of-Trade Rationale

Our results in the previous section indicate that symmetric Nash tariffs are inefficient and provide conditions for efficiency-enhancing tariff liberalization. In this section, we explore the deeper structure leading to the inefficiency of Nash tariffs; in particular, we develop a sense in which terms-of-trade motivations provide the sole rationale for trade agreements in this model. Our anaylsis proceeds in two steps. First, we represent country welfare as a function of local and world prices, and joint welfare as a function of local prices. Second, using these representations, we examine Nash, efficient and "politically optimal" tariffs.

For our first step, we begin by defining several functional relationships. Recall that the overall barrier to trade is represented as $\chi^l = \chi^l(t^l, \tilde{t}^h)$ where $\chi^l(t^l, \tilde{t}^h) \equiv \frac{1+t^l}{1-\tilde{t}^h}$ as noted in (12). Extending our notation in the natural way, we now make the following definitions:

$$\begin{aligned} \rho(\chi) &\equiv (\tau)^{-k}(\chi)^{-(k+1)} \\ c_D(\chi^l, \chi^h) &\equiv [\frac{\phi\gamma(1-\rho(\chi^h))}{1-\rho(\chi^l)\rho(\chi^h)}]^{\frac{1}{k+2}} \\ CS(\chi^l, \chi^h) &\equiv \frac{(\alpha - c_D(\chi^l, \chi^h))}{2\eta}[\alpha - c_D(\chi^l, \chi^h)\frac{k+1}{k+2}], \end{aligned} \tag{46}$$

where we may use (17), (18) and (26) to confirm that $\rho(\chi^l) = \rho^l$, $c_D(\chi^l, \chi^h) = c_D^l$ and $CS(\chi^l, \chi^h) = CS^l$.

[49] We can show that $\frac{dIMP^l}{dt^l}, \frac{dEXP^l}{d\tilde{t}^l} < 0 < \frac{dEXP^l}{dt^l}, \frac{dIMP^l}{d\tilde{t}^l}$, and the proofs are available upon request. Indeed, this result confirms that, starting at global free trade, import and export tariffs exert a complementary relationship on tariff revenue: $\frac{d^2TR^l}{dt^l d\tilde{t}^l} = \frac{dIMP^l}{d\tilde{t}^l} + \frac{dEXP^l}{dt^l} > 0$ at global free trade.

We are now ready to define the relevant functional relationships for prices:

$$\begin{aligned}
\overline{p}^l &= \overline{p}^l(\chi^l,\chi^h) \text{ where } \overline{p}^l(\chi^l,\chi^h) \equiv c_D(\chi^l,\chi^h)\cdot\frac{2k+1}{2k+2} \\
\overline{p}^{wl} &= \overline{p}^{wl}(\chi^l,\chi^h,t^l) \text{ where } \overline{p}^{wl}(\chi^l,\chi^h,t^l) \equiv \frac{\overline{p}^l(\chi^l,\chi^h)}{1+t^l} \\
\widetilde{p}^h &= \widetilde{p}^h(\chi^l,\chi^h) \text{ where } \widetilde{p}^h(\chi^l,\chi^h) \equiv \frac{\overline{p}^l(\chi^l,\chi^h)}{\chi^l}.
\end{aligned} \tag{47}$$

The price $\overline{p}^l$ is the local (average) price in country l, the price $\overline{p}^{wl}$ is the world (average) price for imports into country l, and the price $\widetilde{p}^h$ is the local (average) price received by country h's exporters for exports into country l.[50] As (47) clarifies, the local prices are determined by the overall barriers to trade, but the world price depends directly as well on the associated import tariff.

To express country welfare in terms of local and world prices, we define a new function

$$f(\chi^l,\chi^h) \equiv (1+t^l)IMP^l = (1+t^l)EXP^h, \tag{48}$$

where by (27) and (28) we can understand $f(\chi^l,\chi^h)$ as measuring the value of trade into country l when using delivered (consumer) prices. Using (29), (46) and (48), we may now rewrite the welfare function for country l as

$$U^l = 1 + \frac{t^l}{1+t^l}\cdot f(\chi^l,\chi^h) + \frac{\widetilde{t}^l}{1+t^h}\cdot f(\chi^h,\chi^l) + CS(\chi^l,\chi^h). \tag{49}$$

Referring to the definitions in (47), we note that

$$\frac{t^l}{1+t^l} = \frac{\overline{p}^l - \overline{p}^{wl}}{\overline{p}^l}, \quad \frac{\tilde{t}^l}{1+t^h} = \frac{\overline{p}^{wh} - \widetilde{p}^l}{\overline{p}^h}, \text{ and } \chi^l = \frac{\overline{p}^l}{\widetilde{p}^h}. \tag{50}$$

Using (49) and (50), we can now express the welfare functions for countries H and F, respectively, in terms of prices. To this end, for $l = H, F$, we define

$$\begin{aligned}
V^l(\overline{p}^H,\overline{p}^F,\widetilde{p}^H,\widetilde{p}^F,\overline{p}^{wH},\overline{p}^{wF}) &= 1 + \frac{\overline{p}^l - \overline{p}^{wl}}{\overline{p}^l}\cdot f(\frac{\overline{p}^l}{\widetilde{p}^h},\frac{\overline{p}^h}{\widetilde{p}^l}) + \frac{\overline{p}^{wh} - \widetilde{p}^l}{\overline{p}^h}\cdot f(\frac{\overline{p}^h}{\widetilde{p}^l},\frac{\overline{p}^l}{\widetilde{p}^h}) \\
&\quad + CS(\frac{\overline{p}^l}{\widetilde{p}^h},\frac{\overline{p}^h}{\widetilde{p}^l}),
\end{aligned} \tag{51}$$

where at given tariffs and by construction we have that $V^l = U^l$.

[50] Using $c_D(\chi^l,\chi^h) = c_D^l$ and for any given underlying tariffs, we may confirm that $\overline{p}^l(\chi^l,\chi^h)$ as defined in (47) indeed equals $\overline{p}^l$ as defined in (19) and that $\overline{p}^{wl}(\chi^l,\chi^h,t^l)$ as defined in (47) indeed equals $\overline{p}^{wl}$ as defined in (41).

We turn next to the joint welfare of the two countries. Joint welfare, or $V = V^H + V^F$, can be expressed in terms of prices as

$$\begin{aligned} V(\overline{p}^H,\overline{p}^F,\widetilde{p}^H,\widetilde{p}^F) &= V^H(\overline{p}^H,\overline{p}^F,\widetilde{p}^H,\widetilde{p}^F,\overline{p}^{wH},\overline{p}^{wF}) + V^F(\overline{p}^H,\overline{p}^F,\widetilde{p}^H,\widetilde{p}^F,\overline{p}^{wH},\overline{p}^{wF}) \\ &= 2 + \frac{\overline{p}^H - \widetilde{p}^F}{\overline{p}^H} \cdot f(\chi^H,\chi^F) + \frac{\overline{p}^F - \widetilde{p}^H}{\overline{p}^F} \cdot f(\chi^F,\chi^H) \\ &\quad + CS(\chi^H,\chi^F) + CS(\chi^F,\chi^H), \end{aligned} \tag{52}$$

where at given tariffs and by construction we have that $V = U$ and where we recall from (50) that $\chi^l = \frac{\overline{p}^l}{\widetilde{p}^h}$. A key observation is that the world prices cancel from joint welfare; that is, the function V depends only on local prices. Intuitively, tariff adjustments that preserve χ^H and χ^F, and that thus preserve local prices, may alter world prices but in so doing only facilitate lump-sum transfers between countries as described in Section 5.1.

We now summarize our first-step findings with the following proposition:

Proposition 14 *(welfare and prices) (i). As indicated in (51), the welfare of each country can be expressed as a function of local and world prices, $\overline{p}^H,\overline{p}^F,\widetilde{p}^H,\widetilde{p}^F,\overline{p}^{wH}$ and $\overline{p}^{wF}$. (ii). As indicated in (52), the joint welfare of the two countries can be expressed as a function of just local prices, $\overline{p}^H,\overline{p}^F,\widetilde{p}^H$ and $\widetilde{p}^F$.*

While the model of Melitz and Ottaviano (2008) features selection, price and variety effects, Proposition 14 indicates that, once the appropriate price definitions are identified, the welfare functions take on a familiar representation in the sense that country welfare depends only on local and world prices while joint welfare depends only on local prices.

Using the welfare representations in (51) and (52), we move now to the second step of our analysis and decompose the first-order conditions for Nash and efficient tariffs in order to better understand the efficiency properties of Nash tariffs. We thus generalize our preceding analysis by comparing Nash and efficient tariffs in the absence of any symmetry restrictions. This approach also directs attention to the "politically optimal" tariffs as studied in different modeling contexts by Bagwell and Staiger (1999, 2001, 2012a, 2015, 2016b), and we characterize the efficiency properties of politically optimal tariffs as well.

We begin by considering Nash tariffs. We assume that there exist Nash tariffs that are consistent with the assumptions in Section 2 and interior. The first-order condition for country H with respect to its import tariff is then

$$\begin{aligned} \frac{dV^H}{dt^H} &= \left(\frac{\partial V^H}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^H} + \frac{\partial V^H}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^H} + \frac{\partial V^H}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^H} + \frac{\partial V^H}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^H}\right) \cdot \frac{\partial \chi^H}{\partial t^H} \\ &\quad + \frac{\partial V^H}{\partial \overline{p}^{wH}} \cdot \left(\frac{\partial \overline{p}^{wH}}{\partial \chi^H}\frac{\partial \chi^H}{\partial t^H} + \frac{\partial \overline{p}^{wH}}{\partial t^H}\right) + \frac{\partial V^H}{\partial \overline{p}^{wF}}\frac{\partial \overline{p}^{wF}}{\partial \chi^H}\frac{\partial \chi^H}{\partial t^H} = 0. \end{aligned} \tag{53}$$

The top line in the expression captures the welfare effects for country H of the induced local-price effects of a change in its import tariff, where such effects reach local prices through χ^H. The bottom line captures the welfare effects for country H of the induced world-price effects of a change in its import tariff, where a higher import tariff generates a terms-of-trade gain for country H both by lowering its world import price, $\overline{p}^{wH}$, and by raising its world export price, $\overline{p}^{wF}$.

Consider next the first-order condition for country H with respect to its export tariff:

$$\begin{aligned}\frac{dV^H}{d\tilde{t}^H} &= \left(\frac{\partial V^H}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^F} + \frac{\partial V^H}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^F} + \frac{\partial V^H}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^F} + \frac{\partial V^H}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^F}\right)\cdot\frac{\partial \chi^F}{\partial \tilde{t}^H} \\ &\quad + \frac{\partial V^H}{\partial \overline{p}^{wH}}\frac{\partial \overline{p}^{wH}}{\partial \chi^F}\frac{\partial \chi^F}{\partial \tilde{t}^H} + \frac{\partial V^H}{\partial \overline{p}^{wF}}\frac{\partial \overline{p}^{wF}}{\partial \chi^F}\frac{\partial \chi^F}{\partial \tilde{t}^H} = 0.\end{aligned} \tag{54}$$

The top line in the expression captures the welfare effects for country H of the induced local-price effects of a change in its export tariff, where such effects reach local prices through χ^F. The bottom line captures the welfare effects for country H of the induced world-price effects of a change in its export tariff, where a higher export tariff generates a terms-of-trade gain for country H both by lowering its world import price, $\overline{p}^{wH}$, and by raising its world export price, $\overline{p}^{wF}$.

We can likewise represent the first-order conditions for country F's Nash import and export tariffs:

$$\begin{aligned}\frac{dV^F}{dt^F} &= \left(\frac{\partial V^F}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^F} + \frac{\partial V^F}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^F} + \frac{\partial V^F}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^F} + \frac{\partial V^F}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^F}\right)\cdot\frac{\partial \chi^F}{\partial t^F} \\ &\quad + \frac{\partial V^F}{\partial \overline{p}^{wH}}\frac{\partial \overline{p}^{wH}}{\partial \chi^F}\frac{\partial \chi^F}{\partial t^F} + \frac{\partial V^F}{\partial \overline{p}^{wF}}\cdot\left(\frac{\partial \overline{p}^{wF}}{\partial \chi^F}\frac{\partial \chi^F}{\partial t^F} + \frac{\partial \overline{p}^{wF}}{\partial t^F}\right) = 0\end{aligned} \tag{55}$$

and

$$\begin{aligned}\frac{dV^F}{d\tilde{t}^F} &= \left(\frac{\partial V^F}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^H} + \frac{\partial V^F}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^H} + \frac{\partial V^F}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^H} + \frac{\partial V^F}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^H}\right)\cdot\frac{\partial \chi^H}{\partial \tilde{t}^F} \\ &\quad + \frac{\partial V^F}{\partial \overline{p}^{wH}}\frac{\partial \overline{p}^{wH}}{\partial \chi^H}\frac{\partial \chi^H}{\partial \tilde{t}^F} + \frac{\partial V^F}{\partial \overline{p}^{wF}}\frac{\partial \overline{p}^{wF}}{\partial \chi^H}\frac{\partial \chi^H}{\partial \tilde{t}^F} = 0.\end{aligned} \tag{56}$$

These conditions can be interpreted in a similar manner to that given above for country H's optimal policies.

We next consider efficient tariffs. We assume that there exist efficient tariffs that are consistent with the assumptions in Section 2 and interior. Recall that joint welfare function, V, depends only on local prices, which in turn depend only on χ^H and χ^F. Once the efficient levels for χ^H and χ^F are determined, a continuum of combinations of home and foreign country import and export tariffs exists that delivers χ^H and χ^F. The first-order conditions for efficiency are given as follows:

$$\begin{aligned} \frac{dV}{d\chi^H} &= \left(\frac{\partial V^H}{\partial \overline{p}^H} + \frac{\partial V^F}{\partial \overline{p}^H}\right) \cdot \frac{\partial \overline{p}^H}{\partial \chi^H} + \left(\frac{\partial V^H}{\partial \overline{p}^F} + \frac{\partial V^F}{\partial \overline{p}^F}\right) \cdot \frac{\partial \overline{p}^F}{\partial \chi^H} \\ &\quad + \left(\frac{\partial V^H}{\partial \widetilde{p}^H} + \frac{\partial V^F}{\partial \widetilde{p}^H}\right) \cdot \frac{\partial \widetilde{p}^H}{\partial \chi^H} + \left(\frac{\partial V^H}{\partial \widetilde{p}^F} + \frac{\partial V^F}{\partial \widetilde{p}^F}\right) \cdot \frac{\partial \widetilde{p}^F}{\partial \chi^H} = 0 \end{aligned} \tag{57}$$

and

$$\begin{aligned} \frac{dV}{d\chi^F} &= \left(\frac{\partial V^H}{\partial \overline{p}^H} + \frac{\partial V^F}{\partial \overline{p}^H}\right) \cdot \frac{\partial \overline{p}^H}{\partial \chi^F} + \left(\frac{\partial V^H}{\partial \overline{p}^F} + \frac{\partial V^F}{\partial \overline{p}^F}\right) \cdot \frac{\partial \overline{p}^F}{\partial \chi^F} \\ &\quad + \left(\frac{\partial V^H}{\partial \widetilde{p}^H} + \frac{\partial V^F}{\partial \widetilde{p}^H}\right) \cdot \frac{\partial \widetilde{p}^H}{\partial \chi^F} + \left(\frac{\partial V^H}{\partial \widetilde{p}^F} + \frac{\partial V^F}{\partial \widetilde{p}^F}\right) \cdot \frac{\partial \widetilde{p}^F}{\partial \chi^F} = 0. \end{aligned} \tag{58}$$

We now evaluate the efficiency first-order conditions (57) and (58) at the Nash tariff levels. To this end, we take the Nash first-order condition for t^H captured by (53) and divide through by $\frac{\partial \chi^H}{\partial t^H} > 0$; and we similarly take the Nash first-order condition for $\tilde{t}^F$ captured by (56) and divide through by $\frac{\partial \chi^F}{\partial \tilde{t}^F} > 0$. We then add the two resulting expressions together, use $\frac{\partial V^H}{\partial \overline{p}^{wH}} + \frac{\partial V^F}{\partial \overline{p}^{wH}} = 0 = \frac{\partial V^H}{\partial \overline{p}^{wF}} + \frac{\partial V^F}{\partial \overline{p}^{wF}}$, refer to the efficiency first-order condition for χ^H as captured by (57), and arrive at the following necessary implication for Nash equilibrium:

$$\frac{dV}{d\chi^H}|_{Nash} = -\frac{IMP^H}{\chi^H} < 0. \tag{59}$$

Similarly, we may take the Nash first-order conditions for t^F and $\tilde{t}^H$ as given in (55) and (54), respectively, perform analogous manipulations, refer to the efficiency first-order condition for χ^F as captured by (58), arrive at the following necessary implication for Nash equilibrium:

$$\frac{dV}{d\chi^F}|_{Nash} = -\frac{IMP^F}{\chi^F} < 0. \tag{60}$$

A comparison of (59) and (60) with the efficiency first-order conditions (57) and (58) confirms that the Nash tariffs are inefficient.

Clearly, one reason for the inefficiency of Nash tariffs is the incentive that countries have to manipulate their world prices. To see if other sources of inefficiency are present, we follow Bagwell and Staiger (1999, 2001, 2012a, 2015, 2016b) and examine the *politically optimal tariffs*. These are the tariffs that countries would choose if, hypothetically, they were not motivated by the terms-of-trade implications of their unilateral tariff choices. For the model considered here, when making their respective politically optimal tariff selections, the home country acts as if $\frac{\partial V^H}{\partial \overline{p}^{wH}} = \frac{\partial V^H}{\partial \overline{p}^{wF}} = 0$ while the foreign country acts as if $\frac{\partial V^F}{\partial \overline{p}^{wH}} = \frac{\partial V^F}{\partial \overline{p}^{wF}} = 0$. We assume that there exist politically optimal tariffs that are consistent with the assumptions in Section 2 and interior.

To characterize the politically optimal import and export tariffs for the home country, we refer to the home country's Nash first-order conditions (53) and (54) and impose there that $\frac{\partial V^H}{\partial \overline{p}^{wH}} = \frac{\partial V^H}{\partial \overline{p}^{wF}} = 0$. The politically optimal import and export tariffs for the home country thus satisfy the following first-order conditions:

$$\begin{aligned}
\left(\frac{\partial V^H}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^H} + \frac{\partial V^H}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^H} + \frac{\partial V^H}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^H} + \frac{\partial V^H}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^H}\right) \cdot \frac{\partial \chi^H}{\partial t^H} &= 0 \\
\left(\frac{\partial V^H}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^F} + \frac{\partial V^H}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^F} + \frac{\partial V^H}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^F} + \frac{\partial V^H}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^F}\right) \cdot \frac{\partial \chi^F}{\partial \tilde{t}^H} &= 0
\end{aligned}$$

Equivalently, since $\frac{\partial \chi^H}{\partial t^H} > 0$ and $\frac{\partial \chi^F}{\partial \tilde{t}^H} > 0$, the home country's politically optimal tariffs satisfy

$$\frac{\partial V^H}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^H} + \frac{\partial V^H}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^H} + \frac{\partial V^H}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^H} + \frac{\partial V^H}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^H} = 0 \tag{61}$$

$$\frac{\partial V^H}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^F} + \frac{\partial V^H}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^F} + \frac{\partial V^H}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^F} + \frac{\partial V^H}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^F} = 0. \tag{62}$$

Arguing similarly, the foreign country's politically optimal tariffs satisfy

$$\frac{\partial V^F}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^F} + \frac{\partial V^F}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^F} + \frac{\partial V^F}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^F} + \frac{\partial V^F}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^F} = 0 \tag{63}$$

$$\frac{\partial V^F}{\partial \overline{p}^H}\frac{\partial \overline{p}^H}{\partial \chi^H} + \frac{\partial V^F}{\partial \overline{p}^F}\frac{\partial \overline{p}^F}{\partial \chi^H} + \frac{\partial V^F}{\partial \widetilde{p}^H}\frac{\partial \widetilde{p}^H}{\partial \chi^H} + \frac{\partial V^F}{\partial \widetilde{p}^F}\frac{\partial \widetilde{p}^F}{\partial \chi^H} = 0. \tag{64}$$

It is now direct to confirm that the politically optimal tariffs are efficient. To see this, we simply observe that adding (61) and (64) yields the efficiency first-order condition (57) corresponding to $\frac{dV}{d\chi^H} = 0$, and that adding (62) and (63) likewise yields the efficiency first-order condition (58) corresponding to $\frac{dV}{d\chi^F} = 0$.

We now summarize our findings:

Proposition 15 *(Nash, efficiency and political optimality) After expressing welfare functions in terms of local and world prices, we find that Nash tariffs are inefficient but the politically optimal tariffs are efficient.*

Proposition 15 generalizes Proposition 10 in two ways. First, it establishes that politically optimal tariffs are efficient. Second, it shows that the Nash equilibrium is inefficient in the absence of any symmetry restriction. On the other hand, Proposition 10 imposes symmetry restrictions and the assumption that the joint welfare function $U(\chi, \chi)$ is quasi-concave in χ, and then identifies efficiency-enhancing liberalization paths.

Our finding that the politically optimal tariffs are efficient may be of particular interest. The interpretation is that countries would set their trade policies in an efficient manner if they were not motivated by the terms-of-trade implications of their trade policies. In this sense, the fundamental problem for a trade agreement to address in the Melitz-Ottaviano model is a terms-of-trade problem. From this perspective, the addition of heterogeneous firms does not provide a new problem for a trade agreement to solve. As discussed in the Introduction, this finding is in broad alignment with the findings of Costinot et al (2016), although they consider a different model and use different arguments to highlight the centrality of the terms-of-trade externality.

As Bagwell and Staiger (1999, 2001, 2012a, 2015, 2016b) establish, politically optimal tariffs are efficient in a range of models when governments have a complete set of trade-policy instruments. For the present paper, Bagwell and Staiger (2015) is most relevant. They consider two models of imperfect competition with free entry and trade costs when firms are homogeneous, a linear outside good exists, and import and export tariffs are available.[51] As Maggi (2014) highlights, the models that they consider have no income effects and perfectly substitutable import and export tariffs (i.e., local prices along a given channel of trade depend upon tariffs only via the sum of the import and export tariffs applied on that channel).[52] Relative to the settings studied by Bagwell and Staiger (2015), our model likewise eliminates income effects with the assumption of a linear outside good but differs in other respects. First, we extend the analysis to allow for a heterogeneous-firms model of monopolistic competition with variable markups. Second, in the model considered here, import and export tariffs are not perfectly substitutable. Nevertheless, as indicated in Lemma 1, countries can use their tariffs to effect lump-sum transfers.

7 Quadratic versus CES Preferences

Our trade-policy findings are derived for the Melitz-Ottaviano (MO) model and thus utilize the quadratic preferences defined in (1). A natural question concerns the extent to which our findings would generalize were consumer preferences regarding the differentiated-goods sector instead described by a CES function. We explore this issue in detail in a sequel paper, Bagwell and Lee (2018). In this section, we briefly describe how the results differ across the two formulations, and we also describe a partial perspective concerning the underlying reason for the differences.

Relative to the MO model considered here, the model considered in Bagwell and Lee (2018) has two key differences. The first difference is that preferences for differentiated varieties are described by a CES function. Building on Venables (1987) and following

[51] One model has segmented markets and Cournot competition, and the other model has integrated markets, CES preferences and monopolistic competition. See also Venables (1985, 1987).

[52] For further discussion of conditions under which politically optimal tariffs are efficient, see Bagwell and Staiger (2016b), DeRemer (2012, Appendix E) and Maggi (2014).

Bagwell and Staiger (2015) and Helpman and Krugman (1989), Bagwell and Lee assume that all consumers in country $l \in \{H, F\}$ share the same quasi-linear utility function with CES preferences for the differentiated-goods sector and thus solve the following problem:

$$U^l \equiv \max_{\left\{q_0^l, \left\{q_i^l\right\}_{i \in \Omega^l}\right\}} q_0^l + \frac{1}{\theta} \left(\int_{i \in \Omega^l} \left(q_i^l\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\theta \cdot \sigma}{\sigma - 1}} \text{ s.t. } (2),$$

where $\theta \in (0, 1)$ and $\sigma(1 - \theta) > 1$. The second key difference is that, following Melitz (2003), selection is driven by fixed production costs rather than by a finite demand intercept. Specifically, a firm incurs a fixed cost f_D for domestic sales and a fixed cost f_X for foreign sales (exports), where $f_X > f_D > 0$. Firms are again heterogeneous, where firm productivities are drawn from a Pareto distribution, as in Chaney (2008), with support $[1, \infty)$ and dispersion parameter k where $1 + k - \sigma > 0$.

The two models behave similarly in many respects. Both models feature a selection effect and generate a Metzler paradox, and as a consequence many policy implications hold in common across the two models. The models differ importantly, however, when it comes to the entry-externality effect. For the MO model considered here, the sign of the entry-externality effect depends on model parameters (Proposition 4); by contrast, for the CES model considered by Bagwell and Lee (2018), the entry-externality effect is always positive. Thus, for the benchmark closed-economy setting, it is possible in the MO model that too much entry occurs in the market equilibrium, whereas the market always provides too little entry in the CES model.

Due to this difference, the two models provide different policy implications concerning the use and treatment of export subsidies in the two-country trade model. Starting at global free trade, if a country introduces a small export subsidy, then both models predict that the trading partner is hurt; however, the CES model implies that the introduction of a small export subsidy is always attractive to the intervening country whereas the MO model implies that whether a small export subsidy is attractive to the intervening country depends on parameters (Proposition 6). Likewise, starting at free-trade benchmarks, a small trade subsidy always raises joint welfare in the CES model but raises joint welfare in the MO model if and only if model parameters are such that $\alpha < 2 \cdot c_D^{FT}$ (Propositions 8 and 9).

The models thus provide different views regarding the treatment of export subsidies in the WTO. The MO model identifies circumstances leading to an effective and efficiency-based rationale for a prohibition on export subsidies: if countries start at global free trade and model parameters are such that $\alpha \geq 2 \cdot c_D^{FT}$, then the introduction of a small export subsidy could benefit the intervening country and yet lower joint welfare (Propositions 6 and 8). The CES model, however, fails to deliver a similar efficiency-based rationale for a restriction on export subsidies: in the CES model, if countries start at global free trade, then the introduction of a small export subsidy always benefits the intervening country and *raises* joint welfare.

What is the underlying source of the different entry-externality effects across the two models? In an Online Appendix for the current paper, we explore this issue. We consider there the closed-economy model and offer an intuitive but partial perspective. Our discussion proceeds in three steps, which we now briefly describe.

First, starting at the market equilibrium, we show that the MO model generates a positive entry externality if and only if additional entry raises aggregate profit, $N_E \cdot \bar{\pi}$. Starting at the market equilibrium, we show that the CES model generates a positive entry externality if additional entry raises aggregate profit, and we show also that additional entry indeed raises aggregate profit at the market equilibrium under CES preferences.

Second, we show that additional entry impacts aggregate profit both by changing the number of surviving firms and the expected profit of a firm conditional on its survival. In the MO model, the first channel is positive (more firms survive) and the second channel is negative (due to increased competition, firms expect lower profit conditional on survival). In the CES model, by contrast, the second channel is absent: conditional expected profit is constant with respect to the level of entry. The CES model thus shuts down a channel that lowers aggregate profit and that would otherwise work against a positive entry-externality effect. This finding offers a partial perspective for why additional entry can lower welfare in the MO model even while it always raises welfare in the CES model.

Third, to reinforce this perspective, we consider a firm confronting a demand function from the CREMR (Constant Revenue Elasticity of Marginal Revenue) family defined by Mrazova et al (2017) as $p^{cr}(q) = \frac{\beta}{q}(q-\psi)^{\frac{\sigma-1}{\sigma}}$, where $\beta > 0$, $\sigma > 1$ and $q > \psi\sigma$. As Mrazova et al (2017) observe, this family includes CES demand as a special case: when $\psi = 0$, the elasticity of demand is constant and equal to σ. We focus here on $\psi \geq 0$. We argue in this context that the property of constant expected profit conditional on survival can be explained by the role of a constant markup.

The associated profit-maximization problem is

$$\pi^{cr}(\varphi) = \max\left\{\max_q \left[\left(p^{cr}(q) - \frac{1}{\varphi}\right)q - f_D\right], 0\right\}$$

where $f_D > 0$ and the firm's productivity is φ. We assume that φ follows a Pareto distribution with shape parameter k (i.e. $G(\varphi) = 1 - \varphi^{-k}$) where $1 + k - \sigma > 0$. The resulting markup is variable and increasing in productivity, as in the MO model, when $\psi > 0$. By contrast, if $\psi = 0$, then the markup is constant with respect to productivity, as in the CES model.

Defining the productivity cutoff level φ^* by $\pi^{cr}(\varphi^*) = 0$, we show that the conditional expected profit is decreasing with respect to φ^* for this family except in the special case of $\psi = 0$ for which the conditional expected profit is constant with respect to φ^*. If we were to embed this analysis into a model of monopolistic competition for which greater entry raises φ^*, then the expected profit conditional on survival would be constant with respect to the level of entry only in the CES case. CES demand in this respect defines a knife-edge case.

8 Conclusion

We analyze unilateral, efficient and Nash trade policies in a symmetric, two-country version of the Melitz-Ottaviano (2008) model. Our characterizations are influenced by three driving forces corresponding to the selection effect, the firm-delocation effect, and the entry-externality effect. Starting at global free trade, we show that a country gains from the introduction of (1) a small import tariff; (2) a small export subsidy, if trade costs are low and the dispersion of productivities is high; and (3) an appropriately combined small increase in its import and export tariffs. The welfare of its trading partner, however, falls in each of these three cases. The market may provide too little or too much entry, depending on a simple relationship among model parameters. Correspondingly, global free trade is generally not efficient, even within the class of symmetric trade policies. We also provide conditions under which, starting at the symmetric Nash equilibrium, countries can mutually gain by exchanging small reductions in import tariffs, export tariffs or combinations thereof. A numerical example illustrates our findings and the possibility that Nash import and export tariffs both may be positive. More generally, we show that Nash equilibria are inefficient while "politically optimal" policies are efficient, indicating a central role for the terms-of-trade externality. We also discuss why the model's implications for the treatment of export subsidies in trade agreements differ from those that obtain in a model with CES preferences for the differentiated-goods sector.

9 Appendix

Lemma 2
$$\frac{dc_D^l}{d\tilde{t}^h}, \frac{dc_D^l}{dt^l} < 0 < \frac{dc_D^h}{dt^l}, \frac{dc_D^h}{d\tilde{t}^h}.$$

Proof: Using $\frac{\partial \chi^l}{\partial t^l} = \frac{1}{1-\tilde{t}^h} > 0$ and $\frac{\partial \chi^l}{\partial \tilde{t}^h} = \frac{1+t^l}{\left(1-\tilde{t}^h\right)^2} > 0$, we can sign the terms as follows:

$$\begin{aligned} \frac{dc_D^l}{dt^l} &= -\frac{(k+1)}{(k+2)} \frac{\rho^l \rho^h}{(1-\rho^l \rho^h)} \frac{c_D^l}{\chi^l} \cdot \frac{\partial \chi^l}{\partial t^l} < 0 \\ \frac{dc_D^l}{d\tilde{t}^h} &= -\frac{(k+1)}{(k+2)} \frac{\rho^l \rho^h}{(1-\rho^l \rho^h)} \frac{c_D^l}{\chi^l} \cdot \frac{\partial \chi^l}{\partial \tilde{t}^h} < 0 \end{aligned} \tag{65}$$

$$\begin{aligned} \frac{dc_D^h}{dt^l} &= \frac{(k+1)}{(k+2)} \frac{(1-\rho^h)\rho^l}{(1-\rho^l)(1-\rho^l \rho^h)} \frac{c_D^h}{\chi^l} \frac{\partial \chi^l}{\partial t^l} > 0 \\ \frac{dc_D^h}{d\tilde{t}^h} &= \frac{(k+1)}{(k+2)} \frac{(1-\rho^h)\rho^l}{(1-\rho^l)(1-\rho^l \rho^h)} \frac{c_D^h}{\chi^l} \frac{\partial \chi^l}{\partial \tilde{t}^h} > 0. \end{aligned} \tag{66}$$

■

Lemma 3
$$\frac{dc_X^l}{d\tilde{t}^l}, \frac{dc_X^l}{dt^h} < 0 < \frac{dc_X^l}{dt^l}, \frac{dc_X^l}{d\tilde{t}^h}.$$

Proof: Using $c_X^l = c_D^h(\frac{\rho^h}{\tau})^{\frac{1}{k+1}}$ from (17) along with (18) and Lemma 2, we find that

$$\begin{aligned} \frac{dc_X^l}{dt^l} &= \frac{dc_D^h}{dt^l} \frac{1}{\tau \cdot \chi^h} = \frac{1}{\tau \cdot \chi^h} \frac{(k+1)}{(k+2)} \frac{(1-\rho^h)\rho^l}{(1-\rho^l)(1-\rho^l \rho^h)} \frac{c_D^h}{\chi^l} \frac{\partial \chi^l}{\partial t^l} > 0 \\ \frac{dc_X^l}{d\tilde{t}^h} &= \frac{dc_D^h}{d\tilde{t}^h} \frac{1}{\tau \cdot \chi^h} = \frac{1}{\tau \cdot \chi^h} \frac{(k+1)}{(k+2)} \frac{(1-\rho^h)\rho^l}{(1-\rho^l)(1-\rho^l \rho^h)} \frac{c_D^h}{\chi^l} \frac{\partial \chi^l}{\partial \tilde{t}^h} > 0 \end{aligned}$$

where the inequalities follow given our maintained assumptions that $\chi^l > 0$ and $0 < \rho^l < 1$ for $l \in \{H, F\}$.■

Proof of Proposition 1 (Selection effect): See proof of Lemma 2.■

Proof of Proposition 2 (Firm-delocation effect): Our first step is to establish the comparative statistics results for N^l and N^h. Using (20) and Lemma 2, we obtain

$$\begin{aligned} \frac{dN^l}{dt^l} &= \frac{dN^l}{dc_D^l} \frac{dc_D^l}{dt^l} = -\frac{2\alpha\gamma(k+1)}{\eta(c_D^l)^2} \frac{dc_D^l}{dt^l} > 0 \\ \frac{dN^l}{d\tilde{t}^h} &= \frac{dN^l}{dc_D^l} \frac{dc_D^l}{d\tilde{t}^h} = -\frac{2\alpha\gamma(k+1)}{\eta(c_D^l)^2} \frac{dc_D^l}{d\tilde{t}^h} > 0 \end{aligned} \tag{67}$$

$$\begin{aligned}
\frac{dN^h}{dt^l} &= \frac{dN^h}{dc_D^h}\frac{dc_D^h}{dt^l} = -\frac{2\alpha\gamma(k+1)}{\eta(c_D^h)^2}\frac{dc_D^h}{dt^l} < 0 \\
\frac{dN^h}{d\tilde{t}^h} &= \frac{dN^h}{dc_D^h}\frac{dc_D^h}{d\tilde{t}^h} = -\frac{2\alpha\gamma(k+1)}{\eta(c_D^h)^2}\frac{dc_D^h}{d\tilde{t}^h} < 0
\end{aligned} \tag{68}$$

Our second step is to establish our comparative statics results for N_E^l and N_E^h. We suppose that t^l increases. Then $\chi^l \Uparrow$, $\rho^l \Downarrow$, $c_D^l \Downarrow$, $c_D^h \Uparrow$ and $c_X^h \Downarrow$ by (18), Lemma 2 and Lemma 3. Recalling $\xi^l = \rho^l\chi^l$, we can also easily derive that $\xi^l \Downarrow$.

By $\xi^l \Downarrow$, $c_D^l \Downarrow$ and $c_D^h \Uparrow$, the number of entrants in country h decreases: $N_E^h \Downarrow$. To see this, we use (67) and (68) and refer to (22):

$$N_E^h = \frac{(c_M)^k}{1-\xi^l\xi^h} \Downarrow \cdot \left[\frac{N^h}{(c_D^h)^k} \Downarrow - \frac{\xi^h N^l}{(c_D^l)^k} \Uparrow\right] \Downarrow$$

where the bracketed expression is positive since $N_E^h > 0$. Thus, $N_E^h \Downarrow$.

Referring to (21), we now use (67), $N_E^h \Downarrow$, $c_D^l \Downarrow$ and $c_X^h \Downarrow$ to find that

$$N^l \Uparrow = G\left(c_D^l \Downarrow\right) \cdot N_E^l + G\left(c_X^h \Downarrow\right) \cdot N_E^h \Downarrow .$$

It follows that that $N_E^l \Uparrow$.

Now we suppose that $\tilde{t}^h$ increases. Then $\chi^l \Uparrow$, $\rho^l \Downarrow$, $c_D^l \Downarrow$, $c_D^h \Uparrow$ and $c_X^h \Downarrow$ by (18), Lemma 2 and Lemma 3. Recalling $\xi^l = \rho^l\chi^l$, we can also easily derive that $\xi^l \Downarrow$. Therefore, using (67) and (68) and referring as above to (21) and (22), we may argue as above to conclude that $N_E^h \Downarrow$ and $N_E^l \Uparrow$.■

Proof of Proposition 3 (Metzler paradox): As captured in (19), there is a one-to-one relation between the average price and the cut-off cost level for domestic sales:

$$\bar{p}^l = \frac{2k+1}{2k+2} \cdot c_D^l.$$

The proof now follows directly from Proposition 1. ■

Lemma 4 *Consumer Surplus decreases with the cut-off cost level for domestic sales:* $\frac{dCS^l}{dc_D^l} < 0$.

Proof: Using (26), we find that

$$\frac{dCS^l}{dc_D^l} = \frac{2(1+k)c_D^l - (3+2k)\alpha}{2(2+k)\eta} < 0 \tag{69}$$

where the inequality follows from $\alpha > c_D^l$ (or equivalently from $N^l > 0$).■

Proof of Proposition 4 (Entry-externality effect): We begin by confirming that $\overline{CS}$, VE, N_E and $\bar{\pi}$ can all be regarded as functions of c_D. Following (33) and using $q_D(c) = (c_D - c)/(2\gamma)$, the expected consumer surplus at a single variety, $\overline{CS}$, can be calculated by integration as

$$\overline{CS} = \frac{\gamma}{2} \int_0^{C_D} (q_D(c))^2 dG(c) = \frac{(c_M)^{-k}(c_D)^{k+2}}{4\gamma(k+1)(k+2)}. \tag{70}$$

Referring to (21) and setting $N_E^h \equiv 0$ while replacing N_E^l with N_E, N^l with N and c_D^l with c_D, we have that

$$N_E = \frac{N}{G(c_D)} = \frac{2(1+k)\gamma(c_M)^k}{\eta} \frac{(\alpha - c_D)}{(c_D)^{k+1}}, \tag{71}$$

where we use (20) to express N in terms of c_D after similar variable replacements. The variety effect, VE, is derived as the difference between consumer surplus and the sum of consumer surplus at single varieties. Using the expression for consumer surplus in (26) and that for N_E in (71), and after making similar variable replacements, we get

$$VE = CS - N_E \cdot \overline{CS} = \frac{(\alpha - c_D)^2}{2\eta}, \tag{72}$$

where we also use (70). Finally, referring to (15) after setting $c_X^l = 0$ and after also replacing $\bar{\pi}^l$ with $\bar{\pi}$ and $\pi_D^l(c)$ with $\pi_D(c) = (c_D - c)^2/(4\gamma)$, respectively, we get that

$$\bar{\pi} = \frac{(c_M)^{-k}(c_D)^{k+2}}{2\gamma(k+1)(k+2)}. \tag{73}$$

With these derivations in place, we define

$$EXT \equiv \overline{CS} + \frac{dVE}{dN_E} + N_E \frac{d\overline{CS}}{dN_E} + N_E \frac{d\bar{\pi}}{dN_E}$$

and proceed next to sign each term in this expression. From (70), it is evident that

$$\overline{CS} = \frac{(c_M)^{-k}(c_D)^{k+2}}{4\gamma(k+1)(k+2)} > 0$$

Using the implicit function theorem, and employing (71) while using $\alpha > c_D$ (i.e., $N > 0$), we obtain

$$\frac{dc_D}{dN_E} = (\frac{dN_E}{dc_D})^{-1} = -(\frac{2(1+k)\gamma(c_M)^k(\alpha(1+k) - kc_D)}{\eta(c_D)^{k+2}})^{-1} < 0. \tag{74}$$

Combining (74) with (70), (72) and (73), we find

$$\frac{dVE}{dN_E} = \frac{dVE}{dc_D}\frac{dc_D}{dN_E} = -\frac{(\alpha - c_D)}{\eta}\frac{dc_D}{dN_E} > 0$$

$$\frac{d\overline{CS}}{dN_E} = \frac{d\overline{CS}}{dc_D}\frac{dc_D}{dN_E} = \frac{(c_M)^{-k}(c_D)^{k+1}}{4\gamma(k+1)}\frac{dc_D}{dN_E} < 0$$

$$\frac{d\bar{\pi}}{dN_E} = \frac{d\bar{\pi}}{dc_D}\frac{dc_D}{dN_E} = \frac{(c_M)^{-k}(c_D)^{k+1}}{2\gamma(k+1)}\frac{dc_D}{dN_E} < 0.$$

We have thus now signed each term in EXT.

We now proceed to sign EXT. To this end, we observe from (72) that

$$\frac{dCS}{dN_E} = \overline{CS} + \frac{dVE}{dN_E} + N_E\frac{d\overline{CS}}{dN_E},$$

and so we can write

$$EXT = \frac{dCS}{dN_E} + N_E\frac{d\bar{\pi}}{dN_E} = (\frac{dCS}{dc_D} + N_E\frac{d\bar{\pi}}{dc_D})\frac{dc_D}{dN_E}.$$

It now follows from (74) that

$$sign\{EXT\} = -sign\{\frac{dCS}{dc_D} + N_E\frac{d\bar{\pi}}{dc_D}\}.$$

Using (69) after replacing CS^l with CS and c_D^l with c_D, respectively, and using (71) and (73), we find that $N_E\frac{\partial\bar{\pi}}{\partial c_D} = (\alpha - c_D)/\eta$ and thus

$$\frac{dCS}{dc_D} + N_E\frac{d\bar{\pi}}{dc_D} = \frac{\alpha - 2c_D}{2\eta(k+2)},$$

whence

$$sign\{EXT\} = -sign\{\alpha - 2c_D\}$$

We thus have that $EXT < 0$ if and only if $\alpha - 2c_D > 0$. Finally, to relate this derivation to the statement of Proposition 4, we may fix c_D at the market-equilibrium level determined by the free-entry condition: $c_D = c_D^m$. ■

Proof of Proposition 5 (Small import tariff): We consider first the incentive for country l to impose a small import tariff, given an initial situation of global free trade. Using (29), we find that

$$\frac{dU^l}{dt^l} = \frac{d}{dt^l}[CS^l + t^l \cdot IMP^l + \tilde{t}^l \cdot EXP^l].$$

Thus, given an initial situation of global free trade, we have that

$$\begin{aligned}\frac{dU^l}{dt^l} \quad | \quad {}_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^l}{dt^l} + IMP^l \, |_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \\ &= \frac{dCS^l}{dc_D^l}\frac{dc_D^l}{dt^l} + IMP^l \, |_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} > 0,\end{aligned}$$

where the inequality follows from Lemma 2, Lemma 4 and $IMP^l > 0$ (by $N_E^h > 0$). Thus, country l gains from the introduction of a small import tariff, starting at global free trade.

Next, we consider the effect on country h when country l departs from global free trade and introduces a small import tariff. Using (29), the externality of an increase in country l's import tariff is given by

$$\frac{dU^h}{dt^l} = \frac{d}{dt^l}[CS^h + t^h \cdot IMP^h + \tilde{t}^h \cdot EXP^h].$$

Starting from global free trade, we then have that

$$\begin{aligned}\frac{dU^h}{dt^l} \quad | \quad {}_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^h}{dt^l} \, |_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \\ &= \frac{dCS^h}{dc_D^h}\frac{dc_D^h}{dt^l} \, |_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} < 0\end{aligned}$$

where the inequality follows from Lemma 2 and Lemma 4. Thus, starting at global free trade, country h is harmed when country l introduces a small import tariff.■

Proof of Proposition 6 (Small export subsidy): We begin with part 2 of the proposition and show that, starting at global free trade, country h suffers a welfare loss when country l introduces a small export subsidy. Using (29), we find that the externality of an increase in country l's export tariff is given by

$$\frac{dU^h}{d\tilde{t}^l} = \frac{d}{d\tilde{t}^l}[CS^h + t^h \cdot IMP^h + \tilde{t}^h \cdot EXP^h].$$

Starting from global free trade, we then have that

$$\begin{aligned}\frac{dU^h}{d\tilde{t}^l} \quad | \quad {}_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^h}{d\tilde{t}^l} \, |_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \\ &= \frac{dCS^h}{dc_D^h}\frac{dc_D^h}{d\tilde{t}^l} \, |_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} > 0\end{aligned}$$

where the inequality follows from Lemma 2 and Lemma 4. Thus, starting at global free trade, country h gains when country l introduces a small export tariff. Equivalently, from this starting point, country h loses when country l introduces a small export subsidy.

We turn now to part 1 of the proposition and determine conditions under which country l gains from breaking from global free trade and introducing a small export subsidy. Using (29), we find that

$$\frac{dU^l}{d\tilde{t}^l} = \frac{d}{d\tilde{t}^l}[CS^l + t^l \cdot IMP^l + \tilde{t}^l \cdot EXP^l].$$

Thus, given an initial situation of global free trade, we have that

$$\begin{aligned}\frac{dU^l}{d\tilde{t}^l} \quad | \quad_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^l}{d\tilde{t}^l} + EXP^l \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \\ &= \frac{dCS^l}{dc_D^l}\frac{dc_D^l}{d\tilde{t}^l} + EXP^l \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0},\end{aligned}$$

where the first term is negative under Lemma 2 and Lemma 4 while the second term is positive: $EXP^l > 0$ (by $N_E^l > 0$). Thus, it is not immediately clear whether country l gains from the introduction of a small export subsidy, even when starting at global free trade.

To go further, we use (20), (22), (28), (65) and (69) to get

$$\begin{aligned}\frac{dU^l}{d\tilde{t}^l} \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^l}{d\tilde{t}^l} + EXP^l \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \\ = \frac{(1+k)\,c_D^{FT}}{2\,(2+k)^2\,\eta\,(\tau^{2k}-1)}&[\alpha\left(\tau^k-2k-4\right)+2c_D^{FT}\left(2+k-\tau^k\right)]\end{aligned}$$

where $c_D^{FT} = c_D^l \mid_{t^h=\tilde{t}^h=t^f=\tilde{t}^f=0}$. The sign of the optimal unilateral export policy is thus determined by the expression in the brackets. Country l thus gains from the introduction of a small export subsidy if and only if

$$\alpha\left(\tau^k - 2(k+2)\right) < c_D^{FT}\left(2\tau^k - 2(k+2)\right). \tag{75}$$

Suppose first that $\tau^k \geq 2(k+2)$ or equivalently that $\tau \geq (4+2k)^{\frac{1}{k}}$. If $\tau^k = 2(k+2)$, then the LHS of (75) is zero and the RHS of (75) is positive, whence the introduction of a small export subsidy benefits country l. If $\tau^k > 2(k+2)$, then the LHS and RHS of (75) are both positive, and we may confirm that (75) holds, and thus the introduction of a small export subsidy benefits country l, if and only if

$$\alpha < \left(1 + \frac{\tau^k}{\tau^k - 2(k+2)}\right) c_D^{FT},$$

which is simply inequality (37) in the statement of Proposition 6. We have thus now established that the introduction of a small export subsidy benefits country l in the weak selection effect defined in part 1b of Proposition 6.

Suppose second that $\tau^k < 2(k+2)$ or equivalently that $\tau < (4+2k)^{\frac{1}{k}}$. A first subcase is that $k+2 \leq \tau^k < 2(k+2)$. In this subcase, the LHS of (75) is negative whereas the RHS is non-negative; thus, (75) holds in this subcase. A second subcase is that $\tau^k < k+2 < 2(k+2)$. Both the LHS and RHS of (75) are then negative, so that (75) holds if and only if

$$\alpha > \left(1 + \frac{\tau^k}{\tau^k - 2(k+2)}\right) c_D^{FT}.$$

This inequality is sure to hold for the subcase under consideration, given $\alpha > c_D^{FT}$. Thus, in the strong selection setting where $\tau < [4+2k]^{\frac{1}{k}}$, as considered in part 1a of Proposition 6, the introduction of a small export subsidy benefits country l.■

Proof of Lemma 1: Using (26), (29) and that $IMP^l = EXP^h$ (as may be easily verified from (27) and (28)), we have that

$$U^l + U^h = 2 + (t^l + \widetilde{t}^h) IMP^l + (\widetilde{t}^l + t^h) EXP^l + CS^l + CS^h. \tag{76}$$

Referring to (27) and (28), we thus have that

$$U^l + U^h = 2 + \frac{(t^l + \widetilde{t}^h)}{(1+t^l)} \frac{N_E^h (\tau\chi^l)^{-k} (c_D^l)^{k+2}}{2\gamma(k+2)(c_M)^k} + \frac{(\widetilde{t}^l + t^h)}{(1+t^h)} \frac{N_E^l (\tau\chi^h)^{-k} (c_D^h)^{k+2}}{2\gamma(k+2)(c_M)^k} + CS^l + CS^h.$$

Next, we observe that

$$\frac{(t^l + \widetilde{t}^h)}{(1+t^l)} = \frac{\chi^l - 1}{\chi^l},$$

and so

$$U^l + U^h = 2 + \frac{\chi^l - 1}{\chi^l} \frac{N_E^h (\tau\chi^l)^{-k} (c_D^l)^{k+2}}{2\gamma(k+2)(c_M)^k} + \frac{\chi^h - 1}{\chi^h} \frac{N_E^l (\tau\chi^h)^{-k} (c_D^h)^{k+2}}{2\gamma(k+2)(c_M)^k} + CS^l + CS^h. \tag{77}$$

The result now follows, since c_D^l, c_D^h, CS^l, CS^h, N_E^l and N_E^h likewise depend on tariffs only through χ^l and χ^h.■

Proof of Proposition 8 (Free trade and efficiency): Using (69) and (76), straightforward calculations yield that

$$\frac{dU^l + U^h}{dt^l} \mid_{t^l = t^h = \tilde{t}^l = \tilde{t}^h = 0} = IMP^l + \frac{2(1+k)c_D^{FT} - (3+2k)\alpha}{2(2+k)\eta} \left(\frac{dc_D^l}{dt^l} + \frac{dc_D^h}{dt^l}\right) |_{t^l = t^h = \tilde{t}^l = \tilde{t}^h = 0}$$

$$\frac{dU^l + U^h}{d\tilde{t}^l} \mid_{t^l = t^h = \tilde{t}^l = \tilde{t}^h = 0} = EXP^l + \frac{2(1+k)c_D^{FT} - (3+2k)\alpha}{2(2+k)\eta} \left(\frac{dc_D^l}{d\tilde{t}^l} + \frac{dc_D^h}{d\tilde{t}^l}\right) |_{t^l = t^h = \tilde{t}^l = \tilde{t}^h = 0}.$$

Using (65), (66) and that $EXP^l \mid_{t^h = \tilde{t}^h = t^l = \tilde{t}^l = 0} = IMP^l \mid_{t^h = \tilde{t}^h = t^l = \tilde{t}^l = 0}$ (as may be easily verified from (17), (18), (23), $\xi^l = \xi^h$ given $t^h = \widetilde{t}^h = t^l = \widetilde{t}^l = 0$, (27) and (28)), we now find that

$$\frac{dU^l + U^h}{dt^l}\mid_{t^l=t^h=\tilde{t}^l=\tilde{t}^h=0} = \frac{dU^l + U^h}{d\tilde{t}^l}\mid_{t^l=t^h=\tilde{t}^l=\tilde{t}^h=0} = \frac{(k+1)}{(k+2)^2}\frac{\tau^{-k}(1-\tau^{-k})}{(1-\tau^{-2k})}\frac{c_D^{FT}}{2\eta}\left(\alpha - 2c_D^{FT}\right).$$

Therefore $\frac{dU^l+U^h}{dt^l}\mid_{t^l=t^h=\tilde{t}^l=\tilde{t}^h=0} = \frac{dU^l+U^h}{d\tilde{t}^l}\mid_{t^l=t^h=\tilde{t}^l=\tilde{t}^h=0} > 0$ if and only if $\alpha - 2c_D^{FT} > 0$. ∎

Proof of Proposition 9 (Free trade and efficiency under χ-symmetric policies): It is convenient to introduce notation that captures the dependence of key functions on $\chi = \chi^H = \chi^F$ when χ-symmetric tariffs are used. To this end, we define

$$\begin{aligned}
\rho(\chi) &\equiv (\tau)^{-k}(\chi)^{-(k+1)} \\
\xi(\chi) &\equiv \rho(\chi)\chi \\
c_D(\chi^l, \chi^h) &\equiv [\frac{\phi\gamma(1-\rho(\chi^h))}{1-\rho(\chi^l)\rho(\chi^h)}]^{\frac{1}{k+2}} \\
CS(\chi^l, \chi^h) &\equiv \frac{(\alpha - c_D(\chi^l, \chi^h))}{2\eta}[\alpha - c_D(\chi^l, \chi^h)\frac{k+1}{k+2}].
\end{aligned}$$

Recalling that $\chi^l = (1+t^l)/(1-\tilde{t}^h)$, we may use (17), (18) and (26) to confirm that $\rho(\chi^l) = \rho^l$, $\xi(\chi^l) = \rho(\chi^l)\chi^l = \rho^l\chi^l = \xi^l$, $c_D(\chi^l, \chi^h) = c_D^l$ and $CS(\chi^l, \chi^h) = CS^l$.

Similarly, turning to entry and trade-volume variables, we may define

$$\begin{aligned}
N_E(\chi^l, \chi^h) &\equiv \frac{2(k+1)(c_M)^k\gamma}{\eta[1-\xi(\chi^l)\xi(\chi^h)]}\left(\frac{\alpha - c_D(\chi^l, \chi^h)}{(c_D(\chi^l, \chi^h))^{k+1}} - \frac{\xi(\chi^l)(\alpha - c_D(\chi^h, \chi^l))}{(c_D(\chi^h, \chi^l))^{k+1}}\right) \\
f(\chi^l, \chi^h) &\equiv \frac{N_E(\chi^h, \chi^l)(\tau\chi^l)^{-k}(c_D(\chi^l, \chi^h))^{k+2}}{2\gamma(k+2)(c_M)^k}
\end{aligned}$$

where by (23), (27) and (28), $N_E(\chi^l, \chi^h) = N_E^l$ and $f(\chi^l, \chi^h) = (1+t^l)IMP^l = (1+t^l)EXP^h$. Thus, we may understand $f(\chi^l, \chi^h)$ as measuring the value of trade into country l when using delivered (consumer) prices.

At this point, we may follow the proof of Lemma 1 and use our definitions above to define joint welfare,

$$U(\chi^l, \chi^h) \equiv 2 + \frac{\chi^l - 1}{\chi^l}f(\chi^l, \chi^h) + \frac{\chi^h - 1}{\chi^h}f(\chi^h, \chi^l) + CS(\chi^l, \chi^h) + CS(\chi^h, \chi^l),$$

where $U(\chi^l, \chi^h) = U^l + U^h$. Finally, at χ-symmetric tariffs, we have

$$U(\chi, \chi) = 2[1 + \frac{\chi - 1}{\chi}f(\chi, \chi) + CS(\chi, \chi)], \tag{78}$$

and with this expression we may evaluate efficiency relative to the class of χ-symmetric tariffs.

To this end, we observe from (78) that

$$\frac{dU(\chi,\chi)}{d\chi} = 2[\frac{\chi-1}{\chi}\frac{d}{d\chi}f(\chi,\chi) + \frac{f(\chi,\chi)}{(\chi)^2} + \frac{d}{d\chi}CS(\chi,\chi)] \tag{79}$$

and proceed now to further characterize the bracketed expression.

To characterize $\frac{d}{d\chi}CS(\chi,\chi)$, we begin by showing that

$$\frac{\partial c_D(\chi^l,\chi^h)}{\partial \chi^l} = \frac{c_D(\chi^l,\chi^h)}{k+2}\frac{\rho'(\chi^l)\rho(\chi^h)}{1-\rho(\chi^l)\rho(\chi^h)} < 0 \tag{80}$$

$$\frac{\partial c_D(\chi^l,\chi^h)}{\partial \chi^h} = \frac{c_D(\chi^l,\chi^h)}{k+2}\frac{-\rho'(\chi^h)(1-\rho(\chi^l))}{(1-\rho(\chi^h))(1-\rho(\chi^l)\rho(\chi^h))} > 0, \tag{81}$$

from which it follows that

$$\frac{dc_D(\chi,\chi)}{d\chi} = \frac{-c_D(\chi,\chi)\rho'(\chi)}{(k+2)(1+\rho(\chi))} > 0. \tag{82}$$

We now have that

$$\frac{d}{d\chi}CS(\chi,\chi) = \frac{dc_D(\chi,\chi)}{d\chi}[\frac{2(1+k)c_D(\chi,\chi)-(3+2k)\alpha}{2\eta(k+2)}] < 0, \tag{83}$$

where the inequality follows from (69) given $c_D(\chi^l,\chi^h) = c_D^l$ and (82).

Starting at tariffs that achieve free trade in that $\chi = 1$, and using (18), (23), (79), (82) and (83), we obtain

$$\begin{aligned}
\frac{dU(\chi,\chi)}{d\chi}|_{\chi=1} &= 2[f(1,1) + \frac{d}{d\chi}CS(\chi,\chi)|_{\chi=1}] \\
&= 2[f(1,1) + \frac{dc_D(\chi,\chi)}{d\chi}|_{\chi=1}(\frac{2(1+k)c_D(1,1)-(3+2k)\alpha}{2\eta(k+2)})] \\
&= \frac{c_D(1,1)\tau^{-k}(k+1)}{(k+2)^2\eta(1+\rho(1))}[\alpha - 2c_D(1,1)] \\
&= \frac{c_D^{FT}\tau^{-k}(k+1)}{(k+2)^2\eta(1+\rho(1))}[\alpha - 2c_D^{FT}].
\end{aligned}$$

■

Proof of Proposition 10 (symmetric Nash policies and efficiency): We begin by considering country l's welfare. Referring to (29), we have that

$$U^l = 1 + t^l \cdot IMP^l + \tilde{t}^l \cdot EXP^l + \frac{(\alpha - c_D^l)}{2\eta}[\alpha - c_D^l \frac{k+1}{k+2}].$$

Using the definitions developed in the proof of Proposition 9, we may now re-write country l's welfare as

$$U^l = 1 + \frac{t^l}{1+t^l} \cdot f(\chi^l, \chi^h) + \frac{\tilde{t}^l}{1+t^h} \cdot f(\chi^h, \chi^l) + CS(\chi^l, \chi^h),$$

where we recall that $\chi^l = (1+t^l)/(1-\tilde{t}^h)$. Notice that country l's welfare cannot be expressed as a function only of χ^l and χ^h.

The Nash first-order conditions for country l's optimal import and export tariffs are given as follows:

$$\begin{aligned}
\frac{dU^l}{dt^l} &= \frac{f(\chi^l, \chi^h)}{1+t^l} + t^l \frac{d}{dt^l}[\frac{f(\chi^l, \chi^h)}{1+t^l}] + \frac{\tilde{t}^l}{1+t^h}\frac{df(\chi^h, \chi^l)}{dt^l} + \frac{dCS(\chi^l, \chi^h)}{dt^l} = 0 \\
\frac{dU^l}{d\tilde{t}^l} &= \frac{t^l}{1+t^l}\frac{df(\chi^l, \chi^h)}{d\tilde{t}^l} + \frac{f(\chi^h, \chi^l)}{1+t^h} + \frac{\tilde{t}^l}{1+t^h}\frac{df(\chi^h, \chi^l)}{d\tilde{t}^l} + \frac{dCS(\chi^l, \chi^h)}{d\tilde{t}^l} = 0.
\end{aligned}$$

We can now re-write these first-order conditions as

$$\frac{dU^l}{dt^l} = \frac{f(\chi^l, \chi^h)}{(1+t^l)^2} + [\frac{t^l}{1+t^l} f_1(\chi^l, \chi^h) + \frac{\tilde{t}^l}{1+t^h} f_2(\chi^h, \chi^l) + CS_1(\chi^l, \chi^h)]\frac{\partial \chi^l}{\partial t^l} = 0 \quad (84)$$

$$\frac{dU^l}{d\tilde{t}^l} = \frac{f(\chi^h, \chi^l)}{1+t^h} + [\frac{t^l}{1+t^l} f_2(\chi^l, \chi^h) + \frac{\tilde{t}^l}{1+t^h} f_1(\chi^h, \chi^l) + CS_2(\chi^l, \chi^h)]\frac{\partial \chi^h}{\partial \tilde{t}^l} = 0. \quad (85)$$

Using that $\frac{\partial \chi^l}{\partial t^l} = \frac{1}{1-\tilde{t}^h} > 0$ and $\frac{\partial \chi^h}{\partial \tilde{t}^l} = \frac{1+t^h}{(1-\tilde{t}^l)^2} > 0$ under our assumptions, we may add the Nash first-order conditions, re-arrange terms and find the following necessary condition for the Nash equilibrium:

$$\begin{aligned}
0 &= \frac{f(\chi^l, \chi^h)}{\chi^l(1+t^l)} + \frac{f(\chi^h, \chi^l)}{(\chi^h)^2} + \frac{t^l}{1+t^l}[f_1(\chi^l, \chi^h) + f_2(\chi^l, \chi^h)] \\
&\quad + \frac{\tilde{t}^l}{1+t^h}[f_1(\chi^h, \chi^l) + f_2(\chi^h, \chi^l)] + CS_1(\chi^l, \chi^h) + CS_2(\chi^l, \chi^h).
\end{aligned}$$

We are interested in the symmetric Nash equilibrium tariffs, $t^N = t^H = t^F$ and $\tilde{t}^N = \tilde{t}^H = \tilde{t}^F$. With $\chi^N = \frac{1+t^N}{1-\tilde{t}^N}$, the necessary condition under a symmetric Nash equilibrium takes the form

$$\frac{f(\chi^N,\chi^N)}{\chi^N(1+t^N)}+\frac{f(\chi^N,\chi^N)}{(\chi^N)^2}+\frac{t^N+\tilde{t}^N}{1+t^N}[\frac{df(\chi,\chi)}{d\chi}|_{\chi=\chi^N}]+\frac{dCS(\chi,\chi)}{d\chi}|_{\chi=\chi^N}=0.$$

Finally, using that $\frac{t^N+\tilde{t}^N}{1+t^N}=\frac{\chi^N-1}{\chi^N}$ and re-arranging slightly, we may express the necessary condition for a symmetric Nash equilibrium in the following form:

$$\frac{f(\chi^N,\chi^N)}{\chi^N(1+t^N)}+\frac{\chi^N-1}{\chi^N}[\frac{df(\chi,\chi)}{d\chi}|_{\chi=\chi^N}]+\frac{f(\chi^N,\chi^N)}{(\chi^N)^2}+\frac{dCS(\chi,\chi)}{d\chi}|_{\chi=\chi^N}=0$$

Recalling now (79) from the proof of Proposition 9, we may express the first-order condition for efficient χ-symmetric tariffs in the following form:

$$\frac{dU(\chi,\chi)}{d\chi}=2[\frac{\chi-1}{\chi}\frac{d}{d\chi}f(\chi,\chi)+\frac{f(\chi,\chi)}{(\chi)^2}+\frac{d}{d\chi}CS(\chi,\chi)]=0.$$

This first-order condition determines the efficient value, χ^*. It is now straightforward to see that

$$\frac{dU(\chi,\chi)}{d\chi}|_{\chi=\chi^N}=-2\frac{f(\chi^N,\chi^N)}{\chi^N(1+t^N)}<0.$$

It follows that the symmetric Nash equilibrium is inefficient. Furthermore, if the joint welfare function $U(\chi,\chi)$ is quasi-concave in the symmetric value χ, then $\chi^N>\chi^*$. Accordingly, starting at the symmetric Nash equilibrium, any combination of tariffs changes that results in a symmetric reduction in $\chi^H=\chi^F$ would raise joint welfare. Both countries are then sure to gain if they exchange small and symmetric changes in tariffs that reduce $\chi^H=\chi^F$.■

Lemma 5 *If $\tilde{t}^N\geq 0$, then $t^N>\tilde{t}^N$.*

Proof: The Nash first-order conditions for country l are given by (84) and (85). Evaluating (84) and (85) at symmetric trade policies and multiplying (84) by $(1+t)$ and (85) by $\left(1-\tilde{t}\right)$, respectively, we get

$$\frac{f(\chi,\chi)}{(1+t)}+[\frac{t}{1+t}f_1(\chi,\chi)+\frac{\tilde{t}}{1+t}f_2(\chi,\chi)+CS_1(\chi,\chi)]\chi = 0 \quad (86)$$

$$\frac{f(\chi,\chi)}{\chi}+[\frac{t}{1+t}f_2(\chi,\chi)+\frac{\tilde{t}}{1+t}f_1(\chi,\chi)+CS_2(\chi,\chi)]\chi = 0. \quad (87)$$

We now subtract (87) from (86) to get

$$\frac{\tilde{t}}{1+t}\cdot f(\chi,\chi)+[CS_1(\chi,\chi)-CS_2(\chi,\chi)]\chi=\frac{t-\tilde{t}}{1+t}[f_2(\chi,\chi)-f_1(\chi,\chi)] \quad (88)$$

where using (69), (80) and (81) gives

$$\begin{aligned} CS_1(\chi,\chi) &= \frac{dCS^l}{dc_D^l}\frac{dc_D^l}{d\chi^l}|_{\chi^l=\chi^h=\chi} > 0 \\ CS_2(\chi,\chi) &= \frac{dCS^l}{dc_D^l}\frac{dc_D^l}{d\chi^h}|_{\chi^l=\chi^h=\chi} < 0. \end{aligned}$$

Hence, $CS_1(\chi,\chi) - CS_2(\chi,\chi) > 0$. Now we show $f_2(\chi,\chi) - f_1(\chi,\chi) > 0$. Recall the functional form of $f(\chi^l,\chi^h)$ is given by

$$f(\chi^l,\chi^h) = N_E(\chi^h,\chi^l)\frac{(\tau\chi^l)^{-k}(c_D(\chi^l,\chi^h))^{k+2}}{2\gamma(k+2)(c_M)^k}$$

where $N_E(\chi^h,\chi^l) = N_E^h$ and $c_D(\chi^l,\chi^h) = c_D^l$. Consider the two cases:

1. Suppose χ^l increases. Then $N_E(\chi^h,\chi^l) = N_E^h \Downarrow$ (by Proposition 2 and using $\chi^l = (1+t^l)/(1-\tilde{t}^h)$), $(\tau\chi^l)^{-k} \Downarrow$, and $(c_D(\chi^l,\chi^h))^{k+2} = (c_D^l)^{k+2} \Downarrow$. We conclude that $f_1(\chi^l,\chi^h)|_{\chi^l=\chi^h=\chi} < 0$.
2. Suppose χ^h increases. Then $N_E(\chi^h,\chi^l) = N_E^h \Uparrow$ (by Proposition 2 and using $\chi^h = (1+t^h)/(1-\tilde{t}^l)$), and $(c_D(\chi^l,\chi^h))^{k+2} = (c_D^l)^{k+2} \Uparrow$. We conclude that $f_2(\chi^l,\chi^h)|_{\chi^l=\chi^h=\chi} > 0$.

Therefore, $f_2(\chi,\chi) - f_1(\chi,\chi) > 0$. We now conclude from (88) that, if $\tilde{t} \geq 0$, then $t - \tilde{t} > 0$.■

Proof of Proposition 12 (Nash tariff ranking): Lemma 5 already shows $t^N > \tilde{t}^N$ if $\tilde{t}^N \geq 0$. In order to show $t^N > \tilde{t}^N$ if $\tilde{t}^N < 0$, we assume $\alpha \geq 2 \cdot c_D^{FT}$ and $U^H + U^F$ is quasi-concave in χ. Suppose $\tilde{t}^N < 0$. We know $\chi^N = \frac{1+t^N}{1-\tilde{t}^N} > 1$ by Propositions 9 and 10, and so $t^N \geq -\tilde{t}^N$ follows. Therefore, $t^N \geq -\tilde{t}^N > 0 > \tilde{t}^N$. ■

Proof of Proposition 13 (Nash in the limit): Rescaling the objectives, we note that the symmetric Nash import and export tariffs at $\tau \to \infty$ are characterized by $\lim_{\tau\to\infty}\tau^k\frac{dU^l}{dt^l} = \lim_{\tau\to\infty}\tau^k\frac{dU^l}{d\tilde{t}^l} = 0$, rewritten as:

$$\begin{aligned} \lim_{\tau\to\infty}\tau^k\frac{dCS^l}{dc_D^l}\frac{dc_D^l}{dt^l} + \lim_{\tau\to\infty}\tau^k\cdot IMP^l + t^l\lim_{\tau\to\infty}\tau^k\frac{dIMP^l}{dt^l} + \tilde{t}^l\lim_{\tau\to\infty}\tau^k\frac{dEXP^l}{dt^l} &= 0 \quad (89) \\ \lim_{\tau\to\infty}\tau^k\frac{dCS^l}{dc_D^l}\frac{dc_D^l}{d\tilde{t}^l} + \lim_{\tau\to\infty}\tau^k\cdot EXP^l + t^l\lim_{\tau\to\infty}\tau^k\frac{dIMP^l}{d\tilde{t}^l} + \tilde{t}^l\lim_{\tau\to\infty}\tau^k\frac{dEXP^l}{d\tilde{t}^l} &= 0. \end{aligned}$$

We calculate each term of (89).

To begin, we refer to (65) and (66), and consider the change of the cut-off cost levels:

$$\begin{aligned}
\lim_{\tau\to\infty}\frac{dc_D^l}{dt^l} &= \lim_{\tau\to\infty}\frac{dc_D^h}{dt^l}=\lim_{\tau\to\infty}\frac{dc_D^l}{d\tilde{t}^l}=\lim_{\tau\to\infty}\frac{dc_D^h}{d\tilde{t}^l}=0\\
\lim_{\tau\to\infty}\tau^k\frac{dc_D^l}{dt^l} &= 0\\
\lim_{\tau\to\infty}\tau^k\frac{dc_D^h}{dt^l} &= \frac{(k+1)}{(k+2)}\left(\chi^l\right)^{-(k+2)}c_D^m\frac{\partial\chi^l}{\partial t^l}>0\\
\lim_{\tau\to\infty}\tau^k\frac{dc_D^l}{d\tilde{t}^l} &= \frac{(k+1)}{(k+2)}\left(\chi^h\right)^{-(k+2)}c_D^m\frac{\partial\chi^h}{\partial\tilde{t}^l}>0\\
\lim_{\tau\to\infty}\tau^k\frac{dc_D^h}{d\tilde{t}^l} &= 0
\end{aligned}$$

where we recall that $c_D^m=\lim_{\tau\to\infty}c_D^l$ refers to the cut-off cost level in the market equilibrium under a closed economy. For the rescaled objectives, the above relations show that, as $\tau\to\infty$, t^l works only through c_D^h but not c_D^l whereas $\tilde{t}^l$ works through c_D^l but not c_D^h.

We consider now the impact of tariffs on consumer surplus. As $\tau\to\infty$, and using (69), we see that t^l does not affect CS^l while a reduction in the export tariff (or increase in the export subsidy) still raises CS^l :

$$\lim_{\tau\to\infty}\tau^k\frac{dCS^l}{dc_D^l}\frac{dc_D^l}{dt^l}=0$$

$$\lim_{\tau\to\infty}\tau^k\frac{dCS^l}{dc_D^l}\frac{dc_D^l}{d\tilde{t}^l}=\frac{2(1+k)c_D^m-(3+2k)\alpha}{2\,(2+k)\,\eta}\frac{(k+1)}{(k+2)}\left(\chi^h\right)^{-(k+2)}c_D^m\frac{\partial\chi^h}{\partial\tilde{t}^l}<0.$$

We note that (89) also has terms related to trade values. Using (27) and (28), we find that

$$\lim_{\tau\to\infty}\tau^k\cdot IMP^l=\lim_{\tau\to\infty}\frac{N_E^h}{1+t^l}\frac{(c_D^l)^{k+2}\left(\chi^l\right)^{-k}}{2\gamma(k+2)(c_M)^k}=\frac{N_E^m}{1-\tilde{t}^h}\frac{(c_D^m)^{k+2}\left(\chi^l\right)^{-(k+1)}}{2\gamma(k+2)(c_M)^k}>0$$

$$\lim_{\tau\to\infty}\tau^k\cdot EXP^l=\lim_{\tau\to\infty}\frac{N_E^m}{1+t^h}\frac{(c_D^m)^{k+2}\left(\chi^h\right)^{-k}}{2\gamma(k+2)(c_M)^k}>0$$

where $N_E^m=\lim_{\tau\to\infty}N_E^l=[2(k+1)(c_M)^k\gamma(\alpha-c_D^m)]/[\eta(c_D^m)^{k+1}]$.

Finally, we note that (89) also has terms related to the derivatives of trade values. To evaluate these terms, we begin by reporting that, as $\tau\to\infty$, the impact of any tariff on the level of entry in any country approaches zero:

$$\lim_{\tau\to\infty}\frac{dN_E^l}{dt^l}=\lim_{\tau\to\infty}\frac{dN_E^h}{dt^l}=\lim_{\tau\to\infty}\frac{dN_E^l}{d\tilde{t}^l}=\lim_{\tau\to\infty}\frac{dN_E^h}{d\tilde{t}^l}=0.$$

These limiting results can be directly confirmed using (17), (18) and (23). From here, and using that t^l only works through c_D^h and $\tilde{t}^l$ works through c_D^l, we may derive the following terms:

$$\lim_{\tau\to\infty} \tau^k \frac{dIMP^l}{dt^l} = -\frac{\left(\chi^l\right)^{-(k+1)}}{2\gamma(k+2)(c_M)^k\left(1-\tilde{t}^h\right)} \cdot N_E^m \cdot (c_D^m)^{k+2} \frac{(k+1)}{\chi^l}\frac{\partial \chi^l}{\partial t^l} < 0$$

$$\lim_{\tau\to\infty} \tau^k \frac{dIMP^l}{d\tilde{t}^l} = 0$$

$$\lim_{\tau\to\infty} \tau^k \frac{dEXP^l}{dt^l} = 0.$$

$$\lim_{\tau\to\infty} \tau^k \frac{dEXP^l}{d\tilde{t}^l} = -\frac{\left(\chi^h\right)^{-k}}{2\gamma(k+2)(c_M)^k\left(1+t^h\right)} \cdot N_E^m \cdot (c_D^m)^{k+2} \frac{k}{\chi^h}\frac{\partial \chi^h}{\partial \tilde{t}^l} < 0.$$

Using the above relations, we rewrite (89) as:

$$\lim_{\tau\to\infty} \tau^k \cdot IMP^l + t^l \lim_{\tau\to\infty} \tau^k \frac{dIMP^l}{dt^l} = 0 \tag{90}$$

$$\lim_{\tau\to\infty} \tau^k EXP^l + \tilde{t}^l \lim_{\tau\to\infty} \tau^k \frac{dEXP^l}{d\tilde{t}^l} + \lim_{\tau\to\infty} \tau^k \frac{dCS^l}{dc_D^l}\frac{dc_D^l}{d\tilde{t}^l} = 0. \tag{91}$$

Interestingly, (90) and (91) show that $\lim_{\tau\to\infty} t^N$ is determined to maximize import revenue while $\lim_{\tau\to\infty} \tilde{t}^N$ is determined to maximize the welfare of country l excluding import revenue. By (90) and (91), and using $N_E^m = [2(k+1)(c_M)^k\gamma(\alpha - c_D^m)]/[\eta(c_D^m)^{k+1}]$, we finish our calculations and find that

$$\lim_{\tau\to\infty} t^N = \frac{1}{k} > 0$$

$$\lim_{\tau\to\infty} \frac{\tilde{t}^N}{1-\tilde{t}^N} = \frac{1}{2\,(2+k)\,k}\frac{(\alpha - 2\cdot c_D^m)}{(\alpha - c_D^m)}.$$

■

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