# Online Appendix: Supplemental Materials for ""Nash-in-Nash' Tariff Bargaining"* 

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#### Abstract

Using an endowment economy with Cobb-Douglas preferences, we provide supplemental analysis for our paper entitled "'Nash-in-Nash' Tariff Bargaining."


[^0]
## 1 Set up

In these supplemental notes, we analyze the three-country, two-good general equilibrium model corresponding to the first (homogeneous goods) scenario described in the paper under the further assumptions of an endowment economy with Cobb-Douglas (CD) preferences wherein each product receives equal weight. We assume that each government maximizes the national economic welfare of its country, which is captured as the indirect utility of the representative agent. ${ }^{1,2}$

To begin, we explicitly state the assumption of CD preferences. Let the utility for a representative agent in the home country be given by $U\left(x_{c}, y_{c}\right)$ when $x_{c}$ units of good $x$ and $y_{c}$ units of good $y$ are consumed. Under CD preferences, we have that

$$
\begin{equation*}
U\left(x_{c}, y_{c}\right)=x_{c} \cdot y_{c} . \tag{1}
\end{equation*}
$$

Similarly, under CD preferences, the utility function for a representative consumer in foreign country $* j$, where $j=1,2$, is given by

$$
\begin{equation*}
U^{* j}\left(x_{c}^{* j}, y_{c}^{* j}\right)=x_{c}^{* j} \cdot y_{c}^{* j} \tag{2}
\end{equation*}
$$

when $x_{c}^{* j}$ units of good $x$ and $y_{c}^{* j}$ units of good $y$ are consumed.
The corresponding utility-maximizing consumption levels for the home-country representative agent are

$$
\begin{equation*}
x_{c}(p, I)=I /(2 p) \text { and } y_{c}(p, I)=I / 2 \tag{3}
\end{equation*}
$$

where $p \equiv p_{x} / p_{y}$ is the local price of good $x$ relative to good $y$ in the home country and where $I$ is home country income expressed in local units of good $y$. Similarly, in foreign country $* j$, where $j=1,2$, the utility-maximizing consumption levels are given by

$$
\begin{equation*}
x_{c}^{* j}\left(p^{* j}, I^{* j}\right)=I^{* j} /\left(2 p^{* j}\right) \text { and } y_{c}^{* j}\left(p^{* j}, I^{* j}\right)=I^{* j} / 2 \tag{4}
\end{equation*}
$$

where $p^{* j} \equiv p_{x}^{* j} / p_{y}^{* j}$ is the local price of good $x$ relative to good $y$ in foreign country $* j$ and where $I^{* j}$ is foreign country $* j$ income expressed in local units of good $y$.

Local and world prices are related though ad valorem tariffs. Let $t^{j}>-1$ denote the ad valorem import tariff that the home country imposes on imports from foreign country $* j$ and define $\tau^{j} \equiv 1+t^{j}$. Similarly, let $t^{* j}>-1$ denote the ad valorem import tariff that foreign country $* j$ imposes on imports from

[^1]the home country and define $\tau^{* j} \equiv 1+t^{* j}$. With world prices defined as $p^{w j} \equiv p_{x}^{* j} / p_{y}$, we have the following price relationships:
\[

$$
\begin{equation*}
p=\tau^{j} p^{w j} \text { and } p^{* j}=p^{w j} / \tau^{* j} \tag{5}
\end{equation*}
$$

\]

where these relationships hold for $j=1,2$.
We note that $p^{w j}$ is the terms of trade for foreign country $* j$. Likewise, $1 / p^{w j}$ is the home country's bilateral terms of trade with foreign country $* j$. We define the multilateral terms of trade for the home country below.

## 2 Foreign country $* j$

Our next task is to represent economic relationships within foreign country $* j$, where $j=1,2$. Under our assumption of an endowment economy, the supply side of the model is easily described. In particular, let $Q_{i}^{* j}$ denote the fixed ouput supplied in country $* j$ of good $i$, where $j=1,2$ and $i=x, y$. We thus focus on the demand side. Our approach is to develop some general relationships and then provide further derivations after imposing CD preferences. The end result is a characterization of economic values including income, indirect utility and trade volumes in terms of local and world prices and exogenous endowment levels. We provide a related set of derivations for the home country in the next section.

## Tariff revenue and income

The demand function in foreign country $* j$ for good $i$ is given as $D_{i}^{* j}\left(p^{* j}, R^{* j}\right)$, where $R^{* j}$ denotes tariff revenue in foreign country $* j$. Tariff revenue is a component of income and thus affects demand, but the level of demand influences the volume of imports and thus also affects tariff revenue. Reflecting this interdependence, tariff revenue $R^{* j}$ is implicitly defined by the following relationship:

$$
\begin{equation*}
R^{* j}=\left[D_{y}^{*}\left(p^{* j}, R^{* j}\right)-Q_{y}^{* j}\right]\left[\frac{1}{p^{* j}}-\frac{1}{p^{w j}}\right] p^{* j} \tag{6}
\end{equation*}
$$

which can be solved for $R^{* j}=R^{* j}\left(p^{* j}, p^{w j}\right) .{ }^{3}$ With tariff revenue represented in this fashion, we may define the consumption of good $i$ in foreign country $* j$ as a function of the local price $p^{* j}$ and the world price $p^{w j}$ :

$$
\begin{equation*}
C_{i}^{* j}=C_{i}^{* j}\left(p^{* j}, p^{w j}\right) \equiv D_{i}^{* j}\left(p^{* j}, R^{* j}\left(p^{* j}, p^{w j}\right)\right) \tag{7}
\end{equation*}
$$

We now represent income for foreign country $* j$ with the local price of $y$ (i.e., $p_{y}^{* j}$ ) as the numeraire:

$$
I^{* j}=p^{* j} Q_{x}^{* j}+Q_{y}^{* j}+t^{* j} \frac{p_{y}}{p_{y}^{* j}}\left[C_{y}^{* j}\left(p^{* j}, p^{w j}\right)-Q_{y}^{* j}\right]
$$

[^2]Using $\tau^{* j} \equiv 1+t^{* j},(5), p^{w j} \equiv p_{x}^{* j} / p_{y}$ and $p^{* j} \equiv p_{x}^{* j} / p_{y}^{* j}$, we obtain

$$
\begin{equation*}
I^{* j}=p^{* j} Q_{x}^{* j}+Q_{y}^{* j}+\left[\frac{1}{p^{* j}}-\frac{1}{p^{w j}}\right] p^{* j}\left[C_{y}^{* j}\left(p^{* j}, p^{w j}\right)-Q_{y}^{* j}\right] \tag{8}
\end{equation*}
$$

## Trade balance

We can now confirm that trade balance holds. To this end, we relate income as given in (8) with expenditure. In particular, the consumer budget constraint ensures that consumption expenditures equal income:

$$
\begin{equation*}
I^{* j}=p^{* j} C_{x}^{* j}\left(p^{* j}, p^{w j}\right)+C_{y}^{* j}\left(p^{* j}, p^{w j}\right) \tag{9}
\end{equation*}
$$

Thus, using (8) and (9), income equals expenditure if and only if
$p^{* j} Q_{x}^{* j}+Q_{y}^{* j}+\left[\frac{1}{p^{* j}}-\frac{1}{p^{w j}}\right] p^{* j}\left[C_{y}^{* j}\left(p^{* j}, p^{w j}\right)-Q_{y}^{* j}\right]=p^{* j} C_{x}^{* j}\left(p^{* j}, p^{w j}\right)+C_{y}^{* j}\left(p^{* j}, p^{w j}\right)$
which may be re-written as the trade-balance condition for foreign country $* j$ :

$$
\begin{equation*}
p^{w j} E_{x}^{* j}\left(p^{* j}, p^{w j}\right)=M_{y}^{* j}\left(p^{* j}, p^{w j}\right) \tag{10}
\end{equation*}
$$

where the associated export and import functions for foreign country $* j$ are respectively defined as

$$
\begin{align*}
E_{x}^{* j}\left(p^{* j}, p^{w j}\right) & =Q_{x}^{* j}-C_{x}^{* j}\left(p^{* j}, p^{w j}\right)  \tag{11}\\
M_{y}^{* j}\left(p^{* j}, p^{w j}\right) & =C_{y}^{* j}\left(p^{* j}, p^{w j}\right)-Q_{y}^{* j} \tag{12}
\end{align*}
$$

## Fixed point solution for tariff revenue under CD preferences

We now use CD preferences and solve for the fixed point solution of $R^{* j}$. For the case of CD preferences, we have from (4), (6), (7) and (8) that

$$
D_{y}^{* j}\left(p^{* j}, R^{* j}\right)=\frac{I^{* j}}{2}=\frac{1}{2}\left(p^{* j} Q_{x}^{* j}+Q_{y}^{* j}+R^{* j}\right)
$$

Thus, under CD preferences, we may re-write the fixed point equation for $R^{* j}$ that is captured in (6) as follows:

$$
R^{* j}=\left[\frac{1}{2}\left(p^{* j} Q_{x}^{* j}+Q_{y}^{* j}+R^{* j}\right)-Q_{y}^{* j}\right]\left[\frac{1}{p^{* j}}-\frac{1}{p^{w j}}\right] p^{* j}
$$

which may be explicitly solved, yielding

$$
\begin{equation*}
R^{* j}\left(p^{* j}, p^{w j}\right)=\frac{\frac{1}{2}\left(p^{* j} Q_{x}^{* j}-Q_{y}^{* j}\right)\left(\frac{1}{p^{* j}}-\frac{1}{p^{w j}}\right) p^{* j}}{1-\frac{1}{2}\left(\frac{1}{p^{* j}}-\frac{1}{p^{w j}}\right) p^{* j}} . \tag{13}
\end{equation*}
$$

As (13) reveals, under CD preferences, tariff revenue for foreign country $* j$ in local units of $y$ may thus be expressed as an explicit function of local and world relative prices.

## Income as function of prices under CD preferences

Under CD preferences, we can now express income for foreign country $* j$ as a function of local and world relative prices; in particular, using (6), (7), (8) and (13), we have $I^{* j}=p^{* j} Q_{x}^{* j}+Q_{y}^{* j}+R^{* j}\left(p^{* j}, p^{w j}\right)$ and thus

$$
\begin{equation*}
I^{* j}\left(p^{* j}, p^{w j}\right)=\frac{p^{w j} Q_{x}^{* j}+Q_{y}^{* j}}{\frac{1}{2}\left(\frac{p^{w j}}{p^{* j}}+1\right)} \tag{14}
\end{equation*}
$$

Indirect utility and final expressions in prices under CD preferences
We now provide final expressions of key variables for foreign country $* j$ as functions of local and world relative prices. First, referring to (2) and using (4), we may represent indirect utility in foreign country $* j$ as

$$
\begin{equation*}
V^{* j}\left(p^{* j}, I^{* j}\left(p^{* j}, p^{w j}\right)\right) \equiv C_{x}^{* j}\left(p^{* j}, p^{w j}\right) C_{y}^{* j}\left(p^{* j}, p^{w j}\right)=\left(I^{* j}\left(p^{* j}, p^{w j}\right)\right)^{2} \frac{1}{4 p^{* j}} \tag{15}
\end{equation*}
$$

where we use $x_{c}^{* j}\left(p^{* j}, I^{* j}\left(p^{* j}, p^{w j}\right)\right)=C_{x}^{* j}\left(p^{* j}, p^{w j}\right)$ and $y_{c}^{* j}\left(p^{* j}, I^{* j}\left(p^{* j}, p^{w j}\right)\right)=$ $C_{y}^{* j}\left(p^{* j}, p^{w j}\right)$. Hence, combining (14) and (15), we have this expression for indirect utility in foreign country $* j$ :

$$
\begin{equation*}
V^{* j}\left(p^{* j}, I^{* j}\left(p^{* j}, p^{w j}\right)\right)=\left(\frac{p^{w j} Q_{x}^{* j}+Q_{y}^{* j}}{p^{w j}+p^{* j}}\right)^{2} p^{* j} \tag{16}
\end{equation*}
$$

As an aside, using (16), we may also derive that

$$
\begin{aligned}
\left.\frac{d V^{* j}\left(p^{* j}, I^{* j}\left(p^{* j}, p^{w j}\right)\right)}{d p^{* j}}\right|_{p^{w j}}= & \frac{\partial V^{* j}\left(p^{* j}, I^{* j}\left(p^{* j}, p^{w j}\right)\right)}{\partial p^{* j}} \\
& +\frac{\partial V^{* j}\left(p^{* j}, I^{* j}\left(p^{* j}, p^{w j}\right)\right)}{\partial I^{* j}} \frac{\left.\partial I^{* j}\left(p^{* j}, p^{w j}\right)\right)}{\partial p^{* j}} \\
= & \left(I^{* j}\left(p^{* j}, p^{w j}\right)\right)^{2}\left(1 / p^{* j}\right)^{2}\left[\frac{p^{w j}-p^{* j}}{p^{w j}+p^{* j}}\right],
\end{aligned}
$$

which clearly equals zero if and only if $p^{w j}=p^{* j}$. Thus, the "politically optimal" reaction curve for the national-income maximizing foreign country $* j$ is a policy of free trade. ${ }^{4}$

With all of this, we can also define the underlying consumption and trade volumes:

$$
\begin{gather*}
C_{y}^{* j}\left(p^{* j}, p^{w j}\right)=\frac{I^{* j}\left(p^{* j}, p^{w j}\right)}{2}  \tag{17}\\
C_{x}^{* j}\left(p^{* j}, p^{w j}\right)=\frac{I^{* j}\left(p^{* j}, p^{w j}\right)}{2} \frac{1}{p^{* j}}  \tag{18}\\
E_{x}^{* j}\left(p^{* j}, p^{w j}\right) \equiv Q_{x}^{* j}-C_{x}^{* j}\left(p^{* j}, p^{w j}\right)=Q_{x}^{* j}-\frac{I^{* j}\left(p^{* j}, p^{w j}\right)}{2} \frac{1}{p^{* j}}  \tag{19}\\
M_{y}^{* j}\left(p^{* j}, p^{w j}\right) \equiv C_{y}^{* j}\left(p^{* j}, p^{w j}\right)-Q_{y}^{* j}=\frac{I^{* j}\left(p^{* j}, p^{w j}\right)}{2}-Q_{y}^{* j} \tag{20}
\end{gather*}
$$

where $I^{* j}\left(p^{* j}, p^{w j}\right)$ under CD preferences is given by (14).

[^3]
## 3 Home country

We now make similar derivations for the home country. As before, the supply side is straightforward due to the assumption of an endowment economy. Let $Q_{i}$ denote the exogenous endowment in the home country of good $i$, for $i=x, y$. For the demand side, the definition of the multilateral terms of trade is somewhat complex, since the home country may utilize discriminatory tariffs. After defining the home country's multilateral terms of trade, we again provide some general derivations and then develop further relationships under the assumption of CD preferences. Once again, the end result is a characterization of economic values including income, indirect utility and trade volumes in terms of local and world prices and exogenous endowment levels.

## Multilateral terms of trade, tariff revenue and income

To begin, we define the home country's multilateral terms of trade. Let

$$
\begin{equation*}
T\left(p^{* 1}, p^{* 2}, p^{w 1}, p^{w 2}\right) \equiv \sum_{j=1,2} s_{x}^{* j}\left(p^{* 1}, p^{* 2}, p^{w 1}, p^{w 2}\right) \cdot p^{w j} \tag{21}
\end{equation*}
$$

where

$$
\begin{equation*}
s_{x}^{* j}\left(p^{* 1}, p^{* 2}, p^{w 1}, p^{w 2}\right) \equiv \frac{E_{x}^{* j}\left(p^{* j}, p^{w j}\right)}{\sum_{j=1,2} E_{x}^{* j}\left(p^{* j}, p^{w j}\right)} \tag{22}
\end{equation*}
$$

The home country's multilateral terms of trade is thus $1 / T$, where $T$ is a tradeshare weighted average of bilateral world prices.

We may now define the tariff revenue function, $R=R(p, T)$, implicitly by

$$
\begin{equation*}
R=\left(D_{x}(p, R)-Q_{x}\right)(p-T) \tag{23}
\end{equation*}
$$

where $D_{x}(p, R)$ is the home country demand for good $x$ as a function of the local price $p$ and tariff revenue $R$. ${ }^{5}$

With tariff revenue thus defined using (23), we can define consumption and trade volumes as functions of $p$ and $T$, as follows:

$$
\begin{equation*}
C_{i}(p, T) \equiv D_{i}(p, R(p, T)) \tag{24}
\end{equation*}
$$

for $i=x, y$. Using (24), we can thus define home country import and export volumes, respectively, as

$$
\begin{align*}
M_{x}(p, T) & \equiv C_{x}(p, T)-Q_{x}  \tag{25}\\
E_{y}(p, T) & \equiv Q_{y}-C_{y}(p, T) \tag{26}
\end{align*}
$$

We can now define home country income in terms of the local price of $y$ (i.e., $p_{y}$ ):

$$
I=p Q_{x}+Q_{y}+t^{1} p^{w 1} E_{x}^{* 1}\left(p^{* 1}, p^{w 1}\right)+t^{2} p^{w 2} E_{x}^{* 2}\left(p^{* 2}, p^{w 2}\right)
$$

[^4]where we build in implicitly the market clearing condition defined below in (39) that home imports are utimately comprised of foreign exports, which may face discriminatory home import tariffs. Using $\tau^{j} \equiv 1+t^{j},(5)$, (21) and (22), we may re-write income as
\[

$$
\begin{aligned}
I & =p Q_{x}+Q_{y}+\left(\frac{p}{p^{w 1}}-1\right) p^{w 1} E_{x}^{* 1}\left(p^{* 1}, p^{w 1}\right)+\left(\frac{p}{p^{w 2}}-1\right) p^{w 2} E_{x}^{* 2}\left(p^{* 2}, p^{w 2}\right) \\
& =p Q_{x}+Q_{y}+\left(p-p^{w 1}\right) E_{x}^{* 1}\left(p^{* 1}, p^{w 1}\right)+\left(p-p^{w 2}\right) E_{x}^{* 2}\left(p^{* 2}, p^{w 2}\right) \\
& =p Q_{x}+Q_{y}+(p-T)\left(E_{x}^{* 1}\left(p^{* 1}, p^{w 1}\right)+E_{x}^{* 2}\left(p^{* 2}, p^{w 2}\right)\right)
\end{aligned}
$$
\]

Anticipating the market clearing condition (i.e., $M_{x}(p, T)=E_{x}^{* 1}\left(p^{* 1}, p^{w 1}\right)+$ $E_{x}^{* 2}\left(p^{* 2}, p^{w 2}\right)$ ) as captured below in (39), we are now able to express income as

$$
\begin{equation*}
I(p, T)=p Q_{x}+Q_{y}+(p-T) M_{x}(p, T) \tag{27}
\end{equation*}
$$

or equivalently after using (25) we may simplify and express income as

$$
\begin{equation*}
I(p, T)=p Q_{x}+Q_{y}+(p-T)\left(C_{x}(p, T)-Q_{x}\right) \tag{28}
\end{equation*}
$$

where the last term corresponds to tariff revenue. In expressions (27) and (28), we recall from (21) that $T=T\left(p^{* 1}, p^{* 2}, p^{w 1}, p^{w 2}\right)$.

## Trade balance

We can now confirm that trade balance is implied. From the consumer's budget constraint, we know that income also equals expenditures, so that

$$
I=p D_{x}(p, R(p, T))+D_{y}(p, R(p, T))
$$

Thus, using (24), we have that

$$
\begin{equation*}
I(p, T)=p C_{x}(p, T)+C_{y}(p, T) \tag{29}
\end{equation*}
$$

Combining (28) and (29), we get that income equals expenditure if and only if

$$
0=Q_{y}-C_{y}(p, T)+p\left(Q_{x}-C_{x}(p, T)\right)+(p-T)\left(C_{x}(p, T)-Q_{x}\right)
$$

which can be re-written using (25) and (26) as the trade-balance condition for the home country:

$$
\begin{equation*}
E_{y}(p, T)=T \cdot M_{x}(p, T) \tag{30}
\end{equation*}
$$

## Fixed point solution for tariff revenue

Using CD preferences, we now solve for the fixed point revenue solution, $R(p, T)$. Under CD preferences, we have from (3), (24) and (28) that

$$
D_{x}(p, R)=\frac{p Q_{x}+Q_{y}+R}{2 p}
$$

where $R=\left(D_{x}(p, R)-Q_{x}\right)(p-T)$ by (23). We can thus solve for $R$ from the following equation:

$$
R=\left(\frac{p Q_{x}+Q_{y}+R}{2 p}-Q_{x}\right)(p-T)
$$

whence we obtain the fixed point revenue solution under CD preferences as

$$
\begin{equation*}
R(p, T)=\frac{\left(Q_{y}-p Q_{x}\right)(p-T)}{(p+T)} \tag{31}
\end{equation*}
$$

with the local price of $y$ (i.e., $p_{y}$ ) as the numeraire. Thus, and as (31) confirms, under CD prefernces, we may represent home country tariff revenue as an explicit function of the local relative price $p$ and the multilateral terms of trade $1 / T$.

## Income as a function of prices under CD preferences

We can now use this explicit solution for tariff revenue to arrive at a corresponding expression for income as a function of prices, with the local price of $y$ (i.e., $p_{y}$ ) as the numeraire. Namely, using (23), (24) and (28), we get that

$$
I(p, T)=p Q_{x}+Q_{y}+R(p, T)
$$

Under CD preferences, we may use (31) and represent income as

$$
I(p, T)=p Q_{x}+Q_{y}+\frac{\left(Q_{y}-p Q_{x}\right)(p-T)}{(p+T)}
$$

After simplifying, we get

$$
\begin{equation*}
I(p, T)=2 p\left(\frac{Q_{x} T+Q_{y}}{p+T}\right) . \tag{32}
\end{equation*}
$$

as the fixed point solution for income under CD preferences, with the local price of $y$ as the numeraire.

Indirect utility and final expressions in prices under CD preferences
We next define the indirect utility function as a function of prices. Referring to (1) and using (3), we may represent indirect utility in the home country as

$$
\begin{equation*}
V(p, I(p, T)) \equiv C_{x}(p, T) C_{y}(p, T)=(I(p, T))^{2} \frac{1}{4 p} \tag{33}
\end{equation*}
$$

where we use $x_{c}(p, I(p, T))=C_{x}(p, T)$ and $y_{c}(p, I(p, T))=C_{y}(p, T)$. Hence, combining (32) and (33), we have this expression for indirect utility in the home country:

$$
\begin{equation*}
V(p, I(p, T))=\left(\frac{Q_{x} T+Q_{y}}{p+T}\right)^{2} p \tag{34}
\end{equation*}
$$

As an aside, using (34), we may also derive that

$$
\begin{aligned}
\left.\frac{d V(p, I(p, T))}{d p}\right|_{T} & =\frac{\partial V(p, I(p, T))}{\partial p}+\frac{\partial V(p, I(p, T))}{\partial I} \frac{\partial I(p, T))}{\partial p} \\
& =\left(\frac{I(p, T)}{2}\right)^{2}\left(\frac{1}{p}\right)^{2}\left[\frac{T-p}{p+T}\right]
\end{aligned}
$$

which clearly equals zero if and only if $T=p$. Under MFN, so that by (5), (21) and (22) we have $T=p^{w 1}=p^{w 2}$, we thus may conclude that the MFN political optimum reaction curve for the national-income maximizing home country is necessarily a policy of free trade. ${ }^{6}$

With all of this, we can also represent the underlying consumption and trade volumes under CD preferences:

$$
\begin{gather*}
C_{y}(p, T)=\frac{I(p, T)}{2}  \tag{35}\\
C_{x}(p, T)=\frac{I(p, T)}{2} \frac{1}{p}  \tag{36}\\
E_{y}(p, T) \equiv Q_{y}-C_{y}(p, T)=Q_{y}-\frac{I(p, T)}{2}  \tag{37}\\
M_{x}(p, T) \equiv C_{x}(p, T)-Q_{x}=\frac{I(p, T)}{2} \frac{1}{p}-Q_{x} \tag{38}
\end{gather*}
$$

where $I(p, T)$ under CD preferences is given by (32).

## 4 Market Clearing Prices and Welfares

We are now ready for market clearing conditions to determine world prices. We begin by using the market clearing condition and the definition of $T$ so as to explicitly represent the market clearing condition under CD preferences as an equation in $p^{w 1}$ and $p^{w 2}$. We then use a "linkage condition" based on (5) to give a second equation in $p^{w 1}$ and $p^{w 2}$. With these two equations, we can solve for market clearing world prices, $\widetilde{p}^{w 1}$ and $\widetilde{p}^{w 2}$. We can then represent indirect utilities as functions of market clearing world prices and, thus, tariffs.

## Market clearing condition under CD preferences

We express the market clearing condition in general terms as follows:

$$
\begin{equation*}
M_{x}(p, T)=E_{x}^{* 1}\left(p^{* 1}, p^{w 1}\right)+E_{x}^{* 2}\left(p^{* 2}, p^{w 2}\right) \tag{39}
\end{equation*}
$$

Under CD preferences, the market clearing condition is thus

$$
\begin{equation*}
\frac{I(p, T)}{2} \frac{1}{p}-Q_{x}=Q_{x}^{* 1}-\frac{I^{* 1}\left(p^{* 1}, p^{w 1}\right)}{2} \frac{1}{p^{* 1}}+Q_{x}^{* 2}-\frac{I^{* 2}\left(p^{* 2}, p^{w 2}\right)}{2} \frac{1}{p^{* 2}} \tag{40}
\end{equation*}
$$

[^5]where we use (19) and (38).
The market clearing condition under CD preferences as given in (40) can be further simplified. Substituting for incomes via (14) and (32) and simplifying slightly, we may re-write the market clearing condition (40) as
$$
\left(\frac{Q_{x} T+Q_{y}}{p+T}\right)-Q_{x}=Q_{x}^{* 1}-\frac{p^{w 1} Q_{x}^{* 1}+Q_{y}^{* 1}}{\left(p^{w 1}+p^{* 1}\right)}+Q_{x}^{* 2}-\frac{p^{w 2} Q_{x}^{* 2}+Q_{y}^{* 2}}{\left(p^{w 2}+p^{* 2}\right)}
$$

Simplifying further, we get

$$
\frac{Q_{y}-p Q_{x}}{p+T}=\frac{p^{* 1} Q_{x}^{* 1}-Q_{y}^{* 1}}{p^{w 1}+p^{* 1}}+\frac{p^{* 2} Q_{x}^{* 2}-Q_{y}^{* 2}}{p^{w 2}+p^{* 2}}
$$

To further simplify our expression of the market clearing condition, we introduce the following definition:

$$
\begin{equation*}
A^{* j}\left(p^{* j}, p^{w j}\right) \equiv\left(p^{* j} Q_{x}^{* j}-Q_{y}^{* j}\right)\left(\frac{p^{w j}}{p^{* j}+p^{w j}}\right) \tag{41}
\end{equation*}
$$

Using (41), the market clearing condition takes the form:

$$
\begin{equation*}
\frac{Q_{y}-p Q_{x}}{p+T}=\frac{A^{* 1}}{p^{w 1}}+\frac{A^{* 2}}{p^{w 2}} \tag{42}
\end{equation*}
$$

Our next step is to substitute the definition of $T$ from (21) and (22) into the market clearing condition (42). To this end, we first represent the defined expression for $T$ under CD preferences. We know from (21) and (22) that

$$
\begin{equation*}
T\left(p^{* 1}, p^{* 2}, p^{w 1}, p^{w 2}\right) \equiv \frac{E_{x}^{* 1} \cdot p^{w 1}+E_{x}^{* 2} \cdot p^{w 2}}{E_{x}^{* 1}+E_{x}^{* 2}} \tag{43}
\end{equation*}
$$

Using CD preferences, we may use (14) and (19) to obtain

$$
\begin{equation*}
E_{x}^{* j}\left(p^{* j}, p^{w j}\right)=\frac{Q_{x}^{* j} p^{* j}-Q_{y}^{* j}}{p^{w j}+p^{* j}} \tag{44}
\end{equation*}
$$

Using (5), (41) and (44), we then get that

$$
\begin{equation*}
E_{x}^{* j}\left(p^{* j}, p^{w j}\right)=\left(\frac{1}{p^{w j}}\right) A^{* j}\left(p^{* j}, p^{w j}\right) \tag{45}
\end{equation*}
$$

Thus, under CD preferences, we may use (43) and (45) to obtain that the definition of $T$ can be written as

$$
\begin{equation*}
T\left(p^{* 1}, p^{* 2}, p^{w 1}, p^{w 2}\right) \equiv \frac{A^{* 1}+A^{* 2}}{\left(\frac{A^{* 1}}{p^{w 1}}\right)+\left(\frac{A^{* 2}}{p^{w 2}}\right)}=\frac{\left(A^{* 1}+A^{* 2}\right) p^{w 1} p^{w 2}}{A^{* 1} p^{w 2}+A^{* 2} p^{w 1}} \tag{46}
\end{equation*}
$$

Finally, we may now substitute the expression for the definition of $T$ in (46) into the market clearing condition (42) to get

$$
\frac{Q_{y}-p Q_{x}}{p+\frac{\left(A^{* 1}+A^{* 2}\right) p^{w 1} p^{w 2}}{A^{* 1} p^{w 2}+A^{* 2} p^{w 1}}}=\frac{A^{* 1}}{p^{w 1}}+\frac{A^{* 2}}{p^{w 2}}
$$

Simplifying and using $p=\tau^{1} p^{w 1}$ from (5), we represent the market clearing condition as

$$
\frac{Q_{y}-\tau^{1} p^{w 1} Q_{x}}{\tau^{1}\left(A^{* 1} p^{w 2}+A^{* 2} p^{w 1}\right)+\left(A^{* 1}+A^{* 2}\right) p^{w 2}}=\frac{1}{p^{w 2}}
$$

Using $p^{* j}=p^{w j} / \tau^{* j}$ from (5) and also (41), we may define

$$
\begin{equation*}
\widehat{A}^{* j}\left(\tau^{* j}, p^{w j}\right) \equiv A^{* j}\left(p^{w j} / \tau^{* j}, p^{w j}\right) \tag{47}
\end{equation*}
$$

Utilizing (47), we may now write the market clearing condition as

$$
\begin{equation*}
\frac{Q_{y}-\tau^{1} p^{w 1} Q_{x}}{\tau^{1}\left(\widehat{A}^{* 1} p^{w 2}+\widehat{A}^{* 2} p^{w 1}\right)+\left(\widehat{A}^{* 1}+\widehat{A}^{* 2}\right) p^{w 2}}=\frac{1}{p^{w 2}} . \tag{48}
\end{equation*}
$$

With (48), we have our first (market clearing) equation that relates the two world prices for given tariffs.

## Linkage condition

The second equation on world prices is the linkage condition. This condition comes directly from (5) and is as follows:

$$
\begin{equation*}
p^{w 2}=\frac{\tau^{1}}{\tau^{2}} p^{w 1} \tag{49}
\end{equation*}
$$

Together, (48) and (49) represent two equations that determine the two market clearing world prices, $\widetilde{p}^{w 1}$ and $\widetilde{p}^{w 2}$, for given tariffs.

## Solving for market clearing world prices

We now substitute the linkage equation (49) into the market clearing equation (48) in order to get one equation in one unknown, $p^{w 1}$. Formally, substituting $p^{w 2}=\frac{\tau^{1}}{\tau^{2}} p^{w 1}$, we can rewrite the market clearing condition as

$$
\frac{Q_{y}-\tau^{1} p^{w 1} Q_{x}}{\tau^{1}\left(\widehat{A}^{* 1} \frac{\tau^{1}}{\tau^{2}} p^{w 1}+\widehat{A}^{* 2} p^{w 1}\right)+\left(\widehat{A}^{* 1}+\widehat{A}^{* 2}\right) p^{w 1}\left(\frac{\tau^{1}}{\tau^{2}}\right)}=\frac{\tau^{2}}{\tau^{1} p^{w 1}}
$$

Rearranging, we get

$$
Q_{y}-\tau^{1} p^{w 1} Q_{x}=\widehat{A}^{* 1}\left(1+\tau^{1}\right)+\widehat{A}^{* 2}\left(1+\tau^{2}\right)
$$

Using (41) and (47), we may re-write this equation as

$$
\begin{aligned}
Q_{y}-\tau^{1} p^{w 1} Q_{x}= & {\left[\left(Q_{x}^{* 1}\left(\frac{p^{w 1}}{\tau^{* 1}}\right)-Q_{y}^{* 1}\right)\left(\frac{\tau^{* 1}}{\tau^{* 1}+1}\right)\right]\left(1+\tau^{1}\right) } \\
& +\left[\left(Q_{x}^{* 2}\left(\frac{p^{w 2}}{\tau^{* 2}}\right)-Q_{y}^{* 2}\right)\left(\frac{\tau^{* 2}}{\tau^{* 2}+1}\right)\right]\left(1+\tau^{2}\right)
\end{aligned}
$$

Using (49) once more, we simplify further and obtain

$$
\begin{aligned}
Q_{y}-\tau^{1} p^{w 1} Q_{x}= & {\left[\left(Q_{x}^{* 1}\left(\frac{p^{w 1}}{\tau^{* 1}}\right)-Q_{y}^{* 1}\right)\left(\frac{\tau^{* 1}}{\tau^{* 1}+1}\right)\right]\left(1+\tau^{1}\right) } \\
& +\left[\left(Q_{x}^{* 2}\left(\frac{p^{w 1}}{\tau^{* 2}}\right)\left(\frac{\tau^{1}}{\tau^{2}}\right)-Q_{y}^{* 2}\right)\left(\frac{\tau^{* 2}}{\tau^{* 2}+1}\right)\right]\left(1+\tau^{2}\right)
\end{aligned}
$$

which is one equation in one unknown, $p^{w 1}$.
We can now solve for the market clearing world price $\widetilde{p}^{w 1}$ in terms of tariffs:

$$
\begin{equation*}
\widetilde{p}^{w 1}(\boldsymbol{\tau})=\frac{Q_{y}+Q_{y}^{* 1}\left(\frac{\tau^{* 1}\left(1+\tau^{1}\right)}{1+\tau^{* 1}}\right)+Q_{y}^{* 2}\left(\frac{\tau^{* 2}\left(1+\tau^{2}\right)}{1+\tau^{* 2}}\right)}{\tau^{1} Q_{x}+Q_{x}^{* 1}\left(\frac{1+\tau^{1}}{1+\tau^{* 1}}\right)+Q_{x}^{* 2}\left(\frac{\tau^{1}}{\tau^{2}}\right)\left(\frac{1+\tau^{2}}{1+\tau^{* 2}}\right)} . \tag{50}
\end{equation*}
$$

Similarly, using (49) and (50), we may now solve for the market clearing world price $\widetilde{p}^{w 2}$ in terms of tariffs:

$$
\begin{equation*}
\widetilde{p}^{w 2}(\boldsymbol{\tau})=\frac{Q_{y}+Q_{y}^{* 1}\left(\frac{\tau^{* 1}\left(1+\tau^{1}\right)}{1+\tau^{* 1}}\right)+Q_{y}^{* 2}\left(\frac{\tau^{* 2}\left(1+\tau^{2}\right)}{1+\tau^{* 2}}\right)}{\tau^{2} Q_{x}+Q_{x}^{* 1}\left(\frac{1+\tau^{1}}{1+\tau^{* 1}}\right)\left(\frac{\tau^{2}}{\tau^{1}}\right)+Q_{x}^{* 2}\left(\frac{1+\tau^{2}}{1+\tau^{* 2}}\right)} . \tag{51}
\end{equation*}
$$

In similar fashion, we can use (5) and define

$$
\begin{equation*}
\widehat{T}\left(\tau^{* 1}, \tau^{* 2}, p^{w 1}, p^{w 2}\right) \equiv T\left(p^{w 1} / \tau^{* 1}, p^{w 2} / \tau^{* 2}, p^{w 1}, p^{w 2}\right) \tag{52}
\end{equation*}
$$

From here, we can evaluate at market clearing world prices and represent the multilateral terms of trade function directly in terms of the underling tariff vector:

$$
\begin{equation*}
\widetilde{T}(\boldsymbol{\tau}) \equiv \widehat{T}\left(\tau^{* 1}, \tau^{* 2}, \widetilde{p}^{w 1}, \widetilde{p}^{w 2}\right) \tag{53}
\end{equation*}
$$

Similarly, we may define

$$
\begin{equation*}
\widetilde{A}^{* j}(\boldsymbol{\tau}) \equiv \widehat{A}^{* j}\left(\tau^{* j}, \widetilde{p}^{w j}\right) \tag{54}
\end{equation*}
$$

as the value for $A^{* j}$ at market clearing values and written as a function of the underlying tariff vector.

Indirect utility for foreign country $* i$ at market clearing prices
We can now represent indirect utility for foreign country $* i$ at market clearing prices. Using (16), we recall that the indirect utility in foreign country $* j$ is represented as

$$
V^{* j}\left(p^{* j}, I^{* j}\left(p^{* j}, p^{w j}\right)\right)=\left(\frac{p^{w j} Q_{x}^{* j}+Q_{y}^{* j}}{p^{w j}+p^{* j}}\right)^{2} p^{* j}
$$

Using $p^{* j}=p^{w j} / \tau^{* j}$ from (5) and (16), we may define the welfare function for foreign country $* j$ as

$$
W^{* j}(\boldsymbol{\tau}) \equiv V^{* j}\left(\widetilde{p}^{w j} / \tau^{* j}, I^{* j}\left(\widetilde{p}^{w j} / \tau^{* j}, \widetilde{p}^{w j}\right)\right)
$$

and we thus have that

$$
\begin{equation*}
W^{* j}(\boldsymbol{\tau})=\left(\frac{\widetilde{p}^{w j} Q_{x}^{* j}+Q_{y}^{* j}}{1+\tau^{* j}}\right)^{2} \frac{\tau^{* j}}{\widetilde{p}^{w j}} \tag{55}
\end{equation*}
$$

where

$$
\begin{equation*}
\widetilde{p}^{w j}=\frac{Q_{y}+Q_{y}^{* j}\left(\frac{\tau^{* j}\left(1+\tau^{j}\right)}{1+\tau^{* j}}\right)+Q_{y}^{* i}\left(\frac{\tau^{* i}\left(1+\tau^{i}\right)}{1+\tau^{* i}}\right)}{\tau^{j} Q_{x}+Q_{x}^{* j}\left(\frac{1+\tau^{j}}{1+\tau^{* j}}\right)+Q_{x}^{* i}\left(\frac{\tau^{j}}{\tau^{i}}\right)\left(\frac{1+\tau^{i}}{1+\tau^{* i}}\right)} \tag{56}
\end{equation*}
$$

Indirect utility for the home country at market clearing prices
We can also represent indirect utility for the home country at market clearing prices. Recall from (34) that

$$
V(p, I(p, T))=\left(\frac{Q_{x} T+Q_{y}}{p+T}\right)^{2} p
$$

Using $p=\tau^{1} p^{w 1}$ from (5) and (34), we may define the welfare function for the home country as

$$
W(\boldsymbol{\tau}) \equiv V\left(\tau^{1} \widetilde{p}^{w 1}, I\left(\tau^{1} \widetilde{p}^{w 1}, \widetilde{T}\right)\right)
$$

and we thus have that

$$
\begin{equation*}
W(\boldsymbol{\tau})=\left(\frac{Q_{x} \widetilde{T}+Q_{y}}{\tau^{1} \widetilde{p}^{w 1}+\widetilde{T}}\right)^{2} \tau^{1} \widetilde{p}^{w 1} \tag{57}
\end{equation*}
$$

where we recall from (56) that

$$
\widetilde{p}^{w j}=\frac{Q_{y}+Q_{y}^{* j}\left(\frac{\tau^{* j}\left(1+\tau^{j}\right)}{1+\tau^{* j}}\right)+Q_{y}^{* i}\left(\frac{\tau^{* i}\left(1+\tau^{i}\right)}{1+\tau^{* i}}\right)}{\tau^{j} Q_{x}+Q_{x}^{* j}\left(\frac{1+\tau^{j}}{1+\tau^{* j}}\right)+Q_{x}^{* i}\left(\frac{\tau^{j}}{\tau^{i}}\right)\left(\frac{1+\tau^{i}}{1+\tau^{* i}}\right)}
$$

and from (41), (47) and (54) along with (46), (52) and (53) that

$$
\widetilde{T}(\boldsymbol{\tau})=\frac{\left(\widetilde{A}^{* 1}+\widetilde{A}^{* 2}\right) \widetilde{p}^{w 1} \widetilde{p}^{w 2}}{\widetilde{A}^{* 1} \widetilde{p}^{w 2}+\widetilde{A}^{* 2} \widetilde{p}^{w 1}}
$$

where

$$
\widetilde{A}^{* j}(\boldsymbol{\tau})=\left(Q_{x}^{* j}\left(\frac{\widetilde{p}^{w j}}{\tau^{* j}}\right)-Q_{y}^{* j}\right)\left(\frac{\tau^{* j}}{\tau^{* j}+1}\right)
$$

## 5 Endowment Specifications

At this point, for a CD model with arbitrarily fixed endowment levels, we have a representation for each of the three countries of its welfare as a function of the underlying four tariffs. Note that our derivations are all made under the assumption that the four tariffs are non-prohibitive. Our next step is to specify endowment level relationships and characterize how the resulting welfare
functions depend on tariffs. We consider two specifications: a symmetric specification and an asymmetric specification.

## Symmetric specification

In our symmetric specification, we parameterize endowments with a single parameter $\gamma \in(0,1 / 2)$ according to

$$
\begin{aligned}
Q_{x} & =\gamma, Q_{y}=1-\gamma \\
Q_{x}^{* j} & =\frac{1-\gamma}{2}, Q_{y}^{* j}=\frac{\gamma}{2}, j=1,2 .
\end{aligned}
$$

Then we have, for $\gamma \in(0,1 / 2)$,

$$
\begin{aligned}
W^{* j}(\boldsymbol{\tau}) & =\left(\frac{\widetilde{p}^{w j}\left(\frac{1-\gamma}{2}\right)+\frac{\gamma}{2}}{1+\tau^{* j}}\right)^{2} \frac{\tau^{* j}}{\widetilde{p}^{w j}}, j=1,2 \text { and } \\
W(\boldsymbol{\tau}) & =\left(\frac{\gamma \widetilde{T}+1-\gamma}{\tau^{1} \widetilde{p}^{w 1}+\widetilde{T}}\right)^{2} \tau^{1} \widetilde{p}^{w 1}
\end{aligned}
$$

where

$$
\widetilde{p}^{w j}(\boldsymbol{\tau})=\frac{1-\gamma+\frac{\gamma}{2}\left[\left(\frac{\tau^{* j}\left(1+\tau^{j}\right.}{1+\tau^{* j}}\right)+\left(\frac{\tau^{* i}\left(1+\tau^{i}\right)}{11 \tau^{* i}}\right)\right]}{\tau^{j} \gamma+\left(\frac{1-\gamma}{2}\right)\left[\left(\frac{1+\tau^{j}}{1+\tau^{* j}}\right)+\left(\frac{\tau^{j}}{\tau^{i} i}\right)\left(\frac{1+\tau^{i}}{1+\tau^{* i}}\right)\right]}, i, j=1,2, i \neq j
$$

and where

$$
\widetilde{T}(\boldsymbol{\tau})=\frac{\left(\widetilde{A}^{* 1}+\widetilde{A}^{* 2}\right) \widetilde{p}^{w 1} \widetilde{p}^{w 2}}{\widetilde{A}^{* 1} \widetilde{p}^{w 2}+\widetilde{A}^{* 2} \widetilde{p}^{w 1}}
$$

with

$$
\widetilde{A}^{* j}(\boldsymbol{\tau})=\left(\left(\frac{1-\gamma}{2}\right)\left(\frac{\widetilde{p}^{w j}}{\tau^{* j}}\right)-\frac{\gamma}{2}\right)\left(\frac{\tau^{* j}}{\tau^{* j}+1}\right), i, j=1,2, i \neq j .
$$

Let's now also state the conditions under which the four tariffs are nonprohibitive. To state these conditions, note that the autarky price for the home country, $p_{a}$, is given by

$$
p_{a}=\frac{1-\gamma}{\gamma},
$$

while the autarky price of foreign country $* j, p_{a}^{* j}$, is given by

$$
p_{a}^{* j}=\frac{\gamma}{1-\gamma} \text { for } j=1,2 .
$$

Any set of four tariffs are non-prohibitive as long as they satisfy $p<p_{a}$ and $p^{* j}>p_{a}^{* j}$ for $j=1,2$. Finally, as long as the four tariffs are non-prohibitive, we have from (5) that $p=\tau^{1} p^{w 1}$ and $p^{* j}=p^{w j} / \tau^{* j}$. Using the expressions above for world prices, we can thus state the conditions that define non-prohibitive
combinations of tariffs:

$$
\begin{aligned}
p & <p_{a} \Longleftrightarrow \tau^{1} \frac{1-\gamma+\frac{\gamma}{2}\left[\left(\frac{\tau^{* 1}\left(1+\tau^{1}\right)}{1+\tau^{* 1}}\right)+\left(\frac{\tau^{* 2}\left(1+\tau^{2}\right)}{1+\tau^{* 2}}\right)\right]}{\tau^{1} \gamma+\frac{(1-\gamma)}{2}\left[\left(\frac{1+\tau^{1}}{1+\tau^{* 1}}\right)+\left(\frac{\tau^{1}}{\tau^{2}}\right)\left(\frac{1+\tau^{2}}{1+\tau^{* 2}}\right)\right]}<\frac{1-\gamma}{\gamma} \\
p^{* 1} & >p_{a}^{* 1} \Longleftrightarrow \frac{1-\gamma+\frac{\gamma}{2}\left[\left(\frac{\tau^{* 1}\left(1+\tau^{1}\right)}{1+\tau^{* 1}}\right)+\left(\frac{\tau^{* 2}\left(1+\tau^{2}\right)}{1+\tau^{* 2}}\right)\right]}{\tau^{1} \gamma+\frac{(1-\gamma)}{2}\left[\left(\frac{1+\tau^{1}}{1+\tau^{* 1}}\right)+\left(\frac{\tau^{1}}{\tau^{2}}\right)\left(\frac{1+\tau^{2}}{1+\tau^{* 2}}\right)\right]} \frac{1}{\tau^{* 1}}>\frac{\gamma}{1-\gamma} \\
p^{* 2} & >p_{a}^{* 2} \Longleftrightarrow \frac{1-\gamma+\frac{\gamma}{2}\left[\left(\frac{\tau^{* 2}\left(1+\tau^{2}\right)}{1+\tau^{* 2}}\right)+\left(\frac{\tau^{* 1}\left(1+\tau^{1}\right)}{1+\tau^{* 1}}\right)\right]}{\tau^{2} \gamma+\frac{(1-\gamma)}{2}\left[\left(\frac{1+\tau^{2}}{1+\tau^{* 2}}\right)+\left(\frac{\tau^{2}}{\tau^{1}}\right)\left(\frac{1+\tau^{1}}{1+\tau^{* 1}}\right)\right]} \frac{1}{\tau^{* 2}}>\frac{\gamma}{1-\gamma} .
\end{aligned}
$$

## Asymmetric specification

In our asymmetric specification, we include an additional parameter $\beta \in$ $(0,1)$ that captures any asymmetry in size between the two foreign countries. The asymmetric specification for endowments entails two parameters, $\gamma \in(0,1 / 2)$ and $\beta \in(0,1)$, where

$$
\begin{aligned}
Q_{x} & =\gamma, Q_{y}=1-\gamma \\
Q_{x}^{* 1} & =\beta(1-\gamma), Q_{y}^{* 1}=\beta \gamma \\
Q_{x}^{* 2} & =(1-\beta)(1-\gamma), Q_{y}^{* 2}=(1-\beta) \gamma
\end{aligned}
$$

Then we have, for $\gamma \in(0,1 / 2)$ and $\beta \in(0,1)$,

$$
\begin{aligned}
W^{* 1}(\boldsymbol{\tau}) & =\left(\frac{\widetilde{p}^{w 1} \beta(1-\gamma)+\beta \gamma}{1+\tau^{* 1}}\right)^{2} \frac{\tau^{* 1}}{\widetilde{p}^{w 1}} \\
W^{* 2}(\boldsymbol{\tau}) & =\left(\frac{\widetilde{p}^{w 2}(1-\beta)(1-\gamma)+(1-\beta) \gamma}{1+\tau^{* 2}}\right)^{2} \frac{\tau^{* 2}}{\widetilde{p}^{w 2}}, \text { and } \\
W(\boldsymbol{\tau}) & =\left(\frac{\gamma \widetilde{T}+1-\gamma}{\tau^{1} \widetilde{p}^{w 1}+\widetilde{T}}\right)^{2} \tau^{1} \widetilde{p}^{w 1}
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{p}^{w 1} & =\frac{1-\gamma+\beta \gamma\left(\frac{\tau^{* 1}\left(1+\tau^{1}\right)}{1+\tau^{* 1}}\right)+(1-\beta) \gamma\left(\frac{\tau^{* 2}\left(1+\tau^{2}\right)}{1+\tau^{* 2}}\right)}{\tau^{1} \gamma+\beta(1-\gamma)\left(\frac{1+\tau^{1}}{1+\tau^{* 1}}\right)+(1-\beta)(1-\gamma)\left(\frac{\tau^{1}}{\tau^{2}}\right)\left(\frac{1+\tau^{2}}{1+\tau^{* 2}}\right)} \\
\widetilde{p}^{w 2} & =\frac{1-\gamma+(1-\beta) \gamma\left(\frac{\tau^{* 2}\left(1+\tau^{2}\right)}{1+\tau^{* 2}}\right)+\beta \gamma\left(\frac{\tau^{* 1}\left(1+\tau^{1}\right)}{1+\tau^{* 1}}\right)}{\tau^{2} \gamma+(1-\beta)(1-\gamma)\left(\frac{1+\tau^{2}}{1+\tau^{* 2}}\right)+\beta(1-\gamma)\left(\frac{\tau^{2}}{\tau^{1}}\right)\left(\frac{1+\tau^{1}}{1+\tau^{* 1}}\right)}
\end{aligned}
$$

and where

$$
\widetilde{T}(\boldsymbol{\tau})=\frac{\left(\widetilde{A}^{* 1}+\widetilde{A}^{* 2}\right) \widetilde{p}^{w 1} \widetilde{p}^{w 2}}{\widetilde{A}^{* 1} \widetilde{p}^{w 2}+\widetilde{A}^{* 2} \widetilde{p}^{w 1}}
$$

with

$$
\begin{aligned}
& \widetilde{A}^{* 1}(\boldsymbol{\tau})=\beta\left((1-\gamma)\left(\frac{\widetilde{p}^{w 1}}{\tau^{* 1}}\right)-\gamma\right)\left(\frac{\tau^{* 1}}{\tau^{* 1}+1}\right) \\
& \widetilde{A}^{* 2}(\boldsymbol{\tau})=(1-\beta)\left((1-\gamma)\left(\frac{\widetilde{p}^{w 2}}{\tau^{* 2}}\right)-\gamma\right)\left(\frac{\tau^{* 2}}{\tau^{* 2}+1}\right)
\end{aligned}
$$

Finally, autarky prices and associated conditions for non-prohibitive tariffs may be derived for the asymmetric specification in a manner similar to the derivations above for the symmetric specification.

## 6 Characterizations

We now provide characterizations of Nash, efficient and Horn-Wolinsky tariffs under our symmetric and asymmetric endowment specifications, respectively.

## Symmetric Specification: Nash and Horn-Wolinsky Tariffs

We start with the symmetric specification. Table 1 provides numerical solutions for Nash tariffs at different values for $\gamma$. Observe that lower values for $\gamma$ indicate a greater difference between the endowments of the home country and the foreign countries and are thus associated with higher trade volumes. As the table illustrates, for the symmetric setting, the home-country tariffs are symmetric and the foreign-country tariffs are also symmetric. Symmetric tariffs are an outcome under the symmetric specification; we do not impose a symmetry restriction on the strategy space.

Table 1: Nash Equilibrium Tariff

|  | $\tau_{1}=\tau_{2}$ | $\tau_{1}^{*}=\tau_{2}^{*}$ |
| :--- | :---: | :---: |
| $\gamma=1 \mathrm{e}-6$ | 1225.83 | 1.99 |
| $\gamma=0.05$ | 5.26 | 1.66 |
| $\gamma=0.1$ | 3.54 | 1.50 |
| $\gamma=0.25$ | 1.92 | 1.24 |
| $\gamma=0.4$ | 1.27 | 1.08 |
| $\gamma=0.45$ | 1.13 | 1.04 |

We next consider the Horn-Wolinsky tariffs for the symmetric specification. The Horn-Wolinsky solution for the symmetric specification depends on the endowment parameter $\gamma$, the bargaining power parameter $\alpha$, and the choice of a threat point for bilateral negotiations. Recall also that an interior HornWolinsky solution must be on the bilateral efficiency frontier:

$$
\begin{equation*}
\frac{\partial W}{\partial \tau^{* i}} / \frac{\partial W}{\partial \tau^{i}}=\frac{\partial W^{* i}}{\partial \tau^{* i}} / \frac{\partial W^{* i}}{\partial \tau^{i}}, i=1,2 \tag{58}
\end{equation*}
$$

Table 2 presents our numerical characterization of the Horn-Wolinsky solution for different scenarios. Different values for $\gamma$ are again considered, and we also allow for different values of $\alpha$. We also consider two possible threat points for bilateral negotiations: we allow that a failed bilateral negotiation would
cause the tariffs under negotiation to revert to Nash equilibrium tariffs (NE) or to move to free trade (FT). The NE case can be motivated by a setting in which pre-negotiation tariffs are set at Nash levels, while the FT case can be motivated if we consider a situation in which bilateral negotiations are contemplated even though the intitial tariffs are already at free trade. In each case, the Horn-Wolinsky solution satisfies (58); thus, Table 2 illustrates interior HornWolinsky solutions under different parameter settings and threat-point cases. To see the effect of an increase in the home country's bargaining power $\alpha$, we may refer to the first two parameter specifications in the table. We see that the interior Horn-Wolinsky tariff for the home country (each foreign country) is higher (lower) when $\alpha$ is higher, where the differences are more pronounced under the NE threat point.

Table 2: Horn-Wolinsky Solution to the Tariff Negotiation

|  |  | Threat Point | $\tau_{1}^{h w}=\tau_{2}^{h w}$ | $\tau_{1}^{* h w}=\tau_{2}^{* h w}$ |
| :---: | :--- | :---: | :---: | :---: |
| $\gamma=0.25$ | $\alpha=0.5$ | NE | 0.85 | 0.75 |
|  |  | FT | 0.82 | 0.76 |
| $\gamma=0.25$ | $\alpha=0.75$ | NE | 1.06 | 0.66 |
|  |  | FT | 0.84 | 0.75 |
| $\gamma=0.4$ | $\alpha=0.5$ | NE | 0.922 | 0.895 |
|  |  | FT | 0.919 | 0.897 |
| $\gamma=0.05$ | $\alpha=0.5$ | NE | 1.05 | 0.51 |
|  |  | FT | 0.73 | 0.62 |

Note: NE is for Nash Equilibrium and FT is for Free Trade.

## Asymmetric Specification: Nash and Horn-Wolinsky Tariffs

We now consider the asymmetric specification. Table 3 illustrates the Nash tariffs for different values of $\gamma$ and $\beta$. Notice that the Nash tariffs are no longer symmetric, since the foreign countries are themselves asymmetric when $\beta \neq 1 / 2$. For simplicity, we consider specifications under which $\beta>1 / 2$ so that foreign country $* 1$ is larger than foreign country $* 2$.

Table 4 then provides the corresponding interior Horn-Wolinsky solutions for different values of $\gamma, \alpha$ and $\beta$ under Nash equilbrium and free-trade threat points, respectively.

## Quasi-concavity of the g function

In the Appendix of the main paper, we present an existence result for the Horn-Wolinsky solution. A key sufficient condition in this result is that the Nash bargaining solution objective, $g^{i}\left(s^{1}, s^{2}\right)$, is quasi-concave in $s^{i} \equiv\left(\tau^{i}, \tau^{* i}\right)$. We can use the framework developed here to explore the plausibility of this condition. To this end, we consider the symmetric specification and set $\gamma=.025$

Table 3: Nash Equilibrium Tariff

|  | $\beta$ | $\tau_{1}$ | $\tau_{2}$ | $\tau_{1}^{*}$ | $\tau_{2}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\gamma=0.25$ | $2 / 3$ | 1.85 | 2.00 | 1.35 | 1.15 |
|  | $5 / 6$ | 1.79 | 2.12 | 1.51 | 1.07 |
| $\gamma=0.4$ | $2 / 3$ | 1.26 | 1.30 | 1.12 | 1.05 |
|  | $5 / 6$ | 1.24 | 1.32 | 1.17 | 1.03 |

Table 4: Horn-Wolinsky Solution to the Tariff Negotiation

| Threat Point | $\gamma$ | $\beta$ | $\alpha$ | $\tau_{1}$ | $\tau_{2}$ | $\tau_{1}^{*}$ | $\tau_{2}^{*}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0.25 | $2 / 3$ | 0.5 | 0.876 | 0.849 | 0.797 | 0.708 |
|  | 0.25 | $2 / 3$ | 0.75 | 1.056 | 1.067 | 0.715 | 0.619 |
|  | 0.25 | $5 / 6$ | 0.5 | 0.929 | 0.888 | 0.871 | 0.666 |
| NE | 0.25 | $5 / 6$ | 0.75 | 1.064 | 1.095 | 0.795 | 0.582 |
|  | 0.4 | $2 / 3$ | 0.5 | 0.937 | 0.924 | 0.918 | 0.876 |
|  | 0.4 | $2 / 3$ | 0.75 | 1.002 | 1.004 | 0.881 | 0.832 |
|  | 0.4 | $5 / 6$ | 0.5 | 0.965 | 0.945 | 0.950 | 0.855 |
|  | 0.4 | $5 / 6$ | 0.75 | 1.011 | 1.019 | 0.918 | 0.812 |
|  | 0.25 | $2 / 3$ | 0.5 | 0.844 | 0.814 | 0.815 | 0.724 |
|  | 0.25 | $2 / 3$ | 0.75 | 0.857 | 0.838 | 0.809 | 0.710 |
|  | 0.25 | $5 / 6$ | 0.5 | 0.898 | 0.833 | 0.889 | 0.697 |
| FT | 0.25 | $5 / 6$ | 0.75 | 0.904 | 0.856 | 0.888 | 0.681 |
|  | 0.4 | $2 / 3$ | 0.5 | 0.932 | 0.918 | 0.921 | 0.879 |
|  | 0.4 | $2 / 3$ | 0.75 | 0.937 | 0.929 | 0.919 | 0.872 |
|  | 0.4 | $5 / 6$ | 0.5 | 0.957 | 0.929 | 0.954 | 0.866 |
|  | 0.4 | $5 / 6$ | 0.75 | 0.909 | 0.914 | 0.903 | 0.838 |

Note: NE is for Nash Equilibrium and FT is for Free Trade.
and $\alpha=0.5$. We focus on the graph of $g^{1}\left(s^{1}, s^{2}\right)$ on the $s^{1}$ domain, under the assumption that the tariffs between the home country and foreign country $* 2$ are fixed at their Nash levels. We assume as well that the threat point for the negotiation between the home country and foreign country $* 1$ also corresponds to Nash tariffs. As the numerical graph in Figure 1 illustrates, the function $g^{1}\left(s^{1}, s^{2}\right)$ then assumes a quasi-concave appearance, providing support for the plausibility of the sufficient condition.

Figure 1: Quasi-concavity of the g-function


## 7 References

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[^1]:    ${ }^{1}$ We also assume that tariffs are set at non-prohibitive levels. We re-visit this assumption in Section 5.
    ${ }^{2}$ Our analysis builds on the approach used by Bagwell and Staiger (1999, 2002, 2005, 2018) for settings with production and general consumer and government preferences. By studying an endowment economy with CD consumer preferences and national-welfare maximizing governments, we are able to obtain closed-form solutions for welfare functions in terms of tariffs.

[^2]:    ${ }^{3}$ This expression for tariff revenue can be confirmed following the simplification for income given just below, focusing on the last term.

[^3]:    ${ }^{4}$ See Bagwell and Staiger (1999, 2002, 2005, 2018) for related discussion in settings with production and general consumer and government preferences.

[^4]:    ${ }^{5}$ The expression for tariff revenue can be confirmed following the simplification for income given just below, focusing on the last term.

[^5]:    ${ }^{6}$ See Bagwell and Staiger (1999, 2002, 2005, 2018) for settings with production and general consumer and government preferences.

