Entry and Welfare in General Equilibrium with Heterogeneous Firms and Endogenous Markups*

Kyle Bagwell
Stanford University and NBER

Seung Hoon Lee
Georgia Institute of Technology

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Abstract

We consider the efficiency of market entry in single- and two-sector versions of the Melitz-Ottavanio (MO) model, where differently from the MO model our two-sector model does not involve an outside good. For the one-sector MO model, we show that the market level of entry achieves a (local) welfare maximum. For a two-sector MO model without an outside good, we show that the welfare results are exactly similar to those in the one-sector model when the two sectors are symmetric. When the two sectors are asymmetric and the level of asymmetry is sufficiently small, we identify a perturbation indicating a sense in which the market level of entry into the "high-demand" sector is excessive. This intersectoral misallocation occurs at the market equilibrium even though endogenous average markups are equal across sectors. We also show how the outcomes induced by the planner's direct choice of entry levels alternatively can be induced through the appropriate choice of entry tax/subsidy policies.

*Authors' email addresses: kbagwell@stanford.edu and seunghoon@econ.gatech.edu. We thank Andres Rodriguez-Clare for helpful comments.

1 Introduction

We consider the efficiency of market entry in single- and two-sector versions of the Melitz and Ottavanio (2008) model, where differently from the Melitz-Ottaviano (MO) model our two-sector model does not involve an outside good. For each model version, our goal is to determine whether the market level of entry is efficient. When an inefficiency exists, we also characterize welfare-improving adjustments in entry levels.

The MO model has two sectors, where one sector is a differentiated sector and the other sector is an outside good. Bagwell and Lee (2020) examine trade policy in the MO model. They also examine the efficiency of market entry in a closed-economy model, finding that entry into the differentiated sector is excessive (inadequate) (efficient) if and only if $\alpha > 2 \cdot c_D^m$ ($\alpha < 2 \cdot c_D^m$) ($\alpha = 2 \cdot c_D^m$), where α is a demand parameter with higher values indicating a greater preference for differentiated goods relative to the outside good and where c_D^m is the cutoff cost level for surviving varieties as determined in the market equilibrium. The cutoff level c_D^m is independent of α in this outside-good model. As Bagwell and Lee discuss, an understanding of the efficiency properties of market entry is essential for understanding trade policy and agreements in the two-country MO model of trade.

We focus here on the efficiency of market entry in a closed-economy setting for formulations of the MO model in which the outside good is absent. We first examine a single-sector version of the MO model. Demidova (2017) has previously studied optimal unilateral tariffs in this single-sector model. We focus here on the efficiency properties of the market level of entry.[1] Our first main result is that the market level of entry achieves a (local) welfare maximum in the single-sector model. This result obtains even though entry has external effects on firms and consumers. The result also carries the following specific implication: the entry inefficiency characterized by Bagwell and Lee is attributable to the fact that the MO model has multiple sectors.

We next examine whether the inefficiency of market entry in the MO model is sensitive to the way in which the "second" sector is modeled. To this end, we replace the assumption that the second sector is an outside-good sector with the alternative assumption that the second sector is another differentiated sector, where the upper-tier consumer utility function is additively separable across the two sectors. We conduct two exercises. In the first exercise, we consider a symmetric setting in which the demand paramater α takes the same value in both sectors: $\alpha_1 = \alpha_2$. For this symmetric setting, we analyze the implications of a small perturbation in which the planner symmetrically changes the levels

[1] Demidova (2017) notes that the level of entry in the single-sector model is in fact independent of tariffs and trade costs. As we discuss further in footnote 8, we allow the planner directly to choose the level of entry, where the resulting market outcomes can be replicated with the appropriate selection of entry tax/subsidy policies.

of entry in the two sectors. Our second main result is that, for this symmetric setting and relative to this perturbation class, the market level of entry satisfies the first-order conditions for welfare maximization, just as in the one-sector model. Our second exercise allows that the sectors may be asymmetric, in that the demand parameter α is allowed to take different values across the sectors: $\alpha_1 \neq \alpha_2$ is allowed. For this setting, we consider a small perturbation in which the planner increases the level of entry into sector 1 while simultaneously decreasing the level of entry into sector 2 in such a manner as to ensure that the marginal utility of income λ for the consumer is unaltered. For sufficiently small asymmetries, we show that this perturbation raises (lowers) (does not change) welfare if and only if $\alpha_1 < \alpha_2$ $(\alpha_1 > \alpha_2)$ $(\alpha_1 = \alpha_2)$. Thus, in this sense, the market provides excessive entry into the sector $s \in \{1, 2\}$ with the highest value for α_s.

To interpret our findings, we begin with the single-sector model. Additional entry in this model is consistent with a resource constraint, since additional entry also impacts variety-level consumption through its impact on the marginal utility of income λ and the critical cost cutoff level c_D. We find that the market trades off these considerations in an efficient manner: the market level of entry achieves a (local) welfare maximum in the single sector model. We show that this finding can be understood by considering the impact of additional entry on aggregate output.[2] Starting at the market equilibrium, we find that additional entry introduces offsetting effects on λ and c_D such that the number of surviving varieties (the extensive margin) and the expected variety-level output conditional on survival (the intensive margin) are each unaffected to the first order, ensuring that aggregate output and thus welfare are also unaffected to the first order.

For the two-sector model, we show that our first exercise may be interpreted in an manner that is analogous to the interpretation just given for the one-sector model. Just as in our analysis of the one-sector model, the symmetric change in entry levels induces a change in λ and impacts variety-level consumption through this channel. Indeed, when the two-sector model has a symmetric setting and is subjected to a symmetric change in sectoral entry levels, the results are exactly similar to those in the one-sector model.

Our second exercise for the two-sector model, however, introduces an additional consideration, since in the asymmetric two-sector model the market may misallocate resources *across* sectors. To isolate this consideration, we start at the market equilibrium and increase the level of entry into the first sector while adjusting the level of entry into the second sector so as to ensure that the marginal utility of income λ is unchanged. We show that a small perturbation of this kind necessarily involves a reduction in the level of entry into the second sector. With this experiment, we thus eliminate intensive margin effects that are induced via a change in λ. Assuming that the level of asymmetry is sufficiently

[2] As Demidova (2017) shows, welfare can be expressed as a function of aggregate output in the one-sector MO model.

small, we show that this perturbation lowers welfare when $\alpha_1 > \alpha_2$, a finding that in this sense is consistent with a business-stealing intuition under which the market provides excessive entry into "high-demand" sectors.

Our second exercise shares qualitative features with the closed-economy analysis by Bagwell and Lee (2020); in both cases, the marginal utility of income λ remains fixed, and so variety-level consumption is not impacted by changes in λ. In addition, we find that the market provides excessive entry into the sector s with the highest value for α_s, a finding which is broadly analogous to the findings by Bagwell and Lee (2020) regarding excessive entry into the differentiated sector in the model with an outside good when α is high (namely, when $\alpha > 2 \cdot c_D^m$ in that model).[3]

We do not intend to argue against the value of (partial-equilibrium) models with an outside-good sector. Such models are highly tractable and provide valuable insights for a range of policy analyses. At the same time, some policies may generate income effects that invite a general-equilibrium analysis. We also note that two-sector models with an imperfectly competitive sector and an outside-good sector are typically structured in such a way as to impose intersectoral markup heterogeneity: markups are typically positive in the imperfectly competitive sector and absent in the outside-good sector. This built-in asymmetry can have implications for resource misallocation.

By comparison, in the two-sector model considered in the current paper, the average markup is symmetric across sectors, even when preferences are asymmetric ($\alpha_1 \neq \alpha_2$).[4] In this way, we shut down the possibility that resources are misallocated due to markup asymmetry across sectors. Since both sectors are imperfectly competitive, our approach also differs in that a reallocation of entry across sectors creates potential externalities for consumer and producer interests in both sectors. These externalities account for the welfare gain from entry reallocation that we establish for the two-sector model with asymmetric preferences.

Our research relates interestingly to work by Epifani and Gancia (2011). They examine a multi-sector model that features between- but not within-sector heterogeneity. For a class of models, they show that, under free entry and when the preference for variety differs across sectors, there exists no markup distribution such that the market equilibrium

[3] At a broad level, our second exercise thus suggests directions in which the qualitative findings of Bagwell and Lee (2020) may extend to a multi-sector MO model without an outside good. A complete analysis of this relationship, however, would require placing the multi-sector model considered here into a two-country model of trade policies and agreements. This is beyond the scope of the current effort but is an important direction for future research.

[4] We define the average markup in a sector as the ratio of the average price to the average marginal cost in that sector, where averages are taken over surviving firms. The average markup is equal across sectors in our two-sector model, even after entry is reallocated away from the market equilibrium level. We note as well that, under free entry, the difference between the average price and average marginal cost in a sector is also independent of the sector at the market equilibrium. This difference, however, may become asymmetric across sectors following a reallocation of entry.

replicates the first-best allocation. Markup symmetry is thus not sufficient for first-best efficiency in this setting. Similarly, in our analysis of the two-sector model with asymmetric preferences ($\alpha_1 \neq \alpha_2$) across sectors, we establish a welfare gain from a reallocation of entry across sectors even through the average markup does not differ across sectors. But our analysis also differs in several respects. We include within-sector firm heterogeneity, focus on second-best efficiency and show in this context that market entry is efficient in single-sector and symmetric two-sector models, and establish a specific welfare-improving entry reallocation in our asymmetric two-sector model that entails reducing entry into the "high-demand" sector.

In other related work, Campolmi et al (2014) consider a two-sector monopolistic competition model, where CES preferences are specified for the differentiated sector, the other sector is an outside good sector and the upper-tier utility function takes a Cobb-Douglas form. They find that the market level of entry is inefficient and too low, and they show that a wage subsidy that targets the monopolistic distortion can implement the first-best outcome. Bagwell and Lee (2018) focus on the efficiency of entry in a two-sector model of monopolistic competition, where CES preferences and heterogeneous firms are specified for the differentiated sector, the other sector is an outside-good sector and the upper-tier utility function takes an additively separable form. They also find that the market level of entry is inefficient and too low. Dhingra and Morrow (2019) consider a family of one-sector monopolistic competition models with heterogeneous firms and additively separable preferences, and they show that the market outcome is first best under CES preferences. The preferences that we consider here do not fit in the family that they consider, and we also restrict attention to second-best intervention that targets the number of entrants. Like Bagwell and Lee (2020), Nocco et al (2014) analyze some of the efficiency properties of the market outcome in the original MO model with an outside good. See Bagwell and Lee (2020) for a detailed discussion of the differences between these two papers.[5]

Finally, our paper is related to an Industrial Organization literature that considers the efficiency of entry in an imperfectly competitive sector when firms are symmetric and an outside-good sector also exists. Prominent contibutions to this literature include Mankiw and Whinston (1986) and Spence (1976). We share with this literature a focus on the second-best problem of a planner who can control the number of firms but not the conduct of firms. This literature finds that the level of entry is typically inefficient, due to the associated business-stealing and consumer-surplus externalities. Differently from this research, we eliminate the outside-good sector, include heterogeneous firms, and find that the market level of entry is efficient in the benchmark one-sector model.

Our analysis is organized as follows. In Section 2, we develop the one-sector MO model

[5]See also Spearot (2016) for a multi-sector, multi-country version of the MO model in which the outside-good sector is removed. He provides counterfactual analyses of several trade-policy shocks.

and present our welfare finding for this model. In Section 3, we present the two-sector MO model while allowing for entry policies (i.e., subsidies or taxes for the cost of entry). We then analyze our two welfare exercises for the two-sector MO model in Section 4. In Section 5, we show the outcomes induced by the planner's direct choice of entry levels alternatively can be induced by an appropriate choice of entry tax/subsidy policies, and vice versa. Section 6 concludes. Remaining proofs are contained in the Appendix.

2 One-sector MO model

In this section, we assume:

1. A planner decides the number of entrants N_E.

2. The mass of N_E firms observe their marginal production costs c's and decide how much to produce (including the case of not producing). The decisions of N_E firms determine prices $p(c)$ and the number of varieties $N = G(c_D) N_E$, where G refers to the Pareto distribution function and c_D refers to the marginal cost of a firm indifferent between producing or not.

3. For the given prices and varieties, consumers maximizes utility.

2.1 Consumer's problem

The economy contains a unit mass of identical consumers, each supplying a unit of labor in inelastic fashion to a competitive labor market. We normalize the wage as 1. Consumers also hold symmetric shares of any aggregate net profit, the value of which an individual consumer takes as fixed when choosing consumption.

With respect to #3 above, the consumer's welfare maximization problem can be written as follows

$$\max_{q_i} U = \alpha \int_{i\in\Omega} q_i di - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i)^2 \, di - \frac{1}{2}\eta \left(\int_{i\in\Omega} q_i di \right)^2$$

s.t.

$$\int_{i\in\Omega} p_i q_i di = 1 + \Pi \tag{1}$$

where q_i and p_i represent the consumption and price of variety i in the set Ω of available varieties, wage income is normalized as 1 and Π refers to the aggregate net profit. We assume that the preference parameters α, γ and η are all positive.

To solve the consumer's problem, we consider a Lagrangian equation as follows

$$L = U + \lambda \left(1 + \Pi - \left(\int_{i \in \Omega} p_i q_i di\right)\right),$$

where $\lambda \geq 0$ is the multiplier for the consumer's optimization problem above. Letting $Q \equiv \int_{i \in \Omega} q_i di$, we represent the FOC with respect to q_i as

$$\alpha - \gamma \cdot q_i - \eta \cdot Q = \lambda p_i. \tag{2}$$

Integrating (2) over the set of varieties for which $q_i > 0$ and letting N be the measure of consumed varieties in Ω, we obtain

$$\alpha - \gamma \cdot \frac{Q}{N} - \eta \cdot Q = \lambda \bar{p}$$

and thus

$$\frac{\alpha - \lambda \bar{p}}{\eta + \frac{\gamma}{N}} = Q,$$

where $\bar{p}$ is the average price of consumed varieties.

Using (2), we see that $\frac{(\alpha - \eta \cdot Q - \gamma \cdot q_i)}{\lambda} = p_i$ for consumed varieties. Let us now define $p^{\max}$ as the "choke price." From the foregoing, we may confirm

$$p^{\max} \equiv \frac{\alpha - \eta \cdot Q}{\lambda} = \frac{1}{\lambda}\left(\frac{\gamma \cdot \alpha + \lambda \cdot \eta \cdot N \cdot \bar{p}}{\eta \cdot N + \gamma}\right). \tag{3}$$

Using (2) and (3), the inverse demand can be written as

$$p(q) = p^{\max} - \frac{\gamma}{\lambda} \cdot q. \tag{4}$$

2.2 Firm's problem

With respect to #2 above, profit maximization for a firm with marginal production cost c delivers the profit function[6]

$$\pi(c) = \max_q (p(q) - c) q$$

Using (4), we characterize the solution to the firm's problem as

$$q(c) = \frac{\lambda (p^{\max} - c)}{2\gamma}. \tag{5}$$

[6] Since firms are infinitesimal in size, we assume that each firm takes λ as given when solving its profit maximization problem.

This solution induces profit-maximizing price and maximized profit of the firm

$$p(c) = \frac{p^{\max} + c}{2} \tag{6}$$

$$\pi(c) = \frac{\lambda}{4\gamma}\left(p^{\max} - c\right)^2 .$$

For this model of monopolistic competition, a firm takes the demand intercept as given and produces a positive quantity of its variety provided that its cost realization is not higher than the intercept. In other words, a Zero Cutoff Profit (ZCP) condition determines the cost cutoff c_D as

$$\pi(c_D) = 0.$$

or equivalently

$$p^{\max} = p(c_D) = c_D.$$

Costs are distributed according to a Pareto distribution

$$G(c) = \left(\frac{c}{c_M}\right)^k$$

for $c \in [0, c_M]$ where $k > 1$ and $c_M > 0$.[7] Given this distribution, we have that

$$\bar{c} \equiv E(c|c \leq c_D) = \left(\frac{k}{k+1}\right) c_D.$$

Using (6) and $p^{\max} = c_D$, we find

$$\bar{p} \equiv E(p(c)|c \leq c_D) = \left(\frac{c_D + \bar{c}}{2}\right).$$

It now follows that

$$\bar{p} = \left(\frac{2k+1}{2(k+1)}\right) c_D.$$

Observe that that the average markup, $\bar{\mu} \equiv \bar{p}/\bar{c}$, is a simple function of the parameter k:

$$\bar{\mu} = \frac{2k+1}{2k}.$$

Referring again to (3) and using $p^{\max} = c_D$, we can represent the number of varieties as

$$N = \frac{\gamma\left(\alpha - \lambda \cdot p^{\max}\right)}{\lambda \cdot \eta \cdot \left(p^{\max} - \bar{p}\right)} = \frac{\gamma\left(\alpha - \lambda \cdot c_D\right)}{\lambda \cdot \eta\left(c_D - \bar{p}\right)}.$$

[7] We assume throughout this section that $c_M > c_D$.

Plugging in the expression just derived for $\overline{p}$, we obtain

$$N = \frac{2\,(k+1)\,\gamma}{\eta}\frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot c_D}. \tag{7}$$

Using only (7), we see that, once values for c_D and λ are obtained, the value for the number of available varieties is determined. For $c_D > 0$, we note that N is strictly decreasing with respect to c_D for a given value of $\lambda > 0$.

The number of available varieties can also be expressed as a function of the level of entry and the cost cutoff level as

$$N = N_E \cdot G(c_D). \tag{8}$$

Hence, using (8), the value for the number of available varieties can also be determined given the number of entrants and the cost cutoff level.

Finally, using (7), (8) and the Pareto distribution, we can express the relation between N and N_E as:

$$N_E = \frac{N}{G\,(c_D)} = \frac{2\,(k+1)\,\gamma\,(c_M)^k}{\eta}\frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot (c_D)^{k+1}}. \tag{9}$$

The expression in (9) will be an important ingredient in our analysis below, when we explore the implications of different values for N_E for λ, c_D and consumer welfare.[8]

2.3 Planner's problem

We are now ready to proceed to #1 above and consider the planner's choice of N_E. The planner seeks to choose N_E so as to maximize consumer welfare under (i) a resource constraint derived from (1), (ii) a constraint on the relationship between N_E, c_D and λ as given in (9), and (iii) a profit-maximizing constraint under which the quantity of variety i consumed is determined by the corresponding firm's cost realization and profit-maximizing output (including zero), as implied by (5) and $p^{\max} = c_D$. To state the planner's problem, we proceed by showing that the objective and constraints can be written in terms of α, N_E, c_D and λ.[9]

[8] In Demidova (2017), the number of entrants is shown to be independent of trade costs and tariffs. In this paper, we allow the planner directly to choose the number of entrants. As we show in Section 5 for a two-sector model, the planner's choice can be replicated with the appropriate selection of entry tax/subsidy policies. We thus use a different policy instrument from Demidova.

[9] For later use in a multi-sector setup, we include α as an independent variable for our objective and constraint functions as defined below.

We start with the objective function:

$$U = \alpha \int_{i \in \Omega} q_i di - \frac{1}{2}\gamma \int_{i \in \Omega} (q_i)^2 \, di - \frac{1}{2}\eta \left(\int_{i \in \Omega} q_i di \right)^2 . \tag{10}$$

To simplify U, we use (5), $p^{\max} = c_D$, (9) and the Pareto distribution to show that we may re-write each term in (10) in terms of c_D and λ. This is accomplished through the establishment of two claims.

The first claim is that

$$\int_{i \in \Omega} q_i di = N_E \int_0^{c_D} q(c) \, dG(c) = \frac{1}{\eta} (\alpha - \lambda \cdot c_D) . \tag{11}$$

The first equality in (11) is true, given profit-maximizing behavior. For the second equality, using (5), setting $p^{\max} = c_D$, and using the Pareto distribution, we find that

$$\int_0^{c_D} q(c) \, dG(c) = \frac{(c_D)^{k+1} (c_M)^{-k} \lambda}{2(1+k)\gamma}.$$

Using this expression and (9), we thus have

$$\begin{aligned} N_E \int_0^{c_D} q(c) \, dG(c) &= \frac{2(k+1)\gamma (c_M)^k}{\eta} \frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot (c_D)^{k+1}} \frac{(c_D)^{k+1} (c_M)^{-k} \lambda}{2(1+k)\gamma} \\ &= \frac{\alpha - \lambda \cdot c_D}{\eta}, \end{aligned}$$

confirming (11). With (11) established, we can re-write the first and third terms in (10) in terms of c_D and λ.

The second claim is that

$$\int_{i \in \Omega} (q_i)^2 \, di = N_E \int_0^{c_D} q(c)^2 \, dG(c) = \frac{1}{\eta} \frac{(\alpha - \lambda \cdot c_D)(c_D)\lambda}{\gamma(2+k)}. \tag{12}$$

The first equality in (12) is again true, given profit-maximizing behavior. For the second equality, we again use (5), set $p^{\max} = c_D$, and use the Pareto distribution. We find that

$$\int_0^{c_D} q(c)^2 \, dG(c) = \frac{(c_D)^{k+2} (c_M)^{-k} \lambda^2}{2(1+k)(2+k)\gamma^2}.$$

Using this expression and (9), we thus have

$$
\begin{aligned}
N_E \int_0^{c_D} q(c)^2 \, dG(c) &= \frac{2(k+1)\gamma (c_M)^k}{\eta} \frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot (c_D)^{k+1}} \frac{(c_D)^{k+2} (c_M)^{-k} \lambda^2}{2(1+k)(2+k)\gamma^2} \\
&= \frac{1}{\eta} \frac{(\alpha - \lambda \cdot c_D) \lambda c_D}{\gamma (2+k)},
\end{aligned}
$$

confirming (12). With (12) established, we can re-write the second term in (10) in terms of c_D and λ.

With the two claims established, we now plug (11) and (12) into (10). After simplification, we obtain that

$$
\begin{aligned}
U &= \alpha \int_{i \in \Omega} q_i di - \frac{1}{2}\gamma \int_{i \in \Omega} (q_i)^2 \, di - \frac{1}{2}\eta \left(\int_{i \in \Omega} q_i di \right)^2 \\
&= \frac{(\alpha - \lambda \cdot c_D)}{2\eta} \left(\alpha + \left(\frac{1+k}{2+k} \right) \lambda \cdot c_D \right).
\end{aligned}
$$

We may thus re-write U as a function of (α, c_D, λ). Formally, we write

$$
U = u(\alpha, c_D, \lambda)
$$

where

$$
u(\alpha, c_D, \lambda) \equiv \frac{(\alpha - \lambda \cdot c_D)}{2\eta} \left(\alpha + \left(\frac{1+k}{2+k} \right) \lambda \cdot c_D \right). \tag{13}
$$

We turn next to the resource constraint as given by (1). In order to simplify the planner's problem, we rewrite (1) under utility- and profit-maximizing behavior as

$$
N_E \int_0^{c_D} p(c) \, q(c) \, dG(c) = 1 + N_E \left[\int_0^{c_D} (p(c) - c) \, q(c) \, dG(c) - f_e \right],
$$

where the bracketed term on the RHS of this equation is the expected profit for a firm that incurs the fixed cost $f_e > 0$ to observe its cost realization. After simplification, the resource constraint takes the following form:

$$
N_E \left(\int_0^{c_D} c \cdot q(c) \, dG(c) + f_e \right) = 1. \tag{14}
$$

Using (5), $p^{\max} = c_D$, (9) and the Pareto distribution, and after simplification, we can write the resource constraint (14) as

$$
R(\alpha, c_D, \lambda) = \frac{\eta (2+k)}{\gamma \cdot \phi} \tag{15}
$$

where

$$R(\alpha, c_D, \lambda) \equiv \frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot (c_D)^{k+1}} \left(\frac{(c_D)^{k+2} k \cdot \lambda}{\gamma \cdot \phi} + 1 \right) \tag{16}$$

and $\phi = 2(k+1)(k+2)(c_M)^k f_e$.

Next, using (9), we also define the number of entrants N_E as a function of (α, c_D, λ) as below:

$$N_E = N_e(\alpha, c_D, \lambda) \equiv \frac{2(k+1)\gamma (c_M)^k}{\eta} \frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot (c_D)^{k+1}}. \tag{17}$$

Finally, we have already embedded the profit-maximizing constraints into our representations of the utility function, the resource constraint and the costraint on the relationship between N_E, c_D and λ as given as given by (13), (15), (16) and (17), respectively.

Therefore, *the planner's problem* can be represented as:

$$\max_{N_E} u(\alpha, c_D, \lambda)$$

s.t.

$$R(\alpha, c_D, \lambda) = \frac{\eta(2+k)}{\gamma \cdot \phi} \tag{18}$$

and

$$N_E = N_e(\alpha, c_D, \lambda) > 0 \tag{19}$$

where $u(\alpha, c_D, \lambda)$, $R(\alpha, c_D, \lambda)$ and $N_e(\alpha, c_D, \lambda)$ follow from (13), (16) and (17), respectively.

To understand the planner's problem, suppose that the planner entertains a specific value for N_E. Given this value, we may regard the constraints (18), and (19) as defining a 2×2 system of equations, in which c_D and λ are endogenous while N_E is exogenous. Accordingly, we can conduct a traditional comparative statics exercise to determine how c_D and λ vary with respect to N_E. We can then feed this information into the planner's optimization problem with respect to N_E. Finally, while our representation of the planner's problem does not explicitly include the number of available varieties, N, we recall from (7) that this value can be easily recovered once c_D and λ are determined.

For the planner's problem, our main goal is to determine whether, starting at market equilibrium, additional entry raises welfare or not. To this end, we first define the market equilibrium solution, $(c_D^{mkt}, \lambda^{mkt}, N_E^{mkt})$ as the solution to the 3×3 system of equations that emerges when constraints (18), and (19) are joined with a third equation, the Free Entry condition, which is defined as follows:

$$\int_0^{c_D^{mkt}} (p(c) - c)\, q(c)\, dG(c) = f_e. \tag{20}$$

For our purposes here, the key property of the market solution is that a relationship between c_D^{mkt} and λ^{mkt} is implied:

$$\lambda^{mkt} = \frac{\gamma \cdot \phi}{\left(c_D^{mkt}\right)^{2+k}}. \tag{21}$$

This relationship follows from (20), after using (5), (6), setting $p^{\max} = c_D$, and using the Pareto distribution. By plugging (21) into (18) and (19), we can then pin down c_D^{mkt}, λ^{mkt}, and N_E^{mkt}.

Let us now represent the solutions to the 2×2 system of constraints (18) and (19) as $c_D(N_E)$ and $\lambda(N_E)$. We can then represent the first order condition for the planner's problem as

$$\frac{du}{dN_E}|_{N_E=N_E^{mkt}} = \frac{\partial u}{\partial c_D}\frac{dc_D}{dN_E} + \frac{\partial u}{\partial \lambda}\frac{d\lambda}{dN_E}|_{N_E=N_E^{mkt}}, \tag{22}$$

where for reasons just discussed we focus on evaluating this condition at the market equilibrium solution. We show below that, at the market solution, $\frac{\partial u}{\partial c_D} < 0$, $\frac{\partial u}{\partial \lambda} < 0$, $\frac{dc_D}{dN_E} < 0$ and $\frac{d\lambda}{dN_E} > 0$. We also provide below an interpretation of (22) in terms of the underlying changes induced by an increase in N_E, starting at the market solution.

Formally, we maintain the assumption that a market solution exists satisfying (18), (19) and (20) and at which $N_E^{mkt} > 0$, $c_D^{mkt} > 0$ and $\lambda^{mkt} > 0$.[10] We then appeal to the implicit function theorem to ensure the existence of a solution $(c_D(N_E), \lambda(N_E))$ to (18) and (19) for N_E sufficiently close to N_E^{mkt}. To use this theorem, we require that, at the market solution, the Jacobian determinant associated with (18) and (19) is non-zero. We find that, at the market solution,

$$\begin{aligned} |J| &\equiv \frac{\partial R}{\partial c_D}\frac{\partial N_E}{\partial \lambda} - \frac{\partial R}{\partial \lambda}\frac{\partial N_E}{\partial c_D}|_{N_E=N_E^{mkt}} \\ &= -(\alpha - \lambda \cdot c_D)(\alpha + \lambda k c_D)\lambda k c_D^{k+2}|_{N_E=N_E^{mkt}} < 0, \end{aligned} \tag{23}$$

where the inequality in (23) follows from (17) and our assumption that $N_E^{mkt} > 0$, $c_D^{mkt} > 0$ and $\lambda^{mkt} > 0$.

Using the implicit function theorem, we also know that the derivatives $\frac{dc_D}{dN_E}$ and $\frac{d\lambda}{dN_E}$ exist for N_E sufficiently close to N_E^{mkt}. Calculations reveal that

$$\frac{dc_D}{dN_E}|_{N_E=N_E^{mkt}} = \frac{1}{|J|}\left(k\phi\gamma + \alpha c_D^{k+1}\right)\left(\frac{c_D}{\phi\gamma}\right)^2|_{N_E=N_E^{mkt}} < 0 \tag{24}$$

[10] Given our assumption that $f_e > 0$ and the assumption that $N_E^{mkt} > 0$, we can show that $c_D^{mkt} > 0$ and $\lambda^{mkt} > 0$, with both taking finite values.

and

$$\frac{d\lambda}{dN_E}|_{N_E=N_E^{mkt}} = \frac{-1}{|J|}\left(k\phi\gamma + \alpha c_D^{k+1}\right)\left(\frac{1}{\phi\gamma c_D^{k+1}}\right)|_{N_E=N_E^{mkt}} > 0 \tag{25}$$

where the inequalities follows from (23) and the imposition of (21).

Our next step is to calculate how utility varies with c_D and λ when evaluated at the market solution. Using (13) and imposing (21), we find

$$\frac{\partial u}{\partial c_D}|_{N_E=N_E^{mkt}} = \frac{-\gamma\phi}{2\eta(2+k)c_D^{3+2k}}\left(\alpha c_D^{1+k} + 2(1+k)\gamma\varphi\right)|_{N_E=N_E^{mkt}} < 0 \tag{26}$$

and

$$\frac{\partial u}{\partial \lambda}|_{N_E=N_E^{mkt}} = \frac{-1}{2\eta(2+k)c_D^{k}}\left(\alpha c_D^{1+k} + 2(1+k)\gamma\varphi\right)|_{N_E=N_E^{mkt}} < 0. \tag{27}$$

At this point, we have established the signs of all the derivatives at the market solution as reported just after (22).

We are now ready to evaluate the planner's first order condition at the market solution. Referring to (22), and using (23)-(27), we find that

$$\frac{du}{dN_E}|_{N_E=N_E^{mkt}} = \frac{\partial u}{\partial c_D}\frac{dc_D}{dN_E} + \frac{\partial u}{\partial \lambda}\frac{d\lambda}{dN_E}|_{N_E=N_E^{mkt}} = 0. \tag{28}$$

Thus, the market level of entry satisfies the first order condition for the social planner.

To interpret (28), we first note that (21), (24) and (25) together imply that

$$\frac{d\lambda \cdot c_D}{dN_E}|_{N_E=N_E^{mkt}} = \lambda \cdot \frac{dc_D}{dN_E} + c_D \cdot \frac{d\lambda}{dN_E}|_{N_E=N_E^{mkt}} = 0, \tag{29}$$

which in turn implies from (7) that

$$\frac{dN}{dN_E}|_{N_E=N_E^{mkt}} = 0. \tag{30}$$

Thus, from (29) and (30), we see that starting at the market equilibrium, a higher level of entry has offsetting effects on c_D and λ, which serve to leave the number of varieties, N, unchanged. A higher level of entry also affects the average price and consumer income via the level of aggregate profits. Recalling that $\overline{p} = \left(\frac{2k+1}{2(k+1)}\right)c_D$, we see from (24) that consumers gain from a strictly lower average price; however, they also suffer an income loss associated with the reduction in aggregate profit (to a negative value).[11] The value of the income loss in turn interacts with the implied change in λ. The price and profit

[11] To verify that aggregate profit falls to a strictly negative value starting at the market solution when N_E is slightly increased, we may differentiate the LHS of (20) with respect to c_D while using (6), setting $p^{\max} = c_D$, and using the Pareto distribution. This exercise establishes that aggregate profit strictly rises with c_D. To complete the argument, we refer to (24).

effects apparently balance out as well when evaluated at the market solution, ensuring that the first order condition for social welfare maximization as given by (28) is satisfied.

We can also interpret (28) by considering the impact of entry on the aggregate output enjoyed by consumers. As Demidova (2017) shows, consumer welfare can be expressed as a function of aggregate output, Q. We can rewrite Q as

$$Q = N \cdot \overline{q},$$

where

$$\overline{q} \equiv E[q(c)|c \leq c_D].$$

Using (30), we already know that, starting at the market equilibrium, a slight increase in entry has no effect of the number of available varieties, N. Using (5), setting $p^{\max} = c_D$, and using the Pareto distribution, we find that

$$\overline{q} = \frac{\lambda \cdot c_D}{2\gamma(k+1)}.$$

Thus, by (29), we have that

$$\frac{d\overline{q}}{dN_E}|_{N_E=N_E^{mkt}} = 0.$$

Intuitively, the conditional average variety output is unchanged due to two offsetting forces: starting at the market equilibrium, greater entry lowers the output on any surviving variety but also lowers c_D and thus eliminates the (low) output of the least efficient varieties. Hence, starting at the market solution, a slight increase in entry affects neither the extensive margin N nor the (conditional) intensive margin $\overline{q}$. Aggregate output and thus consumer welfare are therefore also unaffected.

Our analysis so far has not treated the second order condition. We can show, however, that the social planner's welfare function is locally concave when evaluated at the market solution.[12] The following proposition establishes this final point:

Proposition 1 *The entry level at the market equilibrium in the one sector MO model is (locally) efficient.*

Proof. The proof of Proposition 1 is completed in the Appendix. ■

[12] Global optimality may be verified numerically for given parameter specifications. For example, when $\alpha = 2$, $c_M = 1$, $k = 1.1$, $f_e = 0.1$ and $\gamma = \eta = 1$, we may verify that the market entry level is given by $N_E^{mkt} = 4.76$ and is the global optimum.

3 Two-sector MO model without outside good: Market outcomes for given entry policies

We consider the market outcomes in the two-sector MO model without an outside good, when the government may influence the market level of entry by using an entry subsidy or tax. Specifically, we assume

1. For each sector $s \in \{1,2\}$, the government chooses an entry policy t_e^s, where $t_e^s > 0$ indicates an entry subsidy in sector s and $t_e^s < 0$ indicates an entry tax in sector s.

2. The total entry subsidy (tax) is levied on (transferred to) consumers in a lump sum manner.

3. Consumers and firms maximize their objectives as in standard market economy.

4. Entry is determined by the Free Entry condition.

In this section, we take the entry policies, t_e^s for $s \in \{1,2\}$ as given and determine the market outcomes. We begin our analysis by considering the consumer's problem. We then characterize profit maximizing behavior by firms and the resulting free entry condition.

3.1 Consumer's problem

Just as in the one-sector model, the economy contains a unit mass of identical consumers, each supplying a unit of labor in inelastic fashion to a competitive labor market, where we now assume as well that there is costless labor mobility across sectors. We normalize the wage as 1. Consumers also hold symmetric shares of any aggregate net profit or any government transfer, the values of which an individual consumer takes as fixed when choosing consumption.

For the two-sector model, we assume that the consumer's upper level utility function is additively separable, so that the consumer maximizes $U_1 + U_2$ where for $s \in \{1,2\}$

$$U_s = \alpha_s \int_{i \in \Omega_s} q_{si} di - \frac{1}{2}\gamma \int_{i \in \Omega_s} (q_{si})^2 \, di - \frac{1}{2}\eta \left(\int_{i \in \Omega_s} q_{si} di \right)^2 \tag{31}$$

with α_1 possibly different from α_2. We thus represent the consumer's welfare optimization problem as

$$\max_{\{q_{1i}\} \in \Omega_1, \{q_{2i}\} \in \Omega_2} U_1 + U_2$$

s.t.

$$\sum_{s \in \{1,2\}} \int_{i \in \Omega_s} p_{si} q_{si} di = 1 + TR + \sum_{s \in \{1,2\}} \Pi_s \tag{32}$$

where p_{si} and q_{si} are the respective price and quantity of variety i in sector s in the set of available varieties Ω_s in sector s, wage income is normalized as 1, Π_s represents aggregate net profit in sector s and TR refers to the aggregate government transfer. As above, we assume that the preference parameters α, γ and η are all positive.

We consider a Lagrangian equation as follows

$$L = U_1 + U_2 + \lambda \left(1 + TR + \sum_{s \in \{1,2\}} \Pi_s - \sum_{s \in \{1,2\}} \left(\int_{i \in \Omega_s} p_{si} q_{si} di \right) \right),$$

where $\lambda \geq 0$ is the multiplier for the consumer's optimization problem. Letting $Q_s \equiv \int_{i \in \Omega_s} q_{si} di$ denote aggregate output in sector s, we represent the FOC with respect to q_{si} as

$$\alpha_s - \gamma \cdot q_{si} - \eta \cdot Q_s = \lambda p_{si}. \tag{33}$$

As before, we integrate (33) over the set of varieties for which $q_{si} > 0$ and letting N_s be the measure of consumed varieties in Ω_s, we obtain

$$\alpha_s - \gamma \cdot \frac{Q_s}{N_s} - \eta \cdot Q_s = \lambda \bar{p}_s$$

and hence

$$\frac{\alpha_s - \lambda \bar{p}_s}{\eta + \frac{\gamma}{N_s}} = Q_s,$$

where $\bar{p}_s$ is the average price of consumed varieties in sector s.

Using (33), we see that $\frac{(\alpha_s - \eta \cdot Q_s - \gamma \cdot q_{si})}{\lambda} = p_{si}$ for consumed varieties. We now define $p_s^{\max}$ as the "choke price" for varieties in sector s. From the foregoing, we may confirm that

$$p_s^{\max} \equiv \frac{\alpha_s - \eta \cdot Q_s}{\lambda} = \frac{1}{\lambda} \left(\frac{\gamma \cdot \alpha_s + \lambda \cdot \eta \cdot N_s \cdot \bar{p}_s}{\eta \cdot N_s + \gamma} \right). \tag{34}$$

Using (33) and (34), the inverse demand can be written as

$$p_s(q_s) = p_s^{\max} - \frac{\gamma}{\lambda} \cdot q_s. \tag{35}$$

3.2 Firm's problem

We turn now to firm behavior. Profit maximization for a firm with marginal production cost c delivers the profit function[13]

$$\pi_s(c) = \max_q \left(p_s(q) - c \right) q$$

[13] As in the one-sector model, each firm takes λ as given when solving its profit maximization problem.

Using (35), we characterize the solution to firm's problem as

$$q_s(c) = \frac{\lambda\left(p_s^{\max} - c\right)}{2\gamma}. \tag{36}$$

This solution in turn generates a corresponding profit-maximizing price and profit for the firm:

$$p_s(c) = \frac{p_s^{\max} + c}{2} \tag{37}$$
$$\pi_s(c) = \frac{\lambda}{4\gamma}\left(p_s^{\max} - c\right)^2.$$

For this (two-sector) model of monopolistic competition, a firm in a given sector takes the demand intercept as given and produces a positive quantity of its variety provided that its cost realization is no higher than the intercept. In other words, a Zero Cutoff Profit (ZCP) condition determines the cost cutoff c_D^s as

$$\pi_s\left(c_D^s\right) = 0.$$

or equivalently

$$p_s^{\max} = p_s\left(c_D^s\right) = c_D^s.$$

As in the one-sector model examined above, we assume that costs are distributed according to a Pareto distribution that is symmetric across the two sectors:

$$G(c) = \left(\frac{c}{c_M}\right)^k$$

for $c \in [0, c_M]$ where $k > 1$ and $c_M > 0$.[14] Given this distribution, we recall

$$\overline{c}_s \equiv E(c|c \leq c_D^s) = \left(\frac{k}{k+1}\right) c_D^s.$$

Using (37) and $p_s^{\max} = c_D^s$, we find

$$\overline{p}_s \equiv E(p_s(c)|c \leq c_D^s) = \left(\frac{c_D^s + \overline{c}_s}{2}\right).$$

It now follows that

$$\overline{p}_s = \left(\frac{2k+1}{2(k+1)}\right) c_D^s.$$

Notice that the average markup in sector s, $\overline{\mu}_s \equiv \overline{p}_s / \overline{c}_s$, is in fact independent of s and

[14] As for the one-sector model, we assume throughout our analysis of the two-sector MO model that $c_M > c_D^s$.

indeed takes the same value as in the one-sector model: for $s \in \{1, 2\}$,

$$\overline{\mu}_s = \overline{\mu} = \frac{2k+1}{2k}.$$

Thus, the two-sector model considered here does admit markup heterogeneity.[15]

Using (34) and $p_s^{\max} = c_D^s$, we can represent the number of varieties as

$$N_s = \frac{\gamma\left(\alpha_s - \lambda \cdot p_s^{\max}\right)}{\lambda \cdot \eta \cdot \left(p_s^{\max} - \overline{p}_s\right)} = \frac{\gamma\left(\alpha_s - \lambda \cdot c_D^s\right)}{\lambda \cdot \eta\left(c_D^s - \overline{p}_s\right)}.$$

Plugging in the expression just derived for $\overline{p}_s$, we obtain

$$N_s = \frac{2(k+1)\gamma}{\eta}\frac{\left(\alpha_s - \lambda \cdot c_D^s\right)}{\lambda \cdot c_D^s}. \tag{38}$$

Similar to the one-sector model, we see from (38) that, once values for c_D^s and λ are obtained, the value for the number of available varieties is determined. For $c_D^s > 0$, we note that N_s is strictly decreasing with respect to c_D^s for a given value of $\lambda > 0$.

For a given sector $s \in \{1, 2\}$, the number of available varieties can also be represented as a function of the level of entry and the cost cutoff level as

$$N_E^s = \frac{N_s}{G\left(c_D^s\right)}. \tag{39}$$

Hence, using (39), the value for the number of available in sector s can also be determined given the number of entrants and the cost cutoff level for this sector. Finally, using (38), (39) and the Pareto distribution, we can further characterize the relation between N_s and N_E^s as

$$N_E^s = \frac{N_s}{G\left(c_D^s\right)} = \frac{2(k+1)\gamma(c_M)^k}{\eta}\frac{\left(\alpha_s - \lambda \cdot c_D^s\right)}{\lambda \cdot (c_D^s)^{k+1}}. \tag{40}$$

3.3 Free Entry Condition

We focus in this section on the policy-induced market outcome; thus, the level of entry is not a direct choice variable but rather is determined by entry policies via a free entry requirement. Formally, we now impose the Free Entry (FE) condition

$$\int_0^{c_D^s} \pi_s(c)\, dG(c) = f_e - t_e^s \tag{41}$$

[15] By contrast, the difference between $\overline{p}_s$ and $\overline{c}_s$ equals $2c_D^s/(k+1)$ and thus varies across sectors to the extent that cost cutoff c_D^s does.

where $t_e^s > 0$ refers to an entry subsidy in sector s ($t_e^s < 0$ refers to an entry tax). The Free Entry condition pins down c_D^s under given λ. Specifically, using $\pi_s(c) = \frac{\lambda}{4\gamma}\left(p_s^{\max} - c\right)^2$, $p_s^{\max} = c_D^s$ and the Pareto distribution, we find from (41) that the following relationship between c_D^s and λ obtains at the policy-induced market equilibrium:

$$c_D^s = \left(\frac{2(k+1)(k+2)\gamma(c_M)^k(f_e - t_e^s)}{\lambda}\right)^{\frac{1}{2+k}} = \lambda^{-\frac{1}{2+k}}(f_e - t_e^s)^{\frac{1}{2+k}}\gamma^{\frac{1}{2+k}}\tilde{\phi}^{\frac{1}{2+k}}. \quad (42)$$

where $\tilde{\phi} = 2(k+1)(k+2)(c_M)^k$.

As indicated in (40), the market number of available varieties then can be written as

$$N_E^s = \frac{N_s}{G(c_D^s)} = \frac{2(k+1)\gamma(c_M)^k}{\eta}\frac{(\alpha_s - \lambda \cdot c_D^s)}{\lambda \cdot (c_D^s)^{k+1}} \quad (43)$$

3.4 Equilibrium characterization under fixed t_e^s

We may now summarize our characterization of the equilibrium market outcomes for given entry policies, t_e^s.

1. Using (42), we may determine c_D^s for given λ:

$$c_D^s = \left(\frac{2(k+1)(k+2)\gamma(c_M)^k(f_e - t_e^s)}{\lambda}\right)^{\frac{1}{2+k}} = \lambda^{-\frac{1}{2+k}}(f_e - t_e^s)^{\frac{1}{2+k}}\gamma^{\frac{1}{2+k}}\tilde{\phi}^{\frac{1}{2+k}}. \quad (44)$$

2. Using (43), we then may determine N_E^s for given λ :

$$N_E^s = \frac{2(k+1)\gamma(c_M)^k}{\eta}\frac{(\alpha_s - \lambda \cdot c_D^s)}{\lambda \cdot (c_D^s)^{k+1}}. \quad (45)$$

3. Using (36) and (37), we may then determine $q_s(c)$ and $p_s(c)$ for given λ :

$$q_s(c) = \frac{\lambda(c_D^s - c)}{2\gamma} \quad (46)$$

$$p_s(c) = \frac{c_D^s + c}{2} \quad (47)$$

where $p_s^{\max}$ is replaced with c_D^s by the ZCP condition.

4. Using (45)-(47), we can then update the budget constraint (32) to determine λ :

$$\sum_{s\in\{1,2\}} N_E^s \int_0^{c_D^s} \frac{\lambda\left(\left(c_D^s\right)^2 - c^2\right)}{4\gamma} dG\left(c\right) = 1 - \sum_{s\in\{1,2\}} N_E^s t_e^s \tag{48}$$

where $\Pi_s = 0$ for $s \in \{1,2\}$ by the Free Entry condition and $TR = -\sum_{s\in\{1,2\}} N_E^s t_e^s$.[16] Thus, (44), (45), and (48) pin down λ.

5. Hence, (44)-(48) determine a market equilibrium $(c_D^{s*},\ N_E^{s*},\ N^{s*},\ \lambda^*,\ q_s^*\left(c\right),\ p_s^*\left(c\right))$ for given entry policies, (t_e^1, t_e^2).

4 Two-sector MO model without outside good: Planner's problem

In this section, we focus on planner's problem when the planner can choose N_E^1 and N_E^2 in direct fashion. Thus, we put entry policies to the side in this section; however, in the subsequent section, we show how entry policies can be used to replicate the planner's entry level choices. In the current section, we use the (undistorted) market equilibrium as a starting point for comparative statics analyses. We can find the corresponding market equilibrium outcomes by setting $t_e^s = 0$ on results in Section 3.

The planner chooses N_E^1 and N_E^2 to maximize consumer welfare under (i) a resource constraint derived from the budget constraint (32) with $TR = 0$, (ii) a constraint on the relationship between N_E^s, c_D^s and λ as given in (40), and (iii) a profit-maximizing constraint under which the quantity of variety i consumed is determined by the corresponding firm's cost realization and profit-maximizing output (including zero), as implied by (36) and $p_s^{\max} = c_D^s$.

To begin, we note that the planner's problem can be represented as choosing N_E^1 and N_E^2 to maximize consumer's welfare

$$\max_{N_E^1, N_E^2} U \equiv U_1 + U_2$$

[16] To confirm that the updated budget constraint (48) follows from the original budget constraint (32), we rewrite (32) as

$$\sum_{s\in\{1,2\}} N_E^s \int_0^{c_D^s} p_s(c) q_s(c) dG(c) = 1 - \sum_{s\in\{1,2\}} N_E^s t_e^s,$$

where we use that $\Pi_s = 0$ for $s \in \{1,2\}$ by the Free Entry condition and $TR = -\sum_{s\in\{1,2\}} N_E^s t_e^s$. Using (46) and (47), it is now straightforward to confirm (48).

where

$$U_s = \left[\alpha_s \cdot N_E^s \int_0^{c_D^s} q_s(c)\, dG(c) - \frac{\gamma}{2} N_E^s \int_0^{c_D^s} q_s(c)^2\, dG(c) - \frac{\eta}{2} \left(N_E^s \int_0^{c_D^s} q_s(c)\, dG(c) \right)^2 \right] \tag{49}$$

for $s \in \{1,2\}$ s.t.

$$\sum_{s \in \{1,2\}} N_E^s \left(\int_0^{c_D^s} c \cdot q_s(c)\, dG(c) + f_e \right) = 1 \tag{50}$$

where

$$N_E^s = \frac{2(k+1)\gamma (c_M)^k}{\eta} \frac{(\alpha_s - \lambda \cdot c_D^s)}{\lambda \cdot (c_D^s)^{k+1}} \text{ for } s \in \{1,2\} \tag{51}$$

$$q_s(c) = \frac{\lambda (c_D^s - c)}{2\gamma} \text{ for } s \in \{1,2\} \tag{52}$$

and where $\lambda \geq 0$ is the multiplier for the consumer's Lagrangian problem

$$L = U_1 + U_2 + \lambda \left(1 - \sum_{s \in \{1,2\}} \left(N_E^s \int_{i \in \Omega_s} p_s(c)\, q_s(c)\, dG(c) - \Pi_s \right) \right)$$

with $p_s(c) = \frac{c_D^s + c}{2}$.[17]

To interpret this formulation, we note that the consumer utility function represented in (49) follows directly from (31) once profit-maximizing behavior is embedded. We note further that (51) and (52) follow directly from (43) and (36) with $p_s^{\max} = c_D^s$, respectively. Finally, to confirm that the resource constraint (50) follows from the budget constraint (32) with $TR = 0$, we rewrite the latter as

$$\sum_{s \in \{1,2\}} N_E^s \int_0^{c_D^s} p_s(c) q_s(c) dG(c) = 1 + \sum_{s \in \{1,2\}} N_E^s \left(\int_0^{c_D^s} (p_s(c) - c)\, q_s(c) dG(c) - f_e \right)$$

and simplify.

Following the approach taken in Section 2, we now proceed to rewrite the planner's problem with the objective and constraints expressed in terms of α_s, N_E^s, c_D^s and λ. Proceeding as in Section 2 and using (51), we find that U_s from (49) may be rewritten as $u(\alpha_s, c_D^s, \lambda)$ where the function u is defined in (13). Likewise, it is direct that (51) may be rewritten as $N_E^s = N_e(\alpha_s, c_D^s, \lambda)$ where the function N_e is defined in (17). Finally, for the resource constraint (50), we may proceed as in Section 2 while using (51), (52) and the Pareto distribution to rewrite this constraint as $\sum_{s \in \{1,2\}} R(\alpha_s, c_D^s, \lambda) = \frac{\eta(2+k)}{\gamma\phi}$, where

[17] Note also that the value for λ used for our analysis of the two-sector model may differ from that used in the one-sector model.

the function R is defined in (16).

We may thus state *the planner's problem* as

$$\max_{N_E^1, N_E^2} \sum_{s \in \{1,2\}} u\left(\alpha_s, c_D^s, \lambda\right) \tag{53}$$

s.t.

$$\sum_{s \in \{1,2\}} R\left(\alpha_s, c_D^s, \lambda\right) = \frac{\eta\left(2+k\right)}{\gamma\phi} \tag{54}$$

$$N_E^s = N_e\left(\alpha_s, c_D^s, \lambda\right) > 0 \text{ for } s \in \{1,2\} \tag{55}$$

where $u\left(\alpha, \lambda, c_D\right)$, $R\left(\alpha, \lambda, c_D\right)$, and $N_e\left(\alpha, \lambda, c_D\right)$ are defined in (13), (16), and (17), respectively.

The constraints (54) and (55) represent a 3×3 system with endogenous variables c_D^1, c_D^2 and λ. We can thus do comparative statics exercises with respect to changes in the exogenous variables, N_E^1 and N_E^2. With the comparative statics results in place, we can then determine the effects of certain exogenous perturbations on consumer welfare. As before, we consider small perturbations around the market equilibrium.

We maintain the assumption that the (undistorted) market equilibrium represented by the vector $(N_E^{1mkt}, N_E^{2mkt}, c_D^{1mkt}, c_D^{2mkt}, \lambda^{mkt})$ exists satisfying (54), (55) and the Free Entry condition (42) with $t_e^1 = t_e^2 = 0$ imposed, and at which for $s \in \{1,2\}$ we have $N_E^{smkt} > 0$, $c_D^{smkt} > 0$ and $\lambda^{mkt} > 0$.[18] We then appeal to the implicit function theorem to ensure the existence of a solution to (54) and (55) for (N_E^1, N_E^2) sufficiently close to (N_E^{1mkt}, N_E^{2mkt}). To use this theorem, we require that, at the market solution, the Jacobian determinant associated with (54) and (55) is non-zero.

We consider two kinds of comparative statics exercises. In the first exercise, we consider a symmetric setting in which $\alpha_1 = \alpha_2 \equiv \alpha$ and analyze the implications of a small perturbation in which the planner symmetrically changes N_E^1 and N_E^2 (i.e., $dN_E^1 = dN_E^2$). This exercise is similar to that already analyzed above for the one-sector model. We recall that the change in the entry level for that model induced a change in λ and impacted variety-level consumption through this channel. We find exactly analogous results for the two-sector model. Hence, when the two-sector model has a symmetric setting and is subjected to a symmetric change in sectoral entry levels, the results are exactly similar to those in the one-sector model already considered. A second exercise considers a potentially asymmetric setting where α_1 may differ from α_2. For this setting, we allow the

[18] With some abuse of notation, we use λ^{mkt} to represent the market equilibrium value for λ in the two-sector model just as we do above in the one-sector model. Below, we distinguish between symmetric settings ($\alpha_1 = \alpha_2$) and potentially asymmetric settings for the two-sector model, and we introduce additional notation as necessary to distinguish (undistorted) market equilibrium outcomes for these settings from those in the one-sector model.

planner to consider a small change in N_E^1 and N_E^2 where the change is calibrated so that λ is unaltered (i.e., dN_E^1 and dN_E^2 are such that $d\lambda = 0$). We argue that this exercise shares qualitative features with our analysis in Bagwell and Lee (2020) of the welfare effects of an increase in entry into the differentiated sector, where the other sector is an outside good sector. A unifying feature is that, in both cases, the marginal utility of income remains fixed, and so variety-level consumption is not impacted by changes in the marginal utility of income.

First exercise: We start with the first exercise. For this exercise, we assume that the setting is symmetric with $\alpha_1 = \alpha_2 \equiv \alpha$, and we analyze the implications of a small perturbation in which the planner symmetrically changes N_E^1 and N_E^2 (i.e., $dN_E^1 = dN_E^2$). Given the symmetry of the setting, the market solution is also symmetric: $N_E^{1mkt} = N_E^{2mkt}$ and $c_D^{1mkt} = c_D^{2mkt}$. We can thus simplify the constraint set above and represent it with the following 2×2 system:

$$2 \cdot R(\alpha, c_D, \lambda) = \frac{\eta(2+k)}{\gamma\phi}$$

$$N_E = N_e(\alpha, c_D, \lambda) > 0$$

with the symmetric solutions for c_D and λ thus determined given a symmetric entry level N_E. For the symmetric setting, the market solution obtains and satisfies these constraints when $N_E = N_E^{1mkt} = N_E^{2mkt} \equiv \widetilde{N}_E^{mkt}$ and thus $c_D = c_D^{1mkt} = c_D^{2mkt} \equiv \widetilde{c}_D^{mkt}$ with $\lambda = \widetilde{\lambda}^{mkt}$.[19,20] We note further that for this symmetric setting the market equilibrium relationship (42) with $t_s^e = 0$ imposed for $s \in \{1, 2\}$ simplifies and takes the form

$$\widetilde{\lambda}^{mkt} = \frac{\gamma\phi}{(\widetilde{c}_D^{mkt})^{2+k}},$$

which is exactly the same relationship reported in (21) for the one-sector model.

We find that, at the market solution, the Jacobian determinant for this 2×2 system is given as

$$\begin{aligned}|\widetilde{J}| &\equiv 2\left(\frac{\partial R}{\partial c_D}\frac{\partial N_E}{\partial \lambda} - \frac{\partial R}{\partial \lambda}\frac{\partial N_E}{\partial c_D}\right)|_{N_E = \widetilde{N}_E^{mkt}} \\ &= -2(\alpha - \lambda \cdot c_D)(\alpha + \lambda k c_D)\lambda k c_D^{k+2}|_{N_E = \widetilde{N}_E^{mkt}} < 0,\end{aligned}$$

[19] We use a tilde ($\tilde{}$) as necessary to distinguish definitions relating to (undistorted) market equilibrium outcome values in the symmetric setting (first exercise) from those in the one-sector model. Similarly, for the potentially asymmetric setting (second exercise) considered below, we use a hat ($\hat{}$).

[20] For the symmetric two-sector model, we can show that in the market equilibrium the level of entry into any one sector is half the level of entry in the market equilibrium of the one-sector model: $\widetilde{N}_E^{mkt} = (1/2)N_E^{mkt}$.

where the inequality follows from (43) and our assumption that $\widetilde{N}_E^{mkt} > 0$, $\widetilde{c}_D^{mkt} > 0$ and $\widetilde{\lambda}^{mkt} > 0$. Referring to (23), we note that $|\widetilde{J}|$ takes the form $|\widetilde{J}| = 2|J|$. Given $|\widetilde{J}| < 0$ at the market solution, we may apply the implicit function theorem.

By the implicit function theorem, the derivatives that emerge from this comparative statics exercise exist for N_E sufficiently close to $\widetilde{N}_E^{mkt}$. Evaluating at the market solution, we find that the derivatives take the following form:

$$\frac{dc_D}{dN_E}|_{N_E=\widetilde{N}_E^{mkt}} = \frac{1}{|J|}\left(k\phi\gamma + \alpha c_D^{k+1}\right)\left(\frac{c_D}{\phi\gamma}\right)^2 |_{N_E=\widetilde{N}_E^{mkt}} < 0$$

and

$$\frac{d\lambda}{dN_E}|_{N_E=\widetilde{N}_E^{mkt}} = \frac{-1}{|J|}\left(k\phi\gamma + \alpha c_D^{k+1}\right)\left(\frac{1}{\phi\gamma c_D^{k+1}}\right)|_{N_E=\widetilde{N}_E^{mkt}} > 0,$$

where we use that $|\widetilde{J}| = 2|J|$ when $|J|$ is evaluated at the market solution for the (symmetric) two-sector model. Referring to (24) and (25), we see that the derivatives for our first exercise take exactly the same form as they did in the one-sector model.

We can now capture the impact on the planner's objective of a small symmetric change in the level of entry, starting at the market equilibrium, as follows:

$$\frac{d\sum_{s\in\{1,2\}} u\left(\alpha_s, c_D^s, \lambda\right)}{dN_E}|_{N_E=\widetilde{N}_E^{mkt}} = 2\left(\frac{\partial u}{\partial c_D}\frac{dc_D}{dN_E} + \frac{\partial u}{\partial \lambda}\frac{d\lambda}{dN_E}\right)|_{N_E=\widetilde{N}_E^{mkt}},$$

which takes a re-scaled (doubled) form of (22) from the one-sector model. We have already established that the derivatives $\frac{dc_D}{dN_E}$ and $\frac{d\lambda}{dN_E}$ take exactly the same form as they did in the one-sector model; furthermore, we note that the utility partial derivatives $\frac{\partial u}{\partial c_D}$ and $\frac{\partial u}{\partial \lambda}$ are also defined exactly as in the one-sector model and thus again take the form given in (26) and (27).

With these observations in place, we see that the symmetric version of the two-sector model has similar features as the one-sector model. Most importantly, just as the market solution for the one-sector model satisfies the first-order condition for the corresponding planner's problem, the market solution for the two-sector model satisfies the first-order condition for the planner's problem that corresponds to that model. We summarize this point in the following proposition:

Proposition 2 *Suppose $\alpha_1 = \alpha_2$ and that the planner is restricted to consider only symmetric changes in entry levels in both sectors: $dN_E^1 = dN_E^2 \equiv dN_E$. In this restricted policy space, the symmetric market level of entry satisfies the planner's first-order condition, just as in the one-sector model.*

Second exercise: We turn now to our second exercise. Here, we allow that α_1 may differ from α_2. For this potentially asymmetric setting, we allow the planner to consider a small change in N_E^1 and N_E^2 where the change is calibrated so that λ is unaltered (i.e., dN_E^1 and dN_E^2 are such that $d\lambda = 0$). As mentioned above, this exercise shares qualitative features with our analysis in Bagwell and Lee (2020) of the welfare effects of an increase in entry into the differentiated sector, where the other sector is an outside good sector. In both cases, the marginal utility of income remains fixed, and so variety-level consumption is not impacted by changes in the marginal utility of income.

We assume that the (undistorted) market equilibrium represented by the vector (N_E^{1mkt}, N_E^{2mkt}, c_D^{1mkt}, c_D^{2mkt}, $\widehat{\lambda}^{mkt}$) exists satisfying (54), (55) and the Free Entry condition (42) with $t_e^1 = t_e^2 = 0$ imposed, and at which for $s \in \{1,2\}$ we have $N_E^{smkt} > 0$, $c_D^{smkt} > 0$ and $\widehat{\lambda}^{mkt} > 0$. Starting at this solution, the planner imposes a small perturbation to this system, where we now allow the planner to change both N_E^1 and N_E^2 (i.e., $dN_E^1 \neq 0$ and $dN_E^2 \neq 0$) slightly and in a manner that leaves λ unchanged (i.e., $d\lambda = 0$). For a given increase in N_E^1, we thus must determine the corresponding change in N_E^2 that serves to preserve the value of λ.

We consider the following 3×3 system:

$$\sum_{s\in\{1,2\}} R\left(\alpha_s, c_D^s, \lambda\right) = \frac{\eta\left(2+k\right)}{\gamma\phi} \tag{56}$$

$$N_e\left(\alpha_1, c_D^1, \lambda\right) - N_E^1 = 0 \tag{57}$$

$$N_e\left(\alpha_2, c_D^2, \lambda\right) - F(N_E^1) = 0, \tag{58}$$

where the function F is specified so that, at the market equilibrium, $F(N_E^1) = N_E^2$ and

$$F'(N_E^1) = -\frac{\frac{\partial R(\alpha_1, c_D^1, \lambda)}{\partial c_D}\frac{\partial N_e(\alpha_2, c_D^2, \lambda)}{\partial c_D}}{\frac{\partial R(\alpha_2, c_D^2, \lambda)}{\partial c_D}\frac{\partial N_e(\alpha_1, c_D^1, \lambda)}{\partial c_D}}. \tag{59}$$

Starting at the market equilibrium, the function F describes the path of the exogenous change in N_E^2 that accompanies a small change in N_E^1. We note that the market solution satisfies the constraints given by (56)-(58) when $N_E^1 = N_E^{1mkt}$ and thus $c_D^s = c_D^{smkt}$ with $\lambda = \widehat{\lambda}^{mkt}$.

We note further that the market equilibrium relationship (42) with $t_s^e = 0$ imposed for $s \in \{1,2\}$ simplifies and takes the form

$$\widehat{\lambda}^{mkt} = \frac{\gamma\phi}{(c_D^{smkt})^{2+k}} \equiv \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{2+k}}. \tag{60}$$

Thus, even though $\alpha_1 \neq \alpha_2$ is allowed, in the (undistorted) market equilibrium, the cost cutoff level is in fact independent of the sector: $c_D^{1mkt} = c_D^{2mkt} \equiv \widehat{c}_D^{mkt}$. Using (38) and (40), it follows in turn that, at the market equilibrium, the number of available varieties and the number of entrants are each symmetric across sectors, even though $\alpha_1 \neq \alpha_2$ is allowed. Thus, at the market equilibrium, differences in demand as captured by $\alpha_1 \neq \alpha_2$ do not translate into different market entry patterns across sectors.[21]

We next consider the Jacobian $\widehat{J}$ for the 3×3 system described in (56)-(58) when evaluated at the market equilibrium. We are not able to sign this determinant in general, but we can verify that it is non-zero for a tractable special case. In particular, at the market solution when $\alpha_1 = \alpha_2 \equiv \alpha$, we find that the determinant is strictly negative:

$$|\widehat{J}|_{\alpha_1=\alpha_2\equiv\alpha} = -2(\alpha - \widehat{\lambda}^{mkt} \cdot \widehat{c}_D^{mkt})(\alpha + k(\alpha - \widehat{\lambda}^{mkt} \cdot \widehat{c}_D^{mkt}))k(\widehat{c}_D^{mkt})^2 < 0, \tag{61}$$

where the inequality follows from (43) and our assumption that $N_E^{smkt} > 0$, $c_D^{smkt} = \widehat{c}_D^{mkt} > 0$ and $\widehat{\lambda}^{mkt} > 0$. Given $|\widehat{J}| < 0$ at the market solution when $\alpha_1 = \alpha_2$, we know that $|\widehat{J}| < 0$ is sure to hold at the market solution when the level of asymmetry (i.e., $|\alpha_2 - \alpha_1|$) is sufficiently small. In order to apply the implicit function theorem, we thus assume henceforth that the level of asymmetry is sufficiently small. We emphasize, however, that what we require as a general matter is simply that the determinant of the Jacobian is non-zero when evaluated at the market solution.

Totally differentiating the system described in (56)-(58) with respect to N_E^1, using $F(N_E^{1mkt}) = N_E^{2mkt}$ and (59), and evaluating at the market solution, we find that

$$\begin{aligned}
\frac{d\lambda}{dN_E^1}|_{N_E^1=N_E^{1mkt}} &= 0 \\
\frac{dc_D^1}{dN_E^1}|_{N_E^1=N_E^{1mkt}} &= \frac{1}{\frac{\partial N_e(\alpha_1,c_D^1,\lambda)}{\partial c_D}}|_{N_E^1=N_E^{1mkt}} \\
\frac{dc_D^2}{dN_E^1}|_{N_E^1=N_E^{1mkt}} &= -\frac{\frac{\partial R(\alpha_1,c_D^1,\lambda)}{\partial c_D}}{\frac{\partial R(\alpha_2,c_D^2,\lambda)}{\partial c_D}\frac{\partial N_e(\alpha_1,c_D^1,\lambda)}{\partial c_D}}|_{N_E^1=N_E^{1mkt}}.
\end{aligned}$$

Thus, the perturbation captured by our specification in (59) indeed ensures that λ is unchanged.

[21] Recall that the average markup, $\overline{\mu}$, is symmetric across sectors, even away from the (undistorted) market equilibrium (i.e., even when the Free Entry condition is not imposed). Observe also that, at the market equilibrium, $\overline{p}_s = \left(\frac{2k+1}{2(k+1)}\right)\widehat{c}_D^{mkt}$ and $\overline{c}_s = \left(\frac{k}{k+1}\right)\widehat{c}_D^{mkt}$; hence, while $\alpha_1 \neq \alpha_2$ is allowed, the average price, cost and price-cost difference in the market equilibrium are nevertheless independent of the sector. These three values, however, vary across sectors with the cutoff cost level c_D^s, when entry levels are moved away from market equilibrium levels as determined by the Free Entry condition. See also footnote 15.

Using (16), (17) and imposing the market equilibrium condition (60), we find that, for $s \in \{1,2\}$,

$$\begin{aligned}\frac{\partial N_e(\alpha_s, c_D^s, \lambda)}{\partial c_D}|_{N_E^1=N_E^{1mkt}} &= -\frac{2(k+1)(c_M)^k}{\eta\phi}[\alpha_s + k(\alpha_s - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}})] < 0 \\ \frac{\partial R(\alpha_s, c_D^s, \lambda)}{\partial c_D} &= -\frac{1}{\gamma\phi}[\alpha_s + \frac{k\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}}] < 0,\end{aligned}$$

where $\alpha_s - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}} = \alpha_s - \widehat{\lambda}^{mkt} \cdot \widehat{c}_D^{mkt} > 0$ by $N_E^{smkt} > 0$. Referring to (59), we can now verify that our second experiment entails an increase in entry into sector 1 that is accompanied by a decrease in entry into sector 2, where the entry adjustments are balanced to keep λ unaltered.

Gathering our findings, we may further report that

$$\frac{dc_D^1}{dN_E^1}|_{N_E^1=N_E^{1mkt}} = -\frac{\eta\phi}{2(k+1)(c_M)^k}\frac{1}{[\alpha_1 + k(\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}})]} < 0 \tag{62}$$

where $\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}} = \alpha_1 - \widehat{\lambda}^{mkt} \cdot \widehat{c}_D^{mkt} > 0$ by $N_E^{1mkt} > 0$ and

$$\frac{dc_D^2}{dN_E^1}|_{N_E^1=N_E^{1mkt}} = \frac{\eta\phi}{2(k+1)(c_M)^k}\frac{[\alpha_1 + \frac{k\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}}]}{[\alpha_2 + \frac{k\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}}][\alpha_1 + k(\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}})]} > 0. \tag{63}$$

Hence, the reallocation of entry from sector 2 to sector 1 results in a lower cost cutoff level in sector 1 and a higher cost cutoff level in sector 2.

To examine the impact of the described shift in entry levels on consumer welfare, we must first determine the impact of a change in the cutoff cost level for a sector on consumer welfare. Using (13) and imposing the market solution condition (60), we find that

$$\frac{\partial u(\alpha_s, c_D^s, \lambda)}{\partial c_D}|_{N_E^1=N_E^{1mkt}} = \frac{-\gamma\phi}{2\eta(2+k)(\widehat{c}_D^{mkt})^{3+2k}}\left(\alpha_s(\widehat{c}_D^{mkt})^{1+k} + 2(1+k)\gamma\varphi\right) < 0, \tag{64}$$

which parallels the finding (26) for the one-sector model. As expected, an increase in the cost cutoff for a given sector lowers the consumer utility enjoyed in that sector.

We are now prepared to analyze the impact of the proposed shift in entry levels for consumer welfare. Specifically, we seek to evaluate

$$\frac{d}{dN_E^1}\sum_{s\in\{1,2\}} u(\alpha_s, c_D^s, \lambda)|_{N_E^1=N_E^{1mkt}} = \sum_{s\in\{1,2\}}\frac{\partial u(\alpha_s, c_D^s, \lambda)}{\partial c_D} \cdot \frac{dc_D^s}{dN_E^1}|_{N_E^1=N_E^{1mkt}}.$$

Using (62), (63) and (64), we calculate

$$\sum_{s\in\{1,2\}} \frac{\partial u(\alpha_s, c_D^s, \lambda)}{\partial c_D} \cdot \frac{dc_D^s}{dN_E^1}|_{N_E^1=N_E^{1mkt}} = \frac{\eta(\gamma\phi)^3(\widehat{c}_D^{mkt})^{1+k}(2+k)(\alpha_2-\alpha_1)}{D}, \qquad (65)$$

where

$$D \equiv [4\eta(2+k)(k+1)\gamma(c_M)^k(\widehat{c}_D^{mkt})^{3+2k}][\alpha_1 + k(\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}}][\alpha_2(\widehat{c}_D^{mkt})^{1+k} + k\gamma\phi] > 0,$$

with the inequality again following since $\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{2+k}} = \alpha_1 - \widehat{\lambda}^{mkt} \cdot \widehat{c}_D^{mkt} > 0$ by $N_E^{1mkt} > 0$.

Notice that the described shift in entry levels has no effect on welfare in the special case of a symmetric setting, where $\alpha_1 = \alpha_2$. As we can see from (59), in that case, the exercise involves an increase in N_E^1 that induces an equal-sized decrease in N_E^2. When the setting is symmetric with $\alpha_1 = \alpha_2$, it is intuitive that, starting at the market solution, a small zero-sum reallocation of entry from one sector to the other would have no first-order welfare effect. As (65) confirms, however, when $\alpha_1 \neq \alpha_2$, the planner can gain from modifying the market solution and expanding the level of entry into one market at the cost of less entry in the other, where the adjustment is made so as to keep λ constant. Interestingly, the market provides too much entry into the sector s for which α_s is highest,which is suggestive of a business-stealing externality interpretation.[22]

We summarize with the following proposition:

Proposition 3 *Allow $\alpha_1 \neq \alpha_2$ with $|\alpha_2 - \alpha_1|$ sufficiently small so that (61) is sure to hold. Suppose that the planner is restricted to consider only a small increase in entry into sector* 1 *that is accompanied by a decrease in entry into sector* 2 *so as to keep the value for λ fixed: $dN_E^1 > 0 > dN_E^2$ such that $d\lambda = 0$. In this restricted policy space, starting at the market equilibrium, additional entry in sector* 1 *raises (lowers) (does not change) welfare U if and only if $\alpha_1 < \alpha_2$ ($\alpha_1 > \alpha_2$) ($\alpha_1 = \alpha_2$).*

To understand the forces involved, suppose that $\alpha_1 > \alpha_2$ with the difference small.[23] Starting at the market equilibrium, consider a small increase in entry into sector 1 with

[22] If we instead assume $\alpha_1 = \alpha_2$ and allow different fixed entry costs for the two sectors, with $f_e^1 \neq f_e^2$, then we can similarly establish that the market provides excessive entry into the sector with the lowest fixed cost of entry and thus also the sector with the lowest markup. In support of the latter point, we note that if one sector had a lower fixed cost (as captured in (42) by a larger entry subsidy), then that sector would have a lower critical cost cutoff in the market equilibrium; hence, by (37) and $c_D^s = p_s^{\max}$, that sector also would have a lower markup at the market equilibrium.

[23] The assumption of small differences is only used as a sufficient condition for the invertibility of the 3×3 system, so that the implicit function theorem may be applied. It is not otherwise used in our proof.

a corresponding reduction in entry into sector 2 that keeps the value for λ fixed. Due to $\alpha_1 > \alpha_2$, we can show that this perturbation induces (i) a marginal utility gain from a lower value of c_D^1 that is large in magnitude relative to the induced marginal utility loss from a higher value of c_D^2, but also (ii) a reduction in c_D^1 that is small in magnitude relative to the induced increase in c_D^2. When $\alpha_1 > \alpha_2$, we then show that the latter effect dominates, so that the overall level of utility falls. In this way, even though average markups do not vary across sectors, when there are demand differences across sectors ($\alpha_1 \neq \alpha_2$), the interactions in the utility function between the corresponding demand parameters and the cutoff cost levels can support welfare improving interventions.

The finding in Proposition 3 shares qualitative features with that in Bagwell and Lee (2020) of the welfare effects of an increase in entry into the differentiated sector, where the other sector is an outside-good sector. Bagwell and Lee (2020) show that entry into the differentiated sector is too great if the value for α in that sector exceeds a threshold value.[24] In Bagwell and Lee (2020), the presence of an outside good ensures that the level of entry does not impact the marginal utility of consumption; similarly, in the second experiment considered here, an increase of the level of entry into one sector is offset by a decrease in the level of the entry into the other sector, so as to ensure that the marginal utility of income λ is unaltered. Thus, in both cases, the marginal utility of income remains fixed, and so variety-level consumption is not impacted by changes in the marginal utility of income. In addition, and as Proposition 3 confirms, the market provides excessive entry into the sector s with the highest value for α_s, a finding which is broadly analogous to Bagwell and Lee's (2020) finding regarding excessive entry into the differentiated sector when the value for α in that sector exceeds a threshold value

To summarize, we consider two kinds of comparative statics exercises in this section. In the first exercise, we consider a symmetric setting in which $\alpha_1 = \alpha_2$ and analyze the implications of a small perturbation in which the planner symmetrically changes N_E^1 and N_E^2 (i.e., $dN_E^1 = dN_E^2$). Just as in our analysis of the one-sector model, the symmetric change in entry levels induces a change in λ and impacts variety-level consumption through this channel. Indeed, and as Proposition 2 confirms, when the two-sector model has a symmetric setting and is subjected to a symmetric change in sectoral entry levels, the results are exactly similar to those in the one-sector model. In our second exercise, we allow that α_1 may differ from α_2. For this setting, we consider a small perturbation in which the planner increases the level of entry into sector 1 while simultaneously decreasing the level of entry in sector 2 in such a manner as to ensure that λ is unaltered (i.e., dN_E^1 and dN_E^2 are such that $d\lambda = 0$). We show that such an intervention can improve welfare when

[24] Specifically, in Bagwell and Lee (2020), the market generates excessive entry into the differentiated sector when $\alpha > 2 \cdot c_D^m$, where c_D^m is the cutoff cost level for surviving varieties as determined in the market equilibrium.

demand differences are present across sectors ($\alpha_1 \neq \alpha_2$) even though average markups are symmetric across sectors. We also argue that this exercise shares qualitative features with the analysis in Bagwell and Lee (2020) of the welfare effects of an increase in entry into the differentiated sector, where the other sector is an outside good sector.

5 Two-sector MO model without outside good: Replication of planner's problem using entry policies t_e^s

We show in this section that market equilibrium outcomes generated from the planner's direct choice of entry levels (N_E^1, N_E^2) alternatively can be induced by an appropriate choice of entry policies (t_e^1, t_e^2) by a government.

To make our argument, we compare two problems. The first problem is the planner's problem, which we define above in (53)-(55). In this problem, the planner directly chooses $(N_E^1,\, N_E^2)$ to maximize aggregate utility $\sum_{s\in\{1,2\}} u\left(\alpha_s, c_D^s, \lambda\right)$ subject to constraints, where the constraints determine $(c_D^1,\, c_D^2,\, \lambda)$ and thus aggregate utility for a given $(N_E^1,\, N_E^2)$. Below, it will be convenient to represent a candidate choice $(N_E^1,\, N_E^2)$ for the planner and the corresponding values for $(c_D^1,\, c_D^2,\, \lambda)$ as $(N_E^{1**},\, N_E^{2**})$ and $(c_D^{1**},\, c_D^{2**},\, \lambda^{**})$, respectively. Thus, given $(N_E^1, N_E^2) = (N_E^{1**},\, N_E^{2**})$, the corresponding values $(c_D^{1**},\, c_D^{2**},\, \lambda^{**})$ satisfy (54) and (55).

The second problem is *the government's problem*. The government also seeks to maximize aggregate utility $\sum_{s\in\{1,2\}} u\left(\alpha_s, c_D^s, \lambda\right)$, but the government selects entry policies (t_e^1, t_e^2), with the constraints then given by (44)-(48) and corresponding to the market allocations as described in Section 3.4. Below, it will be convenient to represent a candidate choice $(t_e^1,\, t_e^2)$ for the government and the corresponding values for $(c_D^1,\, c_D^2,\, \lambda)$ as $(t_e^{1*},\, t_e^{2*})$ and $(c_D^{1*},\, c_D^{2*},\, \lambda^*)$, respectively.

Formally, we can represent the constraints for the government's problem as

$$\sum_{s\in\{1,2\}} \frac{2\left(k+1\right)\gamma\left(c_M\right)^k}{\eta} \frac{\left(\alpha_s - \lambda\cdot c_D^s\right)}{\lambda\cdot\left(c_D^s\right)^{k+1}} \left(\frac{\lambda\left(c_D^s\right)^{k+2}\left(c_M\right)^{-k}}{2\gamma\left(2+k\right)} + t_e^s\right) = 1 \tag{66}$$

$$c_D^s = \left(\lambda\right)^{-\frac{1}{2+k}} \left(f_e - t_e^s\right)^{\frac{1}{2+k}} \gamma^{\frac{1}{2+k}} \tilde{\phi}^{\frac{1}{2+k}} \text{ for } s\in\{1,2\} \tag{67}$$

where $\tilde{\phi} = 2\left(k+1\right)\left(k+2\right)\left(c_M\right)^k$. Using (45) and the Pareto distribution, we can rewrite (48) as (66). Note that (67) is a restatement of (44). Thus, given $(t_e^1, t_e^2) = (t_e^{1*}, t_e^{2*})$, the corresponding values $(c_D^{1*}, c_D^{2*},\, \lambda^*)$ satisfy (66) and (67). Finally, the corresponding values for $(N_E^{1*},\, N_E^{2*})$ then may be determined using (45).

We now show that any allocation of (c_D^1, c_D^2, λ) generated by a planner's choice over (N_E^1, N_E^2) can be replicated by the government choice of (t_e^1, t_e^2), and vice versa. A maintained assumption is that the policies selected by the planner and government are such that the level of entry in each sector is strictly positive: $(N_E^{1**}, N_E^{2**}) > 0$ and $(N_E^{1*}, N_E^{2*}) > 0$. To state our finding in the simplest possible way, we also assume that for each problem, given the relevant policies, the market equilibrium values for (c_D^1, c_D^2, λ) are uniquely determined by the corresponding constraints.

Proposition 4 *The proposition has two parts:*

(i). Fix a choice (N_E^{1**}, N_E^{2**}) for the planner and let $(c_D^{1**}, c_D^{2**}, \lambda^{**})$ be correspondingly determined by (54) and (55). Then there exists a choice (t_e^{1*}, t_e^{2*}) for the government that correspondingly determines $(c_D^{1*}, c_D^{2*}, \lambda^*)$ by (66) and (67) and then (N_E^{1*}, N_E^{2*}) by (45) where $(c_D^{1*}, c_D^{2*}, \lambda^*) = (c_D^{1**}, c_D^{2**}, \lambda^{**})$ and $(N_E^{1*}, N_E^{2*}) = (N_E^{1**}, N_E^{2**})$.

(ii). Fix a choice (t_e^{1*}, t_e^{2*}) for the government that correspondingly determines $(c_D^{1*}, c_D^{2*}, \lambda^*)$ by (66) and (67) and then (N_E^{1*}, N_E^{2*}) by (45). Then there exists a choice (N_E^{1**}, N_E^{2**}) for the planner that correspondingly determines $(c_D^{1**}, c_D^{2**}, \lambda^{**})$ by (54) and (55) where $(c_D^{1**}, c_D^{2**}, \lambda^{**}) = (c_D^{1*}, c_D^{2*}, \lambda^*)$ and $(N_E^{1**}, N_E^{2**}) = (N_E^{1*}, N_E^{2*})$.

Proof. The proof of Proposition 4 is found in the Appendix. ■

6 Conclusion

We consider the efficiency of market entry in single- and two-sector versions of the Melitz-Ottavanio (MO) model, where differently from the MO model our two-sector model does not involve an outside good. For the one-sector MO model, we show that the market level of entry achieves a (local) welfare maximum. For a two-sector MO model without an outside good, we show that the welfare results are exactly similar to those in the one-sector model when the two sectors are symmetric. When the two sectors are asymmetric and the level of asymmetry is sufficiently small, we identify a perturbation indicating a sense in which the market level of entry into the "high-demand" sector is excessive. This intersectoral misallocation occurs at the market equilibrium even though endogenous average markups are equal across sectors. We also show how the outcomes induced by the planner's direct choice of entry levels alternatively can be induced through the appropriate choice of entry tax/subsidy policies.

7 Appendix

Proof of Proposition 1: As described in the text, we may define the functions $c_D(N_E)$ and $\lambda(N_E)$ as the solutions to (18) and (19), where given (23) we can be assured that solutions exist for N_E sufficiently near N_E^{mkt}. At the market solution, $\frac{dc_D}{dN_E}$ and $\frac{d\lambda}{dN_E}$ are determined by following system:

$$\begin{aligned}\frac{dR(\alpha, c_D, \lambda)}{dN_E}|_{N_E=N_E^{mkt}} &= \frac{\partial R}{\partial c_D}\frac{dc_D}{dN_E} + \frac{\partial R}{\partial \lambda}\frac{d\lambda}{dN_E}|_{N_E=N_E^{mkt}} = 0 \\ \frac{dN_e(\alpha, c_D, \lambda)}{dN_E}|_{N_E=N_E^{mkt}} &= \frac{\partial N_e}{\partial c_D}\frac{dc_D}{dN_E} + \frac{\partial N_e}{\partial \lambda}\frac{d\lambda}{dN_E}|_{N_E=N_E^{mkt}} = 1.\end{aligned}$$

We present solutions for $\frac{dc_D}{dN_E}$ and $\frac{d\lambda}{dN_E}$ at the market equilibrium above in (24) and (25). By using (21) to substitute for λ in $|J|$ as characterized in (23), we can express the solutions alternatively as

$$\frac{dc_D}{dN_E}|_{N_E=N_E^{mkt}} = -\frac{(c_D)^{1+k}(c_M)^{-k}\eta\phi}{2k(1+k)\left((c_D)^{1+k}\alpha - \gamma\phi\right)}|_{N_E=N_E^{mkt}} < 0 \quad (68)$$

$$\frac{d\lambda}{dN_E}|_{N_E=N_E^{mkt}} = \frac{(c_M)^{-k}\gamma\eta\phi^2}{2(c_D)^2 k(1+k)\left((c_D)^{1+k}\alpha - \gamma\phi\right)}|_{N_E=N_E^{mkt}} > 0 \quad (69)$$

where $\left(c_D^{mkt}\right)^{1+k}\alpha - \gamma\phi = \left(c_D^{mkt}\right)^{1+k}[\alpha - \lambda^{mkt}\cdot c_D^{mkt}] > 0$ by $N_E^{mkt} > 0$.

Arguing similarly, at the market equilibrium, $\frac{d^2c_D}{(dN_E)^2}$ and $\frac{d^2\lambda}{(dN_E)^2}$ are determined by following system:

$$\frac{\partial^2 R(c_D(N_E), \lambda(N_E))}{(\partial N_E)^2}|_{N_E=N_E^{mkt}} = 0 \quad (70)$$

$$\frac{\partial^2 N_e(c_D(N_E), \lambda(N_E))}{(\partial N_E)^2}|_{N_E=N_E^{mkt}} = 0. \quad (71)$$

Taking the derivatives in (70) and (71) and then plugging in the market-solution values of $\frac{dc_D}{dN_E}$, $\frac{d\lambda}{dN_E}$ and λ as given in (68), (69) and (21), we find[25]:

$$\begin{aligned}&\frac{d^2c_D}{(dN_E)^2}|_{N_E=N_E^{mkt}} \\ =\ &\frac{(c_D)^{1+2k}(c_M)^{-2k}\gamma\eta^2\phi^3}{4k(1+k)\left((c_D)^{1+k}\alpha - \gamma\phi\right)^2\left((c_D)^{1+k}\alpha + k\gamma\phi\right)}|_{N_E=N_E^{mkt}}\end{aligned}$$

[25] These expressions are generated using Mathematica. Details are available from the authors.

$$
\begin{aligned}
& \frac{d^2\lambda}{(dN_E)^2}|_{N_E=N_E^{mkt}} \\
= \quad & \frac{(c_D)^{-2+k}(c_M)^{-2k}\gamma\eta^2\phi^3\left((c_D)^{1+k}(2+k+k^2)\alpha-2k^2\gamma\phi\right)}{4k^2(1+k)^2\left((c_D)^{1+k}\alpha-\gamma\phi\right)^2\left((c_D)^{1+k}\alpha+k\gamma\phi\right)}|_{N_E=N_E^{mkt}}.
\end{aligned}
$$

Finally, using the solutions above, we compute the second order condition for objective function evaluation at the market equilibrium as:

$$
\begin{aligned}
& \frac{d^2u(\alpha,c_D(N_E),\lambda(N_E))}{(dN_E)^2}|_{N_E=N_E^{mkt}} \\
= \quad & -\frac{(c_M)^{-2k}\gamma\eta\phi^3\left((c_D)^{1+k}\alpha+2(1+k)\gamma\phi\right)}{8(c_D)^2k(1+k)(2+k)\left((c_D)^{1+k}\alpha-\gamma\phi\right)\left((c_D)^{1+k}\alpha+k\gamma\phi\right)}|_{N_E=N_E^{mkt}} \\
< \quad & 0
\end{aligned}
$$

since as shown above $\alpha-\lambda^{mkt}\cdot c_D^{mkt}>0$ implies $\left(c_D^{mkt}\right)^{1+k}\alpha-\gamma\phi>0$. Therefore, N_E^{mkt} is a local maximizer.■

Proof of Proposition 4: To prove part (i), let us first use (16) and (17) and rewrite planner's constraints (54) and (55) as

$$
\sum_{s\in\{1,2\}}\frac{2(k+1)\gamma(c_M)^k}{\eta}\frac{(\alpha_s-\lambda^{**}\cdot c_D^{s**})}{\lambda^{**}\cdot(c_D^{s**})^{k+1}}\left(\frac{\lambda^{**}(c_D^{s**})^{k+2}(c_M)^{-k}k}{2(1+k)(2+k)\gamma}+f_e\right)=1 \tag{72}
$$

$$
N_E^{s**}=\frac{2(k+1)\gamma(c_M)^k}{\eta}\frac{(\alpha_s-\lambda^{**}\cdot c_D^{s**})}{\lambda^{**}\cdot(c_D^{s**})^{k+1}} \tag{73}
$$

where $(c_D^{1**},c_D^{2**},\lambda^{**})$ is determined by (72) and (73) under given (N_E^{1**},N_E^{2**}). Turning now to the government's problem, we let the government pick t_e^{s*} for $s\in\{1,2\}$ such that

$$
f_e=t_e^{s*}+\frac{(\lambda^{**})(c_D^{s**})^{k+2}}{\gamma\tilde{\phi}}. \tag{74}
$$

If we plug (74) into (72) and simplify, then we obtain

$$
\sum_{s\in\{1,2\}}\frac{2(k+1)\gamma(c_M)^k}{\eta}\frac{(\alpha_s-\lambda^{**}\cdot c_D^{s**})}{\lambda^{**}\cdot(c_D^{s**})^{k+1}}\left[\frac{\lambda^{**}(c_D^{s**})^{k+2}(c_M)^{-k}}{2\gamma(k+2)}+t_e^{s*}\right]=1. \tag{75}
$$

For given (t_e^{1*},t_e^{2*}), $(c_D^{1*},c_D^{2*},\lambda^*)$ are determined by (66) and (67) while (N_E^{1*},N_E^{2*}) is determined by (45). Comparing (74) with (67) and likewise (75) with (66), we conclude that

$(c_D^{1*}, c_D^{2*}, \lambda^*) = (c_D^{1**}, c_D^{2**}, \lambda^{**})$. Finally, given this equivalence and comparing (73) with (45), we conclude that $(N_E^{1*}, N_E^{2*}) = (N_E^{1**}, N_E^{2**})$.

The proof of part (ii) is similar and therefore omitted. ■

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