On the Optimality of Tariff Caps*

Manuel Amador
Stanford University

Kyle Bagwell
Stanford University

May 12, 2010

Abstract

We consider a model in which governments negotiate a trade agreement while uncertain about future political pressures. After the agreement is formed and before applying its import tariff, a government privately observes its political economy parameter. This parameter reflects the political pressure faced by the government and determines the welfare weight that the government attaches to the profitability of its import-competing sector. In this private-information setting, we find conditions under which an optimal trade agreement for governments takes the form of a tariff cap: a tariff function that specifies the maximum permissible tariff and grants the importing government flexibility to apply any tariff at or below the cap. This characterization offers an interpretation of WTO rules, under which member governments negotiate tariff bindings (i.e., caps). Our theory also provides an interpretation of "binding overhang" as an implication of an optimally designed trade agreement.

Keywords: Tariff caps, Trade agreements, Political economy, Private information, Mechanism design without transfers.

1 Introduction

Governments negotiate trade agreements in order to obtain higher political economic welfare than they would obtain in the absence of an agreement. An import tariff imposed by the government of a large country lowers the world price at which foreign exporters sell; thus, when one country applies a higher import tariff, a negative international ("terms-of-trade") externality is generated for its trading partner. As a consequence of this externality, and for a wide range of specifications of government preferences, the

*We would like to thank Robert Staiger, Juuso Toikka and Ivan Werning for fruitful comments and suggestions. We also would like to thank seminar participants at London Business School, MIT and Stanford. Manuel Amador acknowledges NSF support.

tariffs that governments set under non-cooperative play are higher than would be efficient, where efficiency is measured relative to the preferences of governments. The central purpose of a trade agreement is then to facilitate reciprocal reductions in tariffs, so that governments can enjoy higher political economic welfare.[1]

The negotiation of a trade agreement, however, is not a simple undertaking. A first complication is that negotiations occur in the presence of uncertainty. In particular, at the time of negotiation, governments may be uncertain about the respective political pressures that they will face in the future. Negotiating governments are then uncertain about the tariffs that will be efficient in the future. If political pressures were publicly observable, then governments might address this complication by negotiating a state-contingent agreement, whereby they apply efficient tariffs once political pressures are realized. Broad movements in political pressures may be publicly observable; however, at the time that tariffs are applied, each government is likely to possess some private information about the extent of the pressures that it faces. A second complication, then, is that governments possess private information when tariffs are applied.

The presence of private information introduces an incentive compatibility problem. Suppose that each government is privately informed as to the magnitude of a political economy parameter in its welfare function, where the parameter determines the welfare weight that the government attaches to the profitability of its import-competing sector. For example, the magnitude of this parameter could reflect the intensity of any of several forms of political pressure that might be applied by firms in this sector. Similarly, the size of the parameter could also reflect the level of political support (lobbying contributions, participation in fund-raising events, etc.) from firms in this sector. If governments attempt to negotiate an agreement that is fully efficient (i.e., first best), then they must ensure that a government applies a higher tariff when its political economy parameter is larger. An incentive compatibility problem then arises, because the higher tariff will also be attractive to a government with a political economy parameter of moderate size. This follows since, for any given political economy parameter, a government's preferred (i.e., Nash) tariff is above its efficient tariff, as a consequence of the aformentioned international externality.

We can imagine that governments might address the incentive compatibility problem with sidepayments (i.e., monetary transfers). If governments design a trade agreement in which sidepayments are used, and if transfers can be made between governments in a lump sum fashion, then governments could implement fully efficient tariffs.[2] A gov-

[1] For further development of this argument, see Bagwell and Staiger (1999).

[2] See, for example, Bagwell and Staiger (2005).

ernment that faces large pressures would find a high tariff so appealing that it would be willing to pay more in exchange for such a tariff than would a government with moderate pressures. Sidepayments do not figure prominently in the rules of the WTO, however, and explicit monetary transfers are almost never used in WTO dispute resolutions.

This discussion motivates the following question: If governments are uncertain of their future political economic preferences at the time that they negotiate a trade agreement but understand that they respectively will be privately informed as to their preferences when tariffs are applied, and if monetary transfers are infeasible, then what form does an optimal trade agreement take? The purpose of this paper is to answer this question. We focus on a static model with two countries, in which each government privately observes the political economy parameter that determines the welfare weight that it attaches to its import-competing sector. A trade agreement is then an import tariff function, which indicates how a government's applied tariff can vary with its "type"(i.e., with the parameter that it observes), where each government has a continuum of possible types. We adopt a simple model of trade, in which consumers have additively separable utility functions and firms are competitive. As we explain, this structure enables us to focus on the optimal trade agreement for the "home"government's tariff function, with the understanding that an analogous characterization applies to the tariff function of the foreign government.[3]

At an intuitive level, it is clear that the choice of an optimal trade agreement must reflect at least three considerations. First, since a higher tariff induces a negative international externality, we may expect that an optimal trade agreement would induce applied tariffs that are lower on average than they would be in the absence of a trade agreement. Second, governments also may be attracted to a trade agreement that allows some flexibility, so that higher tariffs can be matched with higher types, at least over some ranges. A trade agreement that facilitates some matching of this kind may be appealing, since the efficient tariff is increasing in the importing government's type. Finally, governments

[3]To characterize the optimal trade agreement in the absence of transfers, we must be careful to construct our model so as to shut down all means through which governments might achieve transfers. First, governments might achieve transfers using appropriate combinations of import tariffs and export subsidies. In the WTO, export subsidies are prohibited in the Subsidies and Countervailing Measures Agreement. A growing literature addresses the wisdom of this prohibition. It is not our purpose to address this issue here; instead, we consider the optimal form of a trade agreement, when export policies are unavailable. Second, we also rule out the use of tariff-quota schemes, which represent a further means through which governments might transfer revenue (see Feenstra and Lewis, 1991, for an analysis of optimal trade agreements with such transfers). Finally, governments might also achieve transfers via general-equilibrium ("Lerner symmetry") effects, under which a domestic import tariff reduction alters relative prices and thereby expands domestic export volume. To eliminate this channel for revenue transfers, we employ a ("partial-equilibrium") model in which a numeraire good enters utility in a quasi-linear fashion and is freely traded across countries so as to achieve balanced trade.

may also wish to consider the variance of a tariff function. In the model that we analyze, the importing government's welfare function is concave in the tariff, but the exporting government's welfare function may be convex in the import tariff. The optimal trade agreement may thus be expected to depend on relative curvature measures across the two government welfare functions.

Our main finding is that conditions exist under which an optimal trade agreement takes the form of a tariff cap. A tariff cap is a tariff function that specifies a maximum permissible tariff and grants the importing government flexibility to apply any tariff at or below the cap. Intuitively, a cap serves to lower the average tariff, while still allowing a government flexibility when it comes to tariffs below the cap. When the importing government has a low type, its preferred (Nash) tariff is below the cap. The government then applies a tariff that falls strictly below the cap. When the importing government has a higher type, however, the cap binds; thus, for all types above a certain threshold, the government's import tariff function exhibits bunching at the cap. The conditions that we provide relate to the curvature properties of the government welfare functions and the slope of the density function for types. For the domestic import good, the conditions are more likely to be satisfied when the convexity of the foreign welfare function is not too great relative to the concavity of the domestic government welfare function, and when the density function is non-decreasing.

An important benefit of our main finding is that the conditions are developed for a general model, so that we can confirm whether or not the conditions hold in a range of specific examples. To illustrate, we consider a simple example in which the utility function is quadratic and supply functions are linear. In this linear-quadratic example, the conditions underlying our characterization are satisfied if the density is non-decreasing. We also show that these conditions can hold even for densities that decrease over part or all of the support, and for convex and concave densities, provided that the density does not fall too quickly. Similarly, we consider an example with log utility and endowments (inelastic supply) and show that the conditions underlying our characterization are sure to hold if the density is non-decreasing.

Our main finding provides an interpretation of a key design feature of the GATT/ WTO trade agreement, whereby governments negotiate "tariff bindings"or "bound tariff levels"rather than precise tariff levels.[4] A bound tariff is simply a tariff cap. While a tariff cap is a very simple tariff function, our analysis suggests that an optimal trade agreement

[4]GATT Article II.1(a) states "*each contracting party shall accord to the commerce of the other contracting parties treatment no less favourable than that provided for in the appropriate Part of the appropriate Schedule annexed to this Agreement.*"In GATT parlance, a contracting party is a country and the "treatment"provided for in the schedule of concessions is the bound tariff.

among privately informed governments takes this form. Our analysis also provides an interpretation of a practice that is sometimes observed, whereby a WTO member government applies a tariff that falls below its negotiated bound tariff. This phenomenon is sometimes called "binding overhang"or "water in the tariffs."Our analysis indicates conditions under which binding overhang is expected to occur with positive probability in an optimal trade agreement.

Our work relates to two main literatures. The first literature addresses the economic theory of trade agreements. Much of this literature considers the purpose and design of the WTO. Only a few papers, however, have addressed the economics of tariff caps (i.e., bindings) and the associated possibility of binding overhang. Indeed, after surveying work on the economics of tariff bindings, the WTO Report 2009 (World Trade Organization, 2009) concludes that there is "*relatively little theoretical work on this topic....The implications of random tariff regimes...remain largely unexplored*."Recent empirical work suggests that binding overhang is quantitatively important and that the pattern of binding overhang across products and countries is somewhat uneven.[5] The existence of binding overhang can also be a source of tension in negotiations, since an exporting government may question the value of an importing government's offer to reduce its bound tariff when binding overhang is present. Empirical findings and negotiation practice thus provide additional motivation for theoretical analyses of tariff bindings and binding overhang.

In the small and recent theory literature that addresses tariff bindings and binding overhang, our work is most closely related to a pair of papers that considers trade agreements among privately informed governments. Bagwell and Staiger (2005) utilitize the linear-quadratic model of trade and assume that a government has a continuum of possible types. One of their findings is that the optimal tariff cap offers greater expected joint government welfare than is possible under a tariff agreement that specifies a precise (rigid) tariff that is applied by all types. They do not characterize the optimal trade agreement among the full set of incentive compatible tariff functions. Bagwell (2009) also utilizes the linear-quadratic model. In a model with two possible types, one of his findings is that an optimal trade agreement does not take the form of a tariff cap. Both papers establish that binding overhang occurs with positive probability when the optimal tariff cap is used. Relative to these papers, our contribution is to provide general conditions for a model with a continuum of types under which an optimal trade agreement takes the

[5] Bouet and Laborde (2008) estimate that world trade would fall by 7.7% if applied tariffs of all major economies were raised to bound rates. The findings of Bchir, Jean, and Laborde (2006) suggest that binding overhang is often greater in developing countries and for agricultural sectors, but important exceptions exist. Their results (p. 229) "*confirm the importance and unevenness of binding overhang*."

form of a tariff cap.[6]

Tariff bindings and binding overhang have also received some attention in other modeling frameworks. In a model with contracting costs, Horn, Maggi, and Staiger (2010) establish that binding overhang can occur. Maggi and Rodriguez-Clare (2007) analyze a model in which applied tariffs are set at bound levels in equilibrium, and yet the potential to apply a tariff below the bound level induces ex post lobbying that mitigates an ex ante problem of over-investment. Finally, Bagwell and Sykes (2004) suppose that import-competing firms are risk averse and informally argue that governments may set applied tariffs below bound levels, in order to maintain "policy space"in which to adjust tariffs and maintain local-price stability.

Our paper is also related to a second literature, which characterizes the value of rules and caps when agents possess private information. McAfee and McMillan (1992) provide conditions under which an identical-bidding rule represents the optimal form of collusion among bidders, while Athey, Bagwell, and Sanchirico (2004), Athey and Bagwell (2008) and Bagwell and Lee (2010) show that optimal collusion among privately informed firms may entail rigid pricing or advertising rules. Recent work on dynamic consistency and private information identifies settings in which caps are optimal. Athey, Atkeson, and Kehoe (2005) provide conditions under which optimal monetary policy for a privately informed official is characterized by a cap. Amador, Werning, and Angeletos (2006) develop a Lagrangian method and provide a sufficient and necessary condition under which a cap is optimal in a consumption-savings model. Finally, research on delegation in organizations, started by Holmström (1984), has shown that caps may also represent an optimal form of delegation, when a principal faces an informed but biased agent.[7] A common feature of all of the above papers is the finding that an optimal mechanism among privately informed agents with no or limited transfers may entail a rigid rule or cap.

Our main finding that an optimal trade agreement between privately informed governments is described by a tariff cap is clearly related to this second literature. Our model,

[6] We may think of the two type model as a limit of a continuum-type model as the density goes to zero for all types other than the selected two types. Viewed from this perspective, Bagwell's (2009) finding is consistent with our finding here that a tariff cap is optimal when the density satisfies restrictions which ensure that it does not fall too quickly.

[7] Most of the delegation literature has focused on quadratic preferences and one-dimensional action to be delegated, and has restricted the delegation set to be an interval; recent work has relaxed these assumptions (see Alonso and Matouschek, 2008, and references therein). We note as well that our solution method is quite different from the approach commonly used in this literature, and that the sufficient conditions identified by Alonso and Matouschek for "interval delegation" when preferences take a more general form are not satisfied in our trade-policy model, since (i) the agent's welfare, which in our case would correspond to the home government's, cannot generally be written in the form assumed in that paper and (ii) the international (terms-of-trade) externality ensures that Nash tariffs are above efficient tariffs, so that a one-sided "bias" is always present. We thank Ilya Segal for directing our attention to this last paper.

however, has a novel payoff structure: payoffs are separable across actions, and one player's welfare may be convex in the other player's action. Our solution strategy follows Amador et al. (2006); however, the Lagrangian method they propose is not directly applicable in our set up because, under reasonable assumptions regarding preferences and technology, the resulting objective function in our optimization problem may fail to be concave. In spite of this, we extend the methods and obtain sufficient conditions for optimality by explicitly constructing Lagrange multipliers that make the resulting Lagrangian concave and by showing that the tariff cap allocation satisfies the first order conditions for the maximization of this Lagrangian.

The paper is organized as follows. In Section 2, we present our basic model of trade and government welfare. In Section 3, we describe the problem that an optimal trade agreement must solve, and we characterize the optimal trade agreement in the family of agreements that can be described by a tariff cap. Section 4 contains a variational analysis of the linear-quadratic model. Our work in this section suggests that the optimal tariff cap represents a plausible candidate for an optimal trade agreement, under appropriate conditions on the slope of the density and the curvature of the government welfare functions. Returning to the general model, we establish in Section 5 conditions under which an optimal trade agreement takes the form of a tariff cap. In Section 6, we illustrate and interpret this result in the specific context of the linear-quadratic example. Section 7 concludes. Remaining proofs are collected in the Appendix.

2 The Model

We consider a simple model of trade in three goods between two countries. All goods are demanded and produced in both countries. Supply functions differ across countries, however, and international trade arises for this reason. The home country imports good x from the foreign country and exports good y to the foreign country. A numeraire good n is also traded between the two countries.

In both countries, residents share a common utility function which is quasilinear and additively separable across the three products, with the numeraire good entering the utility function in a linear fashion. Within any given country, the consumer demand for good i, where $i = x, y$, thus depends on the local price of good i relative to that of the numeraire good. Each good is supplied under conditions of perfect competition, so that, within any given country, the production of good i, where $i = x, y$, also depends on the local price of good i relative to that of the numeraire good. Finally, the numeraire good is produced in each country under constant returns to scale and is freely traded across countries so as to ensure that trade is balanced. For simplicity, we normalize the price of the numeraire good to unity.[8]

In this setting, which is common in the literature, market outcomes for good x are independent of those for good y. To reduce notational clutter, we may thus put good y to the side and focus on good x. After characterizing trade policies for the home country's import good, we may translate our findings and characterize trade policies for the foreign country's import good.

Let us then put good y to the side and represent the utility function for consumers at home by

$$u(c^x) + c^n,$$

where c^x and c^n represent the respective consumption levels of good x and the numeraire good, respectively. Similarly, the utility function abroad is given by

$$u(c^x_\star) + c^n_\star,$$

where the subscript $\star$ denotes the respective foreign values. The function u is strictly increasing, strictly concave and thrice continuously differentiable. Notice that we assume that home and foreign consumers enjoy symmetric utilities for good x. This assumption simplifies our presentation but is not essential.

Let p denote the home relative price of good x with respect to good n. Similarly, $p_\star$ is the foreign relative price of good x with respect to good n. Let the supply functions of good x at home and abroad be given by $Q(p)$ and $Q_\star(p_\star)$ respectively. For prices that elicit strictly positive supply, the functions $Q(p)$ and $Q_\star(p_\star)$ are assumed to be strictly increasing and twice continuously differentiable. We also assume that the supply functions differ in the following way: $Q(p) < Q_\star(p)$ for any p such that there is strictly positive world supply. One implication of the above is that good x will be imported under free trade by the home country.[9]

We use z to denote the volume of international trade of good x. Home consumers' optimization delivers an inverse demand function for imports:

$$u'(Q(p) + z) = p \Rightarrow p = P(z), \tag{1}$$

[8] We may develop the model further by specifying the underlying factor market. We assume that the numeraire good is produced from labor alone, where one unit of labor produces one unit of the numeraire good. In each country, the supply of labor is infinitely elastic at a unitary wage, and good $i = x, y$ may be produced from labor alone subject to diminishing returns. For further details, see Bagwell and Staiger (2001, 2005).

[9] The symmetric assumption for good y ensures that good y is exported under free trade by the home country.

where $P'(z) < 0$ follows from our assumptions. Likewise, foreign consumers' optimization delivers an inverse supply function for exports:

$$u'(Q_\star(p_\star) - z) = p_\star \Rightarrow p_\star = P_\star(z), \tag{2}$$

where $P'_*(z) > 0$ is implied by our assumptions.

We abstract from export policies and assume that each country has available a specific (i.e., per-unit) import tariff. If the government of the home country selects the import tariff τ, then the implied import volume, $z(\tau)$, is the value of z which satisfies $\tau = P(z) - P_*(z)$. Under our assumptions, it is straightforward to show that $z'(\tau) < 0$. Of course, we may equivalently think of choosing the trade volume z, with an import tariff, $\tau(z)$, then implied by the relationship $\tau = P(z) - P_\star(z)$, where under our assumptions $\tau'(z) < 0$ follows. We use both formulations below, but we find it convenient to begin with the latter formulation.

For a given trade volume z, the associated producer surplus (profit) functions at home and abroad are defined as

$$\Pi(z) = \int_{\underline{p}}^{P(z)} Q(\tilde{p})d\tilde{p}\,, \qquad \Pi_\star(z) = \int_{\underline{p}_*}^{P_\star(z)} Q_\star(\tilde{p})d\tilde{p},$$

where $\underline{p} \geq 0$ is the highest price p at which $Q(p) = 0$ and $\underline{p}_* \geq 0$ is likewise the highest price p_* at which $Q_*(p_*) = 0$. We note that $\Pi(z)$ denotes the producer surplus enjoyed by the import-competing industry in the home country, while $\Pi_\star(z)$ represents the producer surplus for the exporting industry in the foreign country.

We are now prepared to define government welfare functions. The welfare of the foreign government is constructed as the sum of the consumer surplus and the producer surplus in the foreign country:

$$V(z) = u(Q_\star(P_\star(z)) - z) - P_\star(z)(Q_\star(P_\star(z)) - z) + \Pi_\star(z). \tag{3}$$

The welfare of the policy maker at home is constructed in a similar fashion, with two important differences. The first difference is that the welfare of the domestic government includes tariff revenue, and the second difference is that the domestic government welfare function includes a political economy parameter reflecting the greater weight that the government may give to the interests of import-competing firms.[10]. Formally, we

[10]Recall that we put good y to the side. The welfare function of the foreign government includes tariff revenue and a political economy parameter for import-competing firms, when we consider good y.

represent the welfare of the domestic government as

$$W(z|\gamma) = u(Q(P(z)) + z) - P(z)(Q(P(z)) + z) + (P(z) - P_\star(z))z + \gamma\Pi(z), \text{ or}$$
$$W(z|\gamma) = B(z) + \gamma\Pi(z) \tag{4}$$

where we let

$$B(z) \equiv u(Q(P(z)) + z) - P(z)Q(P(z)) - P_\star(z)z \tag{5}$$

and where $\gamma \in \Gamma \equiv [\underline{\gamma}, \bar{\gamma}]$ is the domestic political economy parameter.

We pause now to interpret the political economy parameter, γ. When $\gamma = 1$, the home government maximizes national economic welfare. A value for γ above 1 captures the desire of the politician to transfer resources to the domestic production sector facing import competition. A government may place special weight on the interests of import-competing firms, if such firms have organized lobbies and offer political contributions to the government, as in the model of Grossman and Helpman (1995). More generally, Baldwin (1987) argues that a range of political economy forces can be captured using the specification that we adopt here. Bagwell and Staiger (2001, 2005), for example, analyze a model of this kind, under the assumptions that supply and demand functions are linear.[11] Feenstra and Lewis (1991) also use this specification of political pressure in their analysis of optimal trade agreements when transfers are permitted.

As explained above, different specifications for the import tariff lead to different levels in the equilibrium volume of international trade, where the relationship between τ and z is one to one. For convenience, we use z as the policy variable and report the following proposition:

Proposition 1. *The following holds for all $z > 0$: $V'(z) > 0$; $\Pi'(z) < 0$; and $B''(z) + \Pi''(z) + V''(z) < 0$.*

Proof. See Appendix A. □

A higher value of trade volume is delivered if the import tariff applied by the home government is lower. With $P'_*(z) > 0$ as shown in Appendix A, the foreign country enjoys a terms-of-trade gain. This explains why the welfare of the foreign country strictly increases as trade volume rises. Of course, with $P'(z) < 0$ as confirmed in Appendix A, a higher trade volume depresses the price of the imported good in the home market, leading to a strict decrease in the profit of the import-competing industry. Finally, with $\gamma = 1$,

[11] It is also plausible that a government attaches additional political weight to the well-being of its export interests. The approach that we adopt here has the virture of simplifying the problem somewhat, since along any given trade channel only one political economy parameter is in play.

the joint welfare of the home and foreign government corresponds to global real income, which is maximized at the volume of trade that corresponds to free trade. The third part of the proposition above simply confirms that global real income is strictly concave in the underlying volume of trade.

In the following lemma, we show that, if some further structure is imposed on the production and utility functions, then additional curvature properties hold:

Lemma 1. *Suppose that $Q'' \leq 0$ and that $u''' \geq 0$, then $V''(z) > 0$, $B''(z) < 0$ and $\Pi''(z) > 0$ for all $z > 0$.*

Proof. See Appendix B. □

Under relatively mild conditions, therefore, foreign welfare is strictly convex in the underlying trade volume. By contrast, the domestic government welfare function includes strictly concave and convex components. This lemma foreshadows a challenge we face below, when we define a program and seek to identify multipliers for which the associated Lagrangian is strictly concave.[12]

3 The Incentives Problem

We assume that the home and foreign governments negotiate a trade agreement before realizing their respective political economy parameters. Thus, at the time of negotiation, governments are uncertain about their future preferences. As before, we focus on good x while keeping in mind that a symmetric analysis applies to good y. For good x, the political economy parameter, γ, is embedded in the home government welfare function and has support $\Gamma \equiv [\underline{\gamma}, \bar{\gamma}]$. We represent the c.d.f as $F(\gamma)$ and denote the density as $f(\gamma)$ which we assume to be strictly positive over the support.[13] Once the value of γ is realized, the home government is privately informed of this value.

In this private-information setting, while governments may seek an agreement that assigns different tariffs (i.e., trade volumes) for different realizations of γ, they may choose only among incentive-compatible tariff functions. In their negotiation, therefore, governments select a tariff function, $\tau(\gamma)$, that maximizes their expected joint welfare over

[12]This challenge was not present in Feenstra and Lewis (1991) because the presence of a transfer allows them to obtain a foreign welfare function that is concave with respect to trade volume.

[13]Symmetrically, for good y, we may assume that the foreign government privately observes the realization of its political economy parameter, γ^*, where γ^* defines the weight that the foreign government attaches to the profit of its import-competing firms and where γ^* and γ are independently and identically distributed.

the set of incentive-compatible tariff functions. Equivalently, we may think of governments as selecting the trade volume function, $z(\gamma)$, that maximizes expected joint welfare among those trade volume functions that are incentive compatible. In fact, as we argue just below, it is also equivalent to think of governments as choosing the profit level for the domestic import-competing industry, where the selected profit level is allowed to vary with the political economy parameter, provided that it does so in an incentive-compatible way. In this section, we adopt the latter formulation, define the associated program and consider trade agreements that can be implemented with a cap on the maximum tariff.

3.1 The Problem

In this subsection, we define the program of interest. Recall from Proposition 1 that $\Pi'(z) < 0$. We may thus denote by Π^{-1} the inverse function of Π. Let $z(\pi) = \Pi^{-1}(\pi)$, $b(\pi) \equiv B(\Pi^{-1}(\pi))$ and $v(\pi) \equiv V(\Pi^{-1}(\pi))$. Let the set of feasible π be $[0, \Pi(0)] \equiv [0, \bar{\pi}]$. Building on our work above, we can easily establish that $z'(\pi) < 0$ and $v'(\pi) < 0$ for all $\pi \in [0, \bar{\pi}]$ (i.e., for all $z > 0$). That is, a higher value for domestic profit corresponds to a lower trade volume and thus a higher import tariff; consequently, the welfare of the foreign government strictly falls when the domestic profit is increased. We can also show that, if we strengthen our assumptions in a mild way, so that $Q'' \leq 0$ and $u''' \geq 0$, then $z''(\pi) > 0$ and $v''(\pi) > 0$ for all $\pi \in [0, \bar{\pi}]$. Thus, after the change of variables, it remains reasonable to expect that the foreign welfare function may be strictly convex.

A trade agreement is a function $\pi(\gamma)$ that determines the profit allocated to the domestic producers as a function of γ. We are looking for a trade agreement that is incentive compatible and efficient. That is, we seek the trade agreement that solves the following problem:

$$(P): \qquad \max_{\pi(\gamma)} \left\{ \int_{\gamma \in \Gamma} \Big(\gamma \pi(\gamma) + b(\pi(\gamma)) + v(\pi(\gamma)) \Big) dF(\gamma) \right\} \tag{Obj}$$

subject to:

$$\gamma \in \arg\max_{\gamma' \in \Gamma} \left\{ \gamma \pi(\gamma') + b(\pi(\gamma')) \right\} \tag{IC}$$

Once the optimal profit function is determined, we can easily back out the associated tariff function. We find it convenient to write the problem in terms of profit, since the profit function enters linearly in some terms in the objective and the incentive constraint.

Before analyzing this problem, we note four important features. First, we note that the problem includes terms that are linear, perhaps strictly concave and possibly strictly convex in $\pi(\gamma)$.[14] Second, the set of permissible profit functions is determined by the incentive constraint. As we will see, the feasible set of profit functions includes discontinuous functions, and any feasible profit function must be monotonic (weakly increasing). Third, the statement of the problem reflects our assumption that governments do not have available sidepayments (monetary transfers). The absence of sidepayments makes the analysis of the problem more challenging.[15] Fourth, we assume that the governments seek a trade agreement that maximizes the sum of their expected welfares. The solution generates a particular outcome on the efficiency frontier when only tariffs are allowed, and it is important to keep in mind that an analogous solution applies for good y, where the foreign government has private information about the weight that it attaches to its import-competing industry.[16]

It is useful at this point to define the profits that would be achieved in a "flexible" or Nash allocation, where the home government is not subject to any agreement. To this end, we define the profit under flexibility, $\pi_f(\gamma)$, as follows:

$$\pi_f(\gamma) = \arg\max_{\pi \in [0,\bar{\pi}]} \{\gamma\pi + b(\pi)\} \tag{6}$$

We assume that the above maximization has an interior maximum for all $\gamma \in \Gamma$. This means that, at $\pi = \pi_f(\gamma)$, we have $\gamma + b'(\pi) = 0$ and $b''(\pi) < 0$. It follows then that $\pi'_f(\gamma) = -1/b''(\pi_f(\gamma)) > 0$. Associated with this flexible profit allocation is a flexible or Nash tariff allocation, $\tau_f(\gamma)$, which is also strictly increasing.

3.2 Optimality Within the Set of Tariff Caps

Suppose that we were constrained to look for an optimal trade agreement within the set of tariff functions that are described by a tariff cap. When the home government agrees to a tariff cap, it may not apply a tariff in excess of the cap but it has complete flexibility to choose any tariff that is at or below the cap. Given that domestic profit, π, is strictly

[14] We have already noted that it is reaonable to expect that $v(\pi)$ may be strictly convex. We assume below that $b(\pi)$ is strictly concave for profit levels that are associated with "flexible" outcomes, and we later require in Assumption 1 that $b(\pi)$ is strictly concave for all feasible π.

[15] As discussed in the Introduction, and as Bagwell and Staiger (2005) confirm, a fully efficient ("first-best") outcome can be implemented if sidepayments are allowed.

[16] If the instrument space is expanded so that governments can make sidepayments during their negotiation, and thus before they obtain private information, then all efficient payoffs can be achieved by solving program (P) and specifying an approritate ex ante transfer. Grossman and Helpman (1995) make a similar point, in their analysis of "trade talks."

increasing in the tariff, we may define a tariff cap allocation to be

$$\pi^c(\gamma) = \begin{cases} \pi_f(\gamma) & \text{, for } \gamma < \gamma^c \\ \pi_f(\gamma^c) & \text{, for } \gamma \geq \gamma^c \end{cases}$$

Notice that a tariff cap allocation satisfies (IC).

Under a tariff cap allocation, the objective function, (Obj), becomes:

$$O(\gamma^c) \equiv \int_{\underline{\gamma}}^{\gamma^c} (\gamma \pi_f(\gamma) + b(\pi_f(\gamma)) + v(\pi_f(\gamma)))dF(\gamma)$$
$$+ (1 - F(\gamma^c))\Big(\pi_f(\gamma^c)\mathbb{E}[\gamma|\gamma > \gamma^c] + b(\pi_f(\gamma^c)) + v(\pi_f(\gamma^c)\Big)$$

The first derivative of O, after some algebra, is:

$$\frac{1}{\pi_f'(\gamma_c)} \frac{O'(\gamma^c)}{1 - F(\gamma^c)} = v'(\pi_f(\gamma^c)) - \gamma^c + \mathbb{E}[\gamma|\gamma > \gamma^c] \tag{7}$$

where we use that $b'(\pi_f(\gamma^c)) = -\gamma^c$ which follows from (6).

The following definition will be helpful in the characterization that follows:

Definition 1. *Let* $\gamma^p \in \arg\max_{\gamma \in \Gamma} O(\gamma)$.

We will assume in what follows that γ^p is *unique* and *in the interior* of Γ. Note that given that $\lim_{\gamma^c \to \bar{\gamma}}\{v'(\pi_f(\gamma^c)) - \gamma^c + \mathbb{E}[\gamma|\gamma > \gamma^c]\} = v'(\pi_f(\bar{\gamma})) < 0$, a sufficient condition for an interior maximizer is that $v'(\pi_f(\underline{\gamma})) - \underline{\gamma} + \mathbb{E}\gamma \geq 0$.

The following lemma will prove useful:

Lemma 2. *The value of* γ^p *is such that* $v'(\pi_f(\gamma^p)) - \gamma^p + \mathbb{E}[\gamma|\gamma > \gamma^p] = 0$.

Proof. The stated result follows from the assumption that γ^p is in the interior of Γ, and hence, $O'(\gamma^p) = 0$ in an optimum. □

4 Variational Analysis in a Linear-Quadratic Example

To solve the problem (P), our approach is to define a relaxed problem and use the Lagrangian techniques developed by Amador et al. (2006) This approach is particularly effective for problems in which a conjectured solution is already identified. To motivate and explain our conjecture, we thus now pause and briefly consider the linear-quadratic example studied by Bagwell and Staiger (2005). For this example, we employ simple variational techniques to propose conditions under which tariff functions with certain properties *cannot* be optimal. In particular, when such conditions hold, our findings in this section suggest the optimal tariff cap as a reasonable conjecture for the solution to the problem (P). Accordingly, our discussion here provides an intuitive foundation for some of the assumptions that we make in subsequent sections for our general analysis.[17]

Suppose that the utility function for good x takes a simple quadratic form, $u(c) = c - (c)^2/2$. The resulting demand function for good x in each country is then linear with $1 - p$ units demanded when the local relative price is p. Let the supply function for good x in the home country be $Q(p) = p/2$ and in the foreign country be $Q_*(p_*) = p_*$. For this linear-quadratic example, if we set domestic import demand equal to foreign export supply and use $p = p_* + \tau$, then we may express the market clearing prices as functions of tariffs.[18] We may then compute consumer surplus, profit and tariff revenue, and thereby determine that domestic government welfare may be represented as the following quadratic function of the tariff:

$$\frac{9+8\gamma}{98} + \frac{8\gamma - 5}{49}\tau - \frac{2(17-2\gamma)}{49}\tau^2.$$

Assuming (as we do below) that γ is not too large, the home government welfare function is strictly concave in the tariff. Similary, foreign government welfare takes the following form:

$$\frac{25}{98} - \frac{3\tau}{49} + \frac{9\tau^2}{49}.$$

Notice that foreign welfare is strictly convex in the tariff. These expressions can all be written as functions of the trade volume, z, using the fact that $z(\tau) = (1 - 6\tau)/7$ in this model. For the purposes of the present section, we find it more convenient to work with the tariff as the independent variable.[19]

The flexible or Nash tariff, $\tau_f(\gamma)$, is the tariff that maximizes domestic government welfare, given the realized value of the political economy parameter, γ. For a given value of γ, the fully efficient (i.e., first best) tariff, $\tau_e(\gamma)$, is the tariff that maximizes the sum of home and foreign welfare. For $\gamma \in [1, 7/4)$, the flexible and efficient tariff functions

[17]For a more general analysis of the linear quadratic case in the context of the context of the delegation literature, see Alonso and Matouschek (2008).

[18]The local relative price in the home country is $4(1+\tau)/7$ while the local relative price in the foreign country is $(4-3\tau)/7$. These and other relationships are derived in more detail in Bagwell and Staiger (2005).

[19]In Section 6, we represent the linear-quadratic model while treating the import volume z as the independent variable. This representation matches that taken elsewhere in the paper and faciliates the interpretation of our general assumptions in terms of the linear-quadratic model.

satisfy

$$\tau_f(\gamma) = \frac{8\gamma - 5}{4(17 - 2\gamma)} > \frac{4(\gamma - 1)}{25 - 4\gamma} = \tau_e(\gamma).$$

Thus, for political economy parameters in this range, the flexible tariff is higher than efficient. Intuitively, when contemplating a higher tariff, the domestic government doesn't internalize the negative terms-of-trade externality that is experienced by the foreign government. When $\gamma = 7/4$, the domestic political economy parameter is so high that the efficient tariff eliminates all trade. The flexible and efficient tariffs then agree: $\tau_f(7/4) = 1/6 = \tau_e(7/4)$. While the flexible tariff function is incentive compatible, the efficient tariff function is not.

As previously noted, the tariffs that are applied under a tariff cap are incentive compatible. The optimal tariff cap is the tariff cap that maximizes the expected sum of home and foreign government welfare. Assuming $\underline{\gamma} = 1 < \overline{\gamma} < 7/4$ and that the expected value of γ exceeds $5/4$, Bagwell and Staiger (2005) show that an optimal tariff cap $\overline{\tau}$ for this model satisfies $\overline{\tau} \in (\tau_f(\underline{\gamma}), \tau_e(\overline{\gamma}))$ and is the efficient tariff for the average type among those types that apply the tariff cap. In the particular case of a uniform distribution with $\overline{\gamma} \in [3/2, 7/4)$, the assumptions of their model hold, $\gamma_p = 3\overline{\gamma} - 7/2$, and the optimal tariff cap is $\overline{\tau} = (\overline{\gamma} - 11/8)/(4 - \overline{\gamma})$. The optimal tariff cap is illustrated in Figure 1. Notice that the tariff cap is lower than efficient for the highest type and higher than efficient for lower types. Notice as well that the lowest types apply tariffs below the cap; thus, the optimal tariff cap leads to binding overhang.

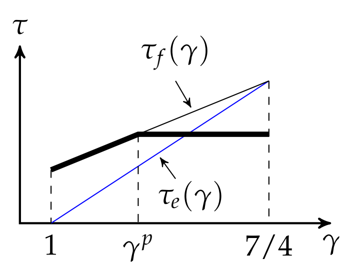

Figure 1: Optimal tariff cap.

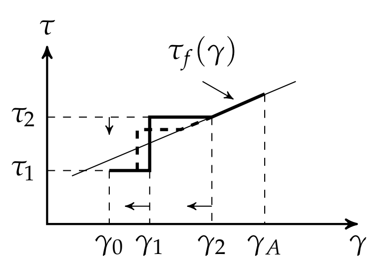

Figure 2: Variation 1.

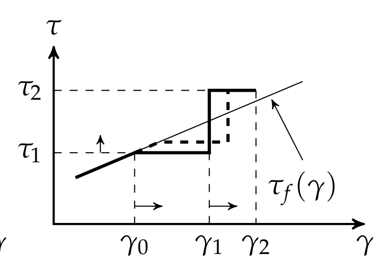

Figure 3: Variation 2.

The tariffs that are applied under an optimal tariff cap represent just one of many incentive compatible tariff functions. Any incentive compatible tariff function $\tau(\gamma)$ must be weakly increasing.[20] In addition, if an incentive compatible tariff function $\tau(\gamma)$ is continuous and strictly increasing over an interval, then $\tau(\gamma) = \tau_f(\gamma)$ on that interval. But an incentive compatible tariff function may also involve steps and points of discontinuity. If

[20]This is a standard finding, which as we discuss in the next section holds as well for the general version of our model.

an incentive compatible tariff function is discontinuous at some breakpoint γ_b and jumps upward at this point from a lower step to a higher step, then the lower step is strictly below $\tau_f(\gamma_b)$ while the higher step is strictly above $\tau_f(\gamma_b)$. Intuitively, in the absence of sidepayments, any jump from a lower to a higher step must take this form in order to ensure that the breakpoint type is indifferent.

To begin our exploration of candidate solutions, we consider any incentive compatible tariff function for which there exists an interval $[\gamma_0, \gamma_A]$ and values γ_1 and γ_2 such that $\underline{\gamma} \leq \gamma_0 < \gamma_1 < \gamma_2 < \gamma_A \leq \overline{\gamma}$ and

$$\tau(\gamma) = \begin{cases} \tau_1 \leq \tau_f(\gamma_0) & \text{, for } \gamma \in [\gamma_0, \gamma_1) \\ \tau_2 = \tau_f(\gamma_2) & \text{, for } \gamma \in (\gamma_1, \gamma_2] \\ \tau_f(\gamma) & \text{, for } \gamma \in [\gamma_2, \gamma_A]. \end{cases}$$

Note that incentive compatibility requires that type γ_1 is indifferent. This implies that $\tau_1 < \tau_f(\gamma_1) < \tau_2$.

As illustrated in Figure 2, over the interval $[\gamma_0, \gamma_A]$, this tariff function starts with a low step and then jumps at γ_1 up to a high step. The tariff function "steps into flexibility," in that it hits the flexible tariff function, $\tau_f(\gamma)$, at γ_2, and rides along the flexible tariff function thereafter for all types up to γ_A. We make no assumptions about the tariffs for types below γ_0 or above γ_A. A special case of interest arises when $\tau_1 = \tau_f(\gamma_0)$. In that case, the tariff function under consideration injects a step into the flexible tariff function.

For a given tariff function of the described kind, we now consider a simple variation in which γ_0, γ_A and τ_1 are held fixed while γ_1 is lowered. As shown in Figure 2, when the breakpoint type is reduced in this way, incentive compatibility requires that the top step be lowered; thus, a reduction in γ_1 results in a lower value for τ_2 and thus a lower value for γ_2.

Does this variation raise or lower expected joint welfare? Figure 2 suggests that three different effects warrant consideration. First, for $\gamma \in (\gamma_1, \gamma_2)$, the variation results in a lower tariff, corresponding to the reduction in τ_2. Since $\tau_f(\gamma) > \tau_e(\gamma)$, the variation results in *more* efficient tariffs for $\gamma \in (\gamma_1, \gamma_2)$. Second, for types just below the original value for γ_1, the variation results in a significantly higher tariff, since these types no longer apply τ_1 but instead apply (the lowered) τ_2. The variation thus results in a significantly *less* efficient tariffs for types just below γ_1. Finally, we can see from Figure 2 that the variation also alters the "risk" properties of the tariff distribution: relative to the baseline of the flexible tariff function, the variation results in a less variable distribution of applied tariffs.

Looking at Figure 2, we may anticipate that this variation is more likely to raise expected joint welfare if (i) the density $f(\gamma)$ increases sufficiently over $[\gamma_0, \gamma_2]$, so that the improved efficiency of tariffs for higher types in this interval contributes more to expected joint welfare than does the significantly diminished efficiency of tariffs for types just below γ_1, and (ii) the foreign welfare function is not too convex in the tariff relative to the concavity of the home welfare function (which is embedded in the slope of the flexible tariff function), so that the reduced variability of the tariff is a source of joint welfare gain.

To examine the variation in a baseline case, we suppose that $F(\gamma)$ is uniform. With this assumption, we pin down the first effect just mentioned, since $f(\gamma)$ is then constant. Our linear-quadratic model has already fixed the relative curvature properties of the foreign and home government welfare functions. In Appendix C, we show that a slight reduction in γ_1 leads to an increase in expected joint welfare in the amount

$$\frac{27(\gamma_2 - \gamma_1)^2}{(17 - 2\gamma_1)^2(17 - 2\gamma_2)(\overline{\gamma} - \underline{\gamma})} > 0.$$

Thus, in the case of a uniform distribution, it is not optimal to step into flexibility, as it would be better to "shrink" the step and extend the region of flexibility.

The result applies as well in the special case in which $\tau_1 = \tau_f(\gamma_0)$. For this special case, we may also consider a related experiment in which we start with the flexible tariff function and then consider introducing a small step of the described kind. In this experiment, we start with $\gamma_0 = \gamma_1 = \gamma_2$ and then increase γ_1 slightly so as to engineer a tiny step. The expression just derived indicates that the first-order effect of this change on joint welfare is zero. As we argue in Appendix C, the second-order effect is zero as well; however, the third-order effect on expected joint welfare of introducing a small step into the flexible tariff function is negative.

For the linear-quadratic model, if the distribution function is uniform, we conclude that it cannot be optimal to step into flexibility, and relatedly it cannot be optimal to inject a step into the flexible tariff function. Furthermore, based on the intuition presented above, we expect that these results would hold as well if the density were increasing rather than constant. Similar findings should also arise in other models of trade, provided that the foreign welfare function is not too convex relative to the concavity of home government welfare function.

We turn now to a second candidate solution. In particular, we consider any incentive compatible tariff function for which there exists an interval $[\gamma_0, \gamma_2]$ and a value γ_1 such that $\underline{\gamma} \leq \gamma_0 < \gamma_1 < \gamma_2 \leq \overline{\gamma}$ and

$$\tau(\gamma) = \begin{cases} \tau_f(\gamma) & \text{, for } \gamma \leq \gamma_0 \\ \tau_1 = \tau_f(\gamma_0) & \text{, for } \gamma \in [\gamma_0, \gamma_1) \\ \tau_2 & \text{, for } \gamma \in (\gamma_1, \gamma_2]. \end{cases}$$

Incentive compatibility again requires that type γ_1 is indifferent, which implies that $\tau_1 < \tau_f(\gamma_1) < \tau_2$.

Figure 3 illustrates this candidate. This tariff function is initially flexible and then "steps out of flexibility," in that it departs from the flexible tariff function with a flat step and then jumps at γ_1 up to a high step. We make no assumptions about the tariffs for types above γ_2.

For this tariff function candidate, we consider a simple variation in which γ_2 and τ_2 are held fixed while γ_1 is raised. As shown in Figure 3, when the breakpoint type is increased in this way, incentive compatibility requires that the bottom step be raised; thus, an increase in γ_1 results in a higher value for τ_1 and thus a higher value for γ_0.

To anticipate the implications of this variation for expected joint welfare, we look to Figure 3. There are again three effects. First, for $\gamma \in (\gamma_0, \gamma_1)$, the variation results in a higher tariff, corresponding to the increase in τ_1. Since $\tau_f(\gamma) > \tau_e(\gamma)$, the variation results in *less* efficient tariffs for $\gamma \in (\gamma_0, \gamma_1)$. Second, for types just above the original value for γ_1, the variation results in a significantly lower tariff, since these types now apply (the increased) τ_1 and no longer apply τ_2. The variation thus results in a significantly *more* efficient tariffs for types just above γ_1. Finally, the variation also results in a less variable distribution of applied tariffs, relative to the baseline of the flexible tariff function.

We may now anticipate that this variation is more likely to raise expected joint welfare if (i) the density $f(\gamma)$ increases sufficiently over $[\gamma_0, \gamma_2]$, so that the significantly improved efficiency of tariffs for types just above γ_1 contributes more to expected joint welfare than does the diminished efficiency of tariffs for types in (γ_0, γ_1), and (ii) the foreign welfare function is not too convex in the tariff relative to the concavity of the home government welfare function, so that the reduced variability of the tariff is a source of joint welfare gain.

We again suppose that $F(\gamma)$ is uniform. In Appendix C, we show that a slight increase in γ_1 leads to an increase in expected joint welfare in the amount

$$\frac{27(\gamma_1 - \gamma_0)^2}{(17 - 2\gamma_1)^2(17 - 2\gamma_0)(\overline{\gamma} - \underline{\gamma})} > 0.$$

Thus, in the case of a uniform distribution, it is not optimal to step out of flexibility, since expected joint welfare would be higher if the initial interval of tariff flexibility were expanded.

For the uniform case, therefore, an optimal tariff function does not step into flexibility nor step out of flexibility. Provided that $\overline{\gamma} < 7/4$ so that $\tau_f(\overline{\gamma}) > \tau_e(\overline{\gamma})$, it also cannot be optimal to use the flexible tariff function everywhere (i.e., for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$). This follows since the optimal tariff cap is less than $\tau_e(\overline{\gamma})$ and thus less than $\tau_f(\overline{\gamma})$. Importantly, we have not ruled out the optimal tariff cap, since in that case the tariff function remains constant upon leaving the flexible tariff function. Other tariff functions remain as well; for example, we have not ruled out tariff functions that utilize two tariffs only, with lower types selecting $\tau_1 < \tau_f(\underline{\gamma})$ and higher types selecting $\tau_2 > \tau_f(\bar{\gamma})$. [21]

In total, our analysis in this section suggests the optimal tariff cap as a plausible solution candidate. The analysis also suggests that this solution may be more likely when the density is non-decreasing and the convexity of the foreign welfare function is not too great relative to the concavity of the home government welfare function. We now return to the general analysis.

5 The Optimal Trade Agreement

In this section, we use the Lagrangian techniques developed by Amador et al. (2006) to obtain sufficient conditions for the optimal tariff cap to be optimal within the set of all feasible and incentive compatible trade agreements. We also provide assumptions under which the sufficient conditions are satisfied. As suggested by our variational analysis in the preceding section, these assumptions relate to the slope of the density and the extent to which the foreign welfare function is convex in relation to the concavity of the home government welfare function. When these assumptions hold, we thus establish that the optimal tariff cap represents an optimal trade agreement.

5.1 The Relaxed Problem

Recall the problem (P) presented above. As usual, the incentive constraint (IC) for this problem is equivalent to a continuum of equality constraints and a monotonicity restriction on the allocation. The standard approach to solve the problem is to substitute the

[21] We may also consider a tariff function $\tau(\gamma)$ such that $\tau(\gamma) = \tau_f(\gamma_1)$ for $\gamma \in [\underline{\gamma}, \gamma_1]$ and $\tau(\gamma) = \tau_f(\gamma)$ for $\gamma \in [\gamma_1, \overline{\gamma}]$. This specification cannot be optimal, however, since it would be more efficient to use the flexible tariff function for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

equality constraints into the objective function and ignore the monotonicity constraint at a first pass. Then, after having optimized the objective function point-wise, ironing techniques can be used to resolve violations of the monotonicity restriction. However, the absence of transfers makes this approach infeasible in our model. Instead, we proceed to relax the problem by introducing a fake variable $\omega(\gamma)$ that linearly reduces the welfare of the home government. We impose that $\omega(\gamma) \geq 0 \; \forall \gamma \in \Gamma$, so that this variable has the economic interpretation of a wasteful punishment that could be imposed on the home government.[22]

Formally, we consider the following relaxed problem:

$$RP: \quad \max_{\pi(\gamma),\omega(\gamma)} \int_\Gamma \Big(\gamma\pi(\gamma) + b(\pi(\gamma)) + v(\pi(\gamma)) - \omega(\gamma) \Big) dF(\gamma) \tag{8}$$

subject to:

$$\gamma\pi(\gamma) + b(\pi(\gamma)) - \omega(\gamma) = \max_{\gamma'} \left\{ \gamma\pi(\gamma') + b(\pi(\gamma')) - \omega(\gamma') \right\} \;\; , \forall \gamma \in \Gamma \tag{9}$$

$$\omega(\gamma) \geq 0 \;\; , \forall \gamma \in \Gamma \tag{10}$$

The following lemma guarantees that a solution to the relaxed problem (RP) above is a solution to our original problem (P) as long as in the solution $\omega(\gamma) = 0$, so that the extra-punishment is not used to provide incentives:

Lemma 3. *If $(\pi^\star(\gamma), \omega^\star(\gamma))$ is a solution to the relaxed problem (RP) and $\omega^\star(\gamma) = 0$ for all $\gamma \in \Gamma$, then $\pi^\star(\gamma)$ is a solution to the original problem (P).*

Proof. The proof follows by noticing that if $\omega^\star(\gamma) = 0$ for all $\gamma \in \Gamma$, then $\pi^\star(\gamma)$ satisfies (IC), and thus the solution to the relaxed problem (RP) is in the constraint set of the original problem (P). □

To analyze the relaxed problem, we begin by characterizing the incentive constraints in a more useful form. In particular, we now rewrite the incentive constraints (9) in their

[22] There are other ways to obtain the sufficient conditions below that do not require the introduction of this fake punishment. For example, it is possible to obtain a similar characterization by replacing the equality constraints that arise from the incentive constraints with two set of weak inequalities with opposite signs (that is, $x = y$ can be replaced with $x \geq y$ and $x \leq y$). The sufficiency theorem can be applied to this case. Alternatively, we can obtain the relaxed problem by solving out for b in the objective function using the equality constraints and then relaxing the equality constraints by changing the equality restriction to an appropriate inequality. However, we prefer to make the introduction of w explicit as it has an economic interpretation: resources will not be wasted in an optimal agreement.

usual integral form:

$$\gamma\pi(\gamma) + b(\pi(\gamma)) - \omega(\gamma) = \int_{\underline{\gamma}}^{\gamma} \pi(\gamma')d\gamma' + \underline{U} \tag{11}$$

where $\underline{U} \equiv \underline{\gamma}\pi(\underline{\gamma}) + b(\pi(\underline{\gamma})) - \omega(\underline{\gamma})$ is the value to the home government with the lowest γ. Equation (11) together with $\pi(\gamma)$ non-decreasing (monotonicity) are equivalent to the incentive constraint (9).[23]

We denote the allocation as a pair of a profit allocation together with the extra-punishment used for the lowest-γ home government: $(\pi(\gamma), \underline{\omega})$. The relaxed problem is then to maximize (8) subject to (10), (11), and that $\pi(\gamma)$ be non-decreasing. Substituting (11) into (8) and into (10), and integrating by parts the objective function, the relaxed problem becomes:

$$RP': \quad \max_{(\pi,\underline{\omega})\in\Phi} \int_{\Gamma} \Big(v(\pi(\gamma))f(\gamma) + (1 - F(\gamma))\pi(\gamma) \Big) d\gamma + \underline{U} \tag{12}$$

subject to:

$$\int_{\underline{\gamma}}^{\gamma} \pi(\gamma')d\gamma' + \underline{U} - \gamma\pi(\gamma) - b(\pi(\gamma)) \leq 0 \text{ for all } \gamma \in \Gamma \tag{13}$$

and where $\Phi = \{(\pi, \underline{\omega}) | \, \underline{\omega} \geq 0, \pi : \Gamma \to [0, \bar{\pi}] \text{ and } \pi(\gamma) \text{ non-decreasing}\}$.

5.2 The Lagrangian and Assumptions

We now define the Lagrangian for the relaxed problem (RP′). We also present two additional assumptions and provide sufficient and necessary conditions associated with the satisfaction of these assumptions.

The Lagrangian function is defined as follows:

$$\begin{aligned}\mathcal{L}(\pi, \underline{\omega}|\Lambda) \equiv & \int_{\gamma\in\Gamma} \Big(v(\pi(\gamma))f(\gamma) + (1 - F(\gamma))\pi(\gamma) \Big) d\gamma + \underline{U} \\ & - \int_{\gamma\in\Gamma} \left(\int_{\underline{\gamma}}^{\gamma} \pi(\gamma')d\gamma' + \underline{U} - \gamma\pi(\gamma) - b(\pi(\gamma)) \right) d\Lambda(\gamma)\end{aligned}$$

where the function Λ is the (cumulative) Lagrange multiplier associated with (13), where $\Lambda(\gamma)$ is non-decreasing. Without loss of generality we set $\Lambda(\bar{\gamma}) = 1$. Integrating by parts

[23] See, for example, Milgrom and Segal (2002).

the above, we have that

$$\mathcal{L}(\pi, \underline{w}|\Lambda) \equiv \int_{\gamma\in\Gamma} \Big(v(\pi(\gamma))f(\gamma) + (\Lambda(\gamma) - F(\gamma))\pi(\gamma) \Big) d\gamma + \int_{\gamma\in\Gamma} (\gamma\pi(\gamma) + b(\pi(\gamma)))\, d\Lambda(\gamma) + \Lambda(\underline{\gamma})\underline{U} \quad (14)$$

We observe that $\pi(\gamma)$ enters the Lagrangian as a linear term in two places. As noted previously, however, $v''(\pi) > 0$ if $Q''(p) \geq 0 \geq u'''(c)$, and so in reasonable circumstances $\pi(\gamma)$ may also enter the Lagrangian in a strictly convex fashion through the v function. Finally, recall as well that $b''(\pi) < 0$ for $\pi \in [\pi_f(\underline{\gamma}), \pi_f(\overline{\gamma})]$. It is thus reasonable to expect that $\pi(\gamma)$ may also enter the Lagrangian in a strictly concave manner through the b function.

Our objective is to determine sufficient conditions for a tariff cap to be optimal. Based on the results in Subsection 3.2 and Section 4, we let the proposed allocation be $(\pi^\star, \underline{w} = 0)$ where:

$$\pi^\star(\gamma) = \begin{cases} \pi_f(\gamma) & \text{, for } \gamma < \gamma^p \\ \pi_f(\gamma^p) & \text{, for } \gamma \geq \gamma^p \end{cases}$$

and where γ^p is as in Definition 1. As expected, the proposed allocation satisfies (13) with equality.[24]

The problem when trying to apply the Lagrangian Theorems in Amador et al. (2006) is that the Lagrangian is not necessarily concave in $\pi(\gamma)$ for $\gamma \in \Gamma$. However, we will show that it is sufficient to construct a multiplier Λ_0 that makes the Lagrangian concave, and that delivers the proposed allocation as a maximizer. To do this, we need two additional assumptions.

Our first assumption is as follows:

Assumption 1. *The function $b(\pi)$ is a concave function for $\pi \in [0, \bar{\pi}]$.*

As noted above, our existing assumptions already ensure that $b''(\pi) < 0$ for $\pi \in [\pi_f(\underline{\gamma}), \pi_f(\overline{\gamma})]$. With Assumption 1, we ensure that $b(\pi)$ is concave over the feasible set of π.

The Lagrangian would be concave if we were to assume that v is a concave function of π; however, as discussed above, under reasonable circumstances v may be strictly convex. Further, our variational analysis suggests that, if v is convex, then the key issue concerns

[24] To see this note that $\int_{\underline{\gamma}}^{\gamma} \pi^\star(\gamma')d\gamma' = \gamma\pi^\star(\gamma) + b(\pi^\star(\gamma)) - \underline{\gamma}\pi^\star(\underline{\gamma}) - b(\pi^\star(\underline{\gamma}))$ for $\gamma \leq \gamma^p$, so that (13) is satisfied for $\gamma \leq \gamma^p$. For $\gamma > \gamma^p$ we note that the left hand side of equation (13) can be written as $\int_{\underline{\gamma}}^{\gamma^p} \pi^\star(\gamma')d\gamma' + \int_{\gamma^p}^{\gamma} \pi^\star(\gamma^p)d\gamma' + \underline{\gamma}\pi^\star(\underline{\gamma}) + b(\pi^\star(\underline{\gamma})) - \gamma\pi^\star(\gamma^p) - b(\pi^\star(\gamma^p))$, which using the previous result can be shown to be equal to zero.

how convex is v relative to the concavity of b. Before stating our second assumption, let us thus denote by κ the following value:

Definition 2. *Let κ be such that:*

$$\kappa \equiv \min\left\{ \min_{\pi\in[0,\bar{\pi}]} \left\{ \frac{v''(\pi)+b''(\pi)}{b''(\pi)} \right\}, 1 \right\}$$

Using Assumption 1, we note that $\kappa = 1$ if v is (weakly) concave and that κ falls as the convexity of v increases relative to the concavity of b.

With this definition in place, we come now to our second assumption:

Assumption 2. *The following holds:*

(a) The function H as given by $H(\gamma) \equiv \kappa F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$ is non-decreasing in γ for $\gamma \in [\underline{\gamma}, \gamma^p]$, and

(b) The function G as given by $G(\gamma) \equiv v'(\pi_f(\gamma^p)) - \gamma + \int_\gamma^{\bar{\gamma}} \gamma' \frac{f(\gamma')}{1-F(\gamma)} d\gamma' + (\gamma - \gamma^p)(1-\kappa)$ is non-positive for all $\gamma \in [\gamma^p, \bar{\gamma}]$.

Since $v'(\pi_f(\gamma)) < 0$, we may observe that Assumption 2(a) is more likely to hold if the density $f(\gamma)$ is non-decreasing and if the function v is (weakly) concave or at least not too convex relative to the concavity of the function b.[25] We observe as well that Assumption 2(b) is more likely to hold if κ is high.

In fact, we can go beyond these simple observations and provide a simple set of sufficient conditions for Assumption 2:

Lemma 4. *Suppose that $f(\gamma)$ is differentiable and non-decreasing and that $\kappa \geq 1/2$. Then, Assumption 2 holds.*

Proof. See Appendix D. □

We now show that a necessary implication of Assumption 2(b) is that κ is bounded below by a positive value (which in turn implies that $v + b$ is concave):

Lemma 5. *The value of κ is such that $\kappa \geq -v'(\pi_f(\gamma^p))\frac{f(\gamma^p)}{1-F(\gamma^p)} > 0$*

[25] To see this, we use (6) to derive that $dv'(\pi_f(\gamma))/d\gamma = -v''(\pi_f(\gamma))/b''(\pi_f(\gamma))$. Assuming that the density function is differentiable, it then follows that $H'(\gamma) = \kappa f(\gamma) + f(\gamma)v''(\pi_f(\gamma))/b''(\pi_f(\gamma)) - v'(\pi_f(\gamma))f'(\gamma)$.

Proof. Note that $G(\gamma^p) = 0$ by Lemma 2. Given that $G(\gamma) \leq 0$ for $\gamma \geq \gamma^p$ it must be that $G'(\gamma^p) \leq 0$. Computing this derivative we get that:

$$G'(\gamma^p) = \Big[\underbrace{-\gamma^p + \int_{\gamma^p}^{\bar{\gamma}} \gamma' \frac{f(\gamma')}{1-F(\gamma^p)} d\gamma'}_{-v'(\pi_f(\gamma^p))}\Big] \frac{f(\gamma^p)}{1-F(\gamma^p)} - \kappa \leq 0$$

and the result of the lemma follows. $\square$

5.3 Our Approach and the Multiplier

To establish that the proposed tariff cap allocation represents an optimal trade agreement, we use the following broad approach. First, we build on a theorem by Luenberger (1969), page 220, so as to conclude that, if there exists a non-decreasing multiplier function, Λ_0, such that $\mathcal{L}(\pi, \underline{\omega}|\Lambda_0)$ is maximized at our proposed allocation, then the proposed allocation solves the relaxed problem (RP') and thus the original problem (P). Second, we employ additional findings in Amador et al. (2006) to establish that the proposed allocation indeed maximizes $\mathcal{L}(\pi, \underline{\omega}|\Lambda_0)$ if $\mathcal{L}(\pi, \underline{\omega}|\Lambda_0)$ is concave in π, and first-order conditions for the maximization of $\mathcal{L}(\pi, \underline{\omega}|\Lambda_0)$ hold at the proposed allocation.

As this outline of our approach suggests, a key step is to identify the correct multiplier function. A particular challenge that we face in the trade application considered here is that the Lagrangian is not automatically concave. Thus, the multiplier must be chosen so as to ensure that the Lagrangian satisfies first-order conditions at the proposed allocation *and* that the Lagrangian is concave.

Our multiplier function Λ_0 is defined as follows:

$$\Lambda_0(\gamma) = \begin{cases} (1-\kappa)F(\gamma) + \kappa & ;\text{if } \gamma \in (\gamma^p, \bar{\gamma}) \\ F(\gamma) - v'(\pi_f(\gamma))f(\gamma) & ;\text{if } \gamma \in (\underline{\gamma}, \gamma^p] \\ 0 & ;\text{if } \gamma = \underline{\gamma} \end{cases} \tag{15}$$

Note that $\Lambda_0(\bar{\gamma}) = 1$, which is consistent with our previous normalization. We now show that $\Lambda_0(\gamma)$ is also non-decreasing for $\gamma \in \Gamma$.

Lemma 6. *The proposed Lagrange multiplier, Λ_0, is non-decreasing for $\gamma \in \Gamma$.*

Proof. Note that for $\gamma \in (\underline{\gamma}, \gamma^p)$, the proposed multiplier equals $F(\gamma) - v'(\pi_f(\gamma))f(\gamma) = H(\gamma) + (1-\kappa)F(\gamma)$. This is non-decreasing for $\gamma \in (\underline{\gamma}, \gamma^p)$ as it is the sum of two non-decreasing functions (by Assumption 2 and that $\kappa \in [0,1]$). For $\gamma \in (\gamma^p, \bar{\gamma})$, we have that the proposed multiplier is also non-decreasing as $\kappa \in [0,1]$. The multiplier is continuous except at two points. At $\gamma = \underline{\gamma}$, the multiplier has a jump, which has a size equal to $-v'(\pi_f(\underline{\gamma}))f(\underline{\gamma}) > 0$. Finally, there is a second jump at γ^p. At this point, the jump is $\kappa(1 - F(\gamma^p)) + v'(\pi_f(\gamma^p))f(\gamma^p)$, which is non negative by Lemma 5. And thus the proposed multiplier is non-decreasing for all $\gamma \in \Gamma$. $\square$

The multiplier function thus satisfies the basic requirement of being non-decreasing. The remaining tasks are to verify that, when this multiplier is used, the Lagrangian is concave and the proposed allocation satisfies the first-order conditions for maximizing the Lagrangian.[26]

5.4 Main Findings

Following the general approach described above, we now confirm that, at the proposed multiplier, the proposed allocation maximizes the Lagrangian and thus solves the problem (P).

We begin by showing that the Lagrangian evaluated at the proposed multiplier is concave.

Lemma 7. *The Lagrangian when evaluated at the proposed multiplier, $\mathcal{L}(\pi, \underline{\omega}|\Lambda_0)$, is concave for $(\pi, \underline{\omega}) \in \Phi$.*

Proof. The Lagrangian at the proposed multiplier is:

$$\mathcal{L}(\pi, \underline{\omega}|\Lambda_0) \equiv \int_{\gamma\in\Gamma} \Big(v(\pi(\gamma))f(\gamma) + (\Lambda_0(\gamma) - F(\gamma))\pi(\gamma)\Big)d\gamma + \int_{\gamma\in\Gamma} (\gamma\pi(\gamma) + b(\pi(\gamma)))\, d\Lambda_0(\gamma)$$

[26] A sketch of the method by which we identify this multiplier function may be of interest to some readers. The first-order conditions for maximization of the Lagrangian with respect to $\pi(\gamma)$ over the region of flexibility (i.e., for $\gamma \in (0, \gamma^p)$, where $\pi^\star(\gamma) = \pi_f(\gamma)$) are satisfied only if $\Lambda_0(\gamma) = F(\gamma) - f(\gamma)v'(\pi_f(\gamma))$. This can be seen by referring to (14) and recalling that $\pi_f(\gamma)$ maximizes $\gamma\pi(\gamma) + b(\pi(\gamma))$. The first-order conditions over the region of pooling (i.e., for $\gamma \geq \gamma^p$, where $\pi^\star(\gamma) = \pi_f(\gamma^p)$) are more complex and require that $\Lambda_0(\gamma)$ is not too low over this range. If we specify that $\Lambda_0(\gamma) = \kappa + (1-\kappa)F(\gamma)$, then κ cannot be too low, say, $\kappa \geq \kappa_{foc}$. Under this specification, concavity of the Lagrangian holds over the region of pooling if κ is not too high, say, $\kappa \leq \kappa^{soc}$. Finally, with $\Lambda_0(\gamma) = F(\gamma) - f(\gamma)v'(\pi_f(\gamma)) \equiv (1-\kappa)F(\gamma) + H(\gamma)$ over the region of flexibility, concavity of the Lagrangian holds over this region provided that κ is not too high (just as in the pooling region) and if $H(\gamma)$ is non-decreasing. The latter requirement means that κ cannot be too low, say, $\kappa \geq \kappa_{soc}$. Our method is then to set $\kappa = \kappa^{soc}$ and to make assumptions which ensure that $\kappa \geq \kappa_{foc}$ (see Assumption 2b) and that $\kappa \geq \kappa_{soc}$ (see Assumption 2a).

Exploiting the proposed multiplier, we can decompose the above into the following six additive terms:

$$
\begin{aligned}
\mathcal{L}(\pi, \underline{\omega} | \Lambda_0) = \int_{\gamma \in \Gamma} (\Lambda_0(\gamma) - F(\gamma))\pi(\gamma) d\gamma + \int_{\gamma \in \Gamma} \gamma \pi(\gamma) d\Lambda_0(\gamma) \\
+ \int_{\gamma \in (\underline{\gamma}, \bar{\gamma})} \Big(v(\pi(\gamma)) + (1-\kappa) b(\pi(\gamma)) \Big) f(\gamma) d\gamma \\
+ \int_{\gamma \in (\underline{\gamma}, \gamma^p]} b(\pi(\gamma)) d\Big(\kappa F(\gamma) - v'(\pi_f(\gamma)) f(\gamma) \Big) \\
+ b(\pi(\underline{\gamma}))(-v'(\pi_f(\underline{\gamma}))) f(\gamma) \\
+ b(\pi(\gamma^p)) \Big(\kappa (1 - F(\gamma^p)) + v'(\pi_f(\gamma^p)) f(\gamma^p) \Big)
\end{aligned}
$$

Note that the first two integrands are linear functions of π, and thus their integrals are concave in π. The third integral is concave as by the definition of κ we have that $v''(\pi) + (1-\kappa)b''(\pi)$ is negative for all π. By Assumption 2 we know that $\kappa F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$ is non-decreasing in $\gamma \in (\underline{\gamma}, \gamma^p]$ and thus the fourth term is concave in π. The fifth term is concave given that $v' < 0$ and Assumption 1 guarantees that b is concave. And finally, the last term is concave, given that by Lemma 5 we have that $\kappa(1 - F(\gamma^p)) + v'(\pi_f(\gamma^p))f(\gamma^p)$ is positive. □

Given that Lemma 7 guarantees that the Lagrangian is concave, we can now state a sufficiency Lemma in term of first order conditions:

Lemma 8 (Sufficiency). *If the following first-order conditions in terms of Gateaux differentials*[27]*:*

$$
\begin{aligned}
\partial \mathcal{L}(\pi^\star, 0; \pi^\star, 0 | \Lambda_0) = 0 \\
\partial \mathcal{L}(\pi^\star, 0; x, y | \Lambda_0) \leq 0
\end{aligned}
$$

hold for all $(x, y) \in \Phi$, then the proposed allocation, $(\pi^\star, 0)$, solves the relaxed problem (RP).

Proof. In the Appendix E. □

As discussed in further detail in the Appendix, in the proof of this lemma, we build on a theorem by Luenberger (1969), page 220, and conclude that our proposed allocation

[27]Given a function $T : \Omega \to Y$, where $\Omega \subset X$ and X and Y are normed spaces, if for $x \in \Omega$ and $h \in X$ the limit

$$\lim_{\alpha \downarrow 0} \frac{1}{\alpha} \left[T(x + \alpha h) - T(x) \right]$$

exists, then it is called the Gateaux differential at x with direction h and is denoted by $\partial T(x; h)$.

solves the relaxed problem (RP′) and thus our original problem (P), if our proposed allocation maximizes $\mathcal{L}(\pi, \underline{\omega}|\Lambda_0)$. We then utilize additional findings in Amador et al. (2006) to establish that the stated first-order conditions ensure that our proposed allocation maximizes $\mathcal{L}(\pi, \underline{\omega}|\Lambda_0)$, given our finding in Lemma 7 that $\mathcal{L}(\pi, \underline{\omega}|\Lambda_0)$ is concave.

We are now ready to state and prove our main proposition.

Proposition 2. *The proposed allocation $(\pi^\star, 0)$ solves the relaxed problem RP.*

Proof. The Gateaux differential of the Lagrangian evaluated at the proposed multiplier Λ_0 is

$$d\mathcal{L}(\pi^\star, 0; x, y|\Lambda_0) = \int_{\gamma^p}^{\bar{\gamma}} \Big[v'(\pi_f(\gamma^p)) f(\gamma) + \kappa(1 - F(\gamma)) + (\gamma - \gamma^p)(1-\kappa) f(\gamma) \Big] x(\gamma) d\gamma$$

where $(x, y) \in \Phi$. Integrating by parts the above:

$$\begin{aligned} & d\mathcal{L}(\pi^\star, 0; x, y|\Lambda_0) = \\ & \quad = \left(\int_{\gamma^p}^{\bar{\gamma}} \Big[v'(\pi_f(\gamma^p)) f(\gamma) + \kappa(1 - F(\gamma)) + (\gamma - \gamma^p)(1-\kappa) f(\gamma) \Big] d\gamma \right) x(\gamma^p) \\ & \qquad + \int_{\gamma^p}^{\bar{\gamma}} \int_{\gamma}^{\bar{\gamma}} \Big[v'(\pi_f(\gamma^p)) f(\gamma') + \kappa(1 - F(\gamma')) + (\gamma' - \gamma^p)(1-\kappa) f(\gamma') \Big] d\gamma' dx(\gamma) \end{aligned}$$

Now, using that $\int_{\gamma}^{\bar{\gamma}} (1 - F(\gamma')) d\gamma' = \int_{\gamma}^{\bar{\gamma}} (\gamma' - \gamma) f(\gamma') d\gamma'$, we have that:

$$\begin{aligned} & \int_{\gamma}^{\bar{\gamma}} \Big[v'(\pi_f(\gamma^p)) f(\gamma') + \kappa(1 - F(\gamma')) + (\gamma' - \gamma^p)(1-\kappa) f(\gamma') \Big] d\gamma' \\ & \quad = \int_{\gamma}^{\bar{\gamma}} \Big[v'(\pi_f(\gamma^p)) f(\gamma') + (\gamma' - \gamma) f(\gamma') + (\gamma - \gamma^p)(1-\kappa) f(\gamma') \Big] d\gamma' \\ & \quad = (1 - F(\gamma)) \Big[v'(\pi_f(\gamma^p)) - \gamma + \int_{\gamma}^{\gamma^p} \gamma' \frac{f(\gamma')}{1 - F(\gamma)} d\gamma' + (\gamma - \gamma^p)(1-\kappa) \Big] \\ & \quad = (1 - F(\gamma)) G(\gamma) \end{aligned}$$

Note that evaluated at $\gamma = \gamma^p$ the above expression equals zero, given Lemma 2. Even more, by Assumption 2, $G(\gamma) \leq 0$ for $\gamma \geq \gamma^p$ and thus te expression above is always negative for $\gamma \geq \gamma^p$. And thus, we have that $d\mathcal{L}(\pi^\star, 0; x, y|\Lambda_0) \leq 0$. Finally note that at the proposed allocation $dx = d\pi^\star = 0$ for $\gamma > \gamma^p$ and thus $d\mathcal{L}(\pi^\star, 0; \pi^\star, 0|\Lambda_0) = 0$. That is, the conditions of Lemma 8 hold and $(\pi^\star, 0)$ solves the relaxed problem. □

Recalling that the proposed allocation corresponds to the optimal tariff cap, we may thus now conclude that, under Assumptions 1 and 2, the optimal tariff cap represents an optimal trade agreement.

6 The Linear-Quadratic Example

To confirm the value of Proposition 2, we return to the linear-quadratic example studied by Bagwell and Staiger (2005) and considered in Section 4. We confirm for this example that Assumptions 1 and 2 hold, if the density is differentiable and non-decreasing. Thus, provided the density satisfies these conditions, an optimal trade agreement is represented as an optimal tariff cap. We provide additional findings for the case of a uniform distribution. Finally, we establish as well that an optimal trade agreement takes the form of an optimal tariff cap, under a condition on the density function that allows for decreasing density functions.

To begin, we recall from Section 4 that, in the linear-quadratic example, $Q(p) = p/2$, $Q_\star(p) = p$, and $u(c) = c - c^2/2$. The political shocks are distributed over $[\underline{\gamma}, \bar{\gamma}]$ where $1 \leq \underline{\gamma} < \bar{\gamma} < 7/4$. Note as well that in this example a tariff higher than $1/6$ is prohibitive.[28]

When trade volume z is treated as the independent variable, we find that $P(z) = \frac{2}{3}(1-z)$, $P_\star(z) = \frac{1}{2}(1+z)$, $\Pi(z) = (1-z)^2/9$, $\Pi_\star(z) = (1+z)^2/8$, $V(z) = \frac{1}{4}(1+z^2)$ and $B(z) = \frac{1}{18}(1+7z-17z^2)$. Letting π denote domestic profits as before, we can rewrite these functions in terms of π:

$$b(\pi) = \frac{1}{2}(-1 + 9\sqrt{\pi} - 17\pi)$$
$$v(\pi) = \frac{1}{4}(2 - 6\sqrt{\pi} + 9\pi)$$

Inspecting the above, it follows that Assumption 1 holds. Further, the definition of κ implies that $\kappa = 2/3$. Using Lemma 4, we thus conclude that Assumption 2 holds for any differentiable and non-decreasing density, $f(\gamma)$. Thus, if $f(\gamma)$ is differentiable and non-decreasing, then an optimal trade agreement is represented by the optimal tariff cap.

An interesting special case is that the density is uniform. The value of γ^p is then $\gamma^p = -7/2 + 3\bar{\gamma}$. One can also show that

$$H(\gamma) = \frac{2}{3}\left(\frac{7/8 - \underline{\gamma}}{\bar{\gamma} - \underline{\gamma}}\right) + \frac{1}{3}\frac{\gamma}{\bar{\gamma} - \underline{\gamma}}$$

which is increasing in γ as required. Finally, $G(\gamma) = -\frac{1}{12}(7 + 2\gamma - 6\bar{\gamma}) = -\frac{1}{6}(\gamma - \gamma^p)$,

[28] A government with a political pressure γ higher than or equal to $7/4$ prefers a prohibitive tariff.

which is non-positive for $\gamma \in [\gamma^p, \bar{\gamma}]$ as required. So for this special case, we can easily confirm directly that Assumption 2 holds.

For the special case of a uniform distribution, we can also confirm that the tariff associated with γ^p is:

$$\bar{\tau} = P(\Pi^{-1}(\pi_f(\gamma^p))) - P_\star(\Pi^{-1}(\pi_f(\gamma^p))) = \frac{1}{6} - \frac{7(7-4\bar{\gamma})}{24(4-\bar{\gamma})}.$$

Recalling that a tariff of $1/6$ is prohibitive, we see that the optimal tariff cap allows for positive trade volume since $\bar{\gamma} < 7/4$. The optimal tariff cap binds for higher types (i.e., for $\gamma \geq \gamma^p$), while lower types apply their flexible (Nash) tariffs and thus exhibit binding overhang. A partial intuition is that, for the very highest types, the flexible tariff exceeds the efficient tariff for any type (i.e., for γ near $\bar{\gamma}$, $\tau_f(\gamma) > \tau_e(\bar{\gamma})$). It cannot be optimal to provide flexibility to the very highest types, since it would be more efficient to cap their tariff at the maximal efficient tariff, $\tau_e(\bar{\gamma})$.[29]

Continuing with the uniform distribution, we consider next the agreement as $\bar{\gamma}$ approaches $7/4$. In the limit, the highest level of pressure is sufficiently high that the efficient tariff leads to a trade volume of zero. The flexible and efficient tariffs agree at the limit, since there is then no trade volume with which to impose a terms-of-trade externality; in other words, $\tau_f(7/4) = \tau_e(7/4) = 1/6$. Of course, as discussed in Section 4 and as depicted in Figure 1, for all $\gamma < 7/4$, $\tau_f(\gamma) > \tau_e(\gamma)$. The findings above indicate that, as $\bar{\gamma}$ approaches $7/4$, we have that γ^p approaches $\bar{\gamma}$, and so $\bar{\tau}$ approaches $1/6$. Thus, when the distribution function is uniform and the highest level of support approaches the limiting case in which zero trade volume is efficient, the optimal trade agreement entails full flexibility for all types! In this limiting case, governments with private information are unable to design a trade agreement that improves upon the non-cooperative (Nash) benchmark.

The following proposition summarizes the above results:

Proposition 3. *If $Q(p) = p/2$, $Q_\star(p) = p$, $u(c) = c - c^2/2$, and the political shocks are distributed over $[\underline{\gamma}, \bar{\gamma}]$ where $1 \leq \underline{\gamma} < \bar{\gamma} < 7/4$ according to a non-decreasing and differentiable density, then an optimal trade agreement is represented as the optimal tariff cap. In the special case of a uniform distribution, the optimal tariff cap is at $\bar{\tau} = \frac{1}{6} - \frac{7(7-4\bar{\gamma})}{24(4-\bar{\gamma})}$, and full flexibility is thus used for all types as $\bar{\gamma}$ approaches $7/4$.*

This proposition is usefully compared with our variational analysis in Section 4, where we also consider the linear-quadratic example, address the special case of a uniform disti-

[29]See Amador et al. (2006) for a related intuition.

bution and develop a conjecture as to the optimal tariff agreement. The Lagrangian techniques developed in Section 5 provide a global method of confirming conditions under which the optimal tariff cap is an optimal trade agreement.

Finally, the linear-quadratic example also provides a tractable setting in which to explore the possibility of non-increasing densities. For this example, we can establish weaker sufficient conditions for Assumption 2.

Proposition 4. *If $Q(p) = p/2$, $Q_\star(p) = p$, $u(c) = c - c^2/2$, and the political shocks are distributed over $[\underline{\gamma}, \bar{\gamma}]$ where $1 \leq \underline{\gamma} < \bar{\gamma} < 7/4$ according to a differentiable density that satisfies $f(\gamma) - 3v'(\pi_f(\gamma))f'(\gamma) \geq 0$, then an optimal trade agreement is represented as the optimal tariff cap.*

Proof. In Appendix F. □

Notice that the assumption on the density holds if the density is non-decreasing, since $v' < 0$. Proposition 4, however, includes as well densities that are decreasing over ranges or even over the entire support, provided that the rate of decrease is not so great as to violate the stated inequality. In particular, in the linear-quadratic example, we can derive that $v'(\pi_f(\gamma)) = -1/3(7/4 - \gamma)$ and thus re-write the inequality in Proposition 4 as $f(\gamma) + \left(\frac{7}{4} - \gamma\right) f'(\gamma) \geq 0$. This condition clearly holds even for densities that decline over the entire support, provided that the rate of decline is sufficiently small. As well, the condition holds for any concave density for which $f(\bar{\gamma}) + (\frac{7}{4} - \bar{\gamma})f'(\bar{\gamma}) \geq 0$, or for any convex density for which $f(\underline{\gamma}) + (\frac{7}{4} - \underline{\gamma})f'(\underline{\gamma}) \geq 0$.[30]

The linear-quadratic example is tractable and offers a convenient setting with which to illustrate our findings. An important benefit of our general analysis is that we can employ our findings to characterize an optimal trade agreement for other examples, too. In the Appendix G, we consider an example with log utility and endowments (inelastic supply), where the endowment of good x in the foreign country exceeds that in the home country. Similar results apply for this example as well.

7 Conclusion

In this paper, we characterize an optimal trade agreement among privately informed governments. In particular, we provide conditions under which an optimal trade agreement takes the form of an optimal tariff cap: tariffs above the cap are not permitted, but a

[30] To see this, note that the derivative of $f(\gamma) + (\frac{7}{4} - \gamma)f'(\gamma)$ with respect to γ is $(\frac{7}{4} - \gamma)f''(\gamma)$.

government may apply any import tariff at or below the cap. In the optimal trade agreement, with positive probability, a government applies a tariff strictly below the cap. The optimal trade agreement thus corresponds well with actual GATT/WTO rules, under which member governments negotiate tariff bindings (i.e., caps). Our theory also provides an interpretation of binding overhang as an implication of an optimally designed trade agreement.

We develop our findings using a static model, but we can also interpret our results in a dynamic setting. In particular, if the distribution of types is iid across governments and over time, and if governments are sufficiently patient, then there exists an optimal strongly symmetric perfect public equilibrium (SSPPE) for the repeated game in which the optimal tariff cap is used in each period. The SSPPE solution concept allows that governments may provide incentives through trade wars (low continuation values), provided that both governments experience the same expected continuation value. Trade wars then represent "wasteful"transfers. That such wasteful transfers are suboptimal is intuitive in light of our finding that wasteful transfers (fake extra-punishments) are not used in the solution of our Relaxed Program above.

We emphasize, though, that governments may be able to improve upon the optimal tariff cap in a dynamic setting if the solution concept is expanded to allow for asymmetric continuation values.[31] An asymmetric continuation value plays the role of a transfer from the government that anticipates a lower continuation value to the government that expects a higher continuation value. As Bagwell and Staiger (2005) confirm for the linear-quadratic example when the distribution function is uniform, if governments are sufficiently patient, asymmetric perfect public equilibria can be constructed in which governments enjoy expected joint welfare in excess of that which would be provided by the stationary application of the optimal tariff cap. Such asymmetric equilibria provide an interpretation of the WTO Safeguard Agreement, whereby a government that applies a tariff in excess of its bound rate loses the option to do so again in the immediate future. Martin and Vergote (2008) also consider the repeated interaction of privately informed governments in a linear-quadratic model in which sidepayments and export policies are infeasible. They show that equilibrium-path retaliation in tariffs is a necessary feature of an efficient equilibrium among arbitrarily patient governments, and they use this finding to interpret the retaliatory use of anti-dumping duties.

In this broad context, the present paper may be understood as establishing a strong benchmark result for the optimal tariff cap. In the absence of transfers or asymmetric

[31] Indeed, for the repeated game with iid types, if the action and type spaces are finite, then the folk theorem of Fudenberg, Levine, and Maskin (1994) applies.

continuation values, and under the conditions that we identify, governments can do no better than to form an agreement in which they impose the optimal tariff cap. Therefore, if governments manage to do even better through repeated interaction, then we may now understand that the source of the gain must be associated with the use of asymmetric continuation values as transfers. Future work addressing WTO safeguard rules or dynamic tariff retaliation strategies may proceed from this foundation.

A Proof of Proposition 1

As a first step we will show that $P'(z) < 0$. To see this note that taking the derivative with respect to z in equation (1) is:

$$u''(Q(P(z)) + z)\Big(Q'(P(z))P'(z) + 1\Big) = P'(z) \tag{16}$$

which solves to $P'(z) = -\left(Q'(P(z)) - \frac{1}{u''(Q(P(z))+z)}\right)^{-1} < 0$.

We know show that $P'_\star(z) > 0$. To see this, note that the derivative with respect to z in (2) is:

$$u''(Q_\star(P_\star(z)) - z)\Big(Q'_\star(P_\star(z))P'_\star(z) - 1\Big) = P'_\star(z) \tag{17}$$

which solves to $P'_\star(z) = \left(Q'_\star(P_\star(z)) - \frac{1}{u''(Q_\star(P_\star(z))+z)}\right)^{-1} > 0$.

Now note that $\Pi'(z) = Q(P(z))P'(z) < 0$ and using equation (3) and (2), one finds that $V'(z) = zP'_\star(z) > 0$ where we used that $\Pi'_\star(z) = Q_\star(P_\star(z))P'_\star(z)$.

Using equation (5), together with (1), one finds that:

$$\begin{aligned}
B'(z) &= P(z) - P_\star(z) - Q(P(z))P'(z) - zP'_\star(z) \\
B''(z) &= P'(z) - 2P'_\star(z) - Q'(P(z))(P'(z))^2 - Q(P(z))P''(z) - zP''_\star(z)
\end{aligned}$$

From the definition of the profits, we have that $\Pi''(z) = Q'(P(z))(P'(z))^2 + Q(P(z))P''(z)$. Using $V''(z) = zP''_\star(z) + P'_\star(z)$, we have that $B''(z) + \Pi''(z) + V''(z) = P'(z) - P'_\star(z) < 0$.

B Proof of Lemma 1

Taking another derivative in equation (16) we obtain that:

$$P''(z) = -\frac{Q''(P(z)) + \frac{u'''(Q(P(z))+z)}{(u''(Q(P(z))+z)^3}}{\left(Q'(P(z)) - \frac{1}{u''(Q(P(z))+z)}\right)^3} \geq 0$$

and $P''_\star(z) \geq 0$. And taking another derivative in equation (17) we obtain that:

$$P''_\star(z) = -\frac{Q''_\star(P_\star(z)) + \frac{u'''(Q_\star(P_\star(z))-z)}{(u''(Q_\star(P_\star(z))+z))^3}}{\left(Q'_\star(P_\star(z)) - \frac{1}{u''(Q_\star(P_\star(z)))}\right)^3} \geq 0$$

Using our equation for $V''(z)$, it follows that $V''(z) = zP''_\star(z) + P'_\star(z) > 0$. Recall also the equation for $B''(z)$:

$$B''(z) = P'(z) - 2P'_\star(z) - Q'(P(z))(P'(z))^2 - Q(P(z))P''(z) - zP''_\star(z) < 0$$

And finally, $\Pi''(z) = Q'(P(z))(P'(z))^2 + Q(P(z))P''(z) > 0$.

C Linear-Quadratic Example: Variational Analysis

As discussed in the text, we may write the welfare for the home government and the foreign government respectively as:

$$w(\tau,\gamma) \equiv \frac{9+8\gamma}{98} + \frac{8\gamma-5}{49}\tau + \frac{2[2\gamma-17]}{49}\tau^2, \quad \text{and} \quad w_*(\tau) \equiv \frac{25}{98} - \frac{3\tau}{49} + \frac{9\tau^2}{49}.$$

We may also define the joint welfare function: $J(\tau,\gamma) = w(\tau,\gamma) + w_*(\tau)$.

The flexible (Nash) tariff maximizes $w(\tau,\gamma)$ and is given by $\tau_f(\gamma) = \frac{8\gamma-5}{4(17-2\gamma)}$. The efficient tariff maximizes $J(\tau,\gamma)$ and is given by $\tau_e(\gamma) = \frac{4(\gamma-1)}{25-4\gamma}$. Let the support of γ be $[\underline{\gamma},\overline{\gamma}]$ where $\underline{\gamma} = 1 < \overline{\gamma} < \frac{7}{4}$. Over this support, we may easily verify that $\tau_f(\gamma) > \tau_e(\gamma)$. It is also convenient to define

$$g(\tau) \equiv \frac{9}{98} - \frac{5}{49}\tau - \frac{34}{49}\tau^2\,; \quad \text{and} \quad r(\tau) \equiv \frac{4}{49} + \frac{8}{49}\tau + \frac{4}{49}\tau^2$$

so that $w(\tau,\gamma) = g(\tau) + \gamma r(\tau)$, where $r'(\tau) > 0$ for all $\tau > -1$.

Incentive compatibility implies that any candidate tariff function $\tau(\gamma)$ is nondecreasing. In our variational analysis, we posit a candidate incentive compatible tariff function $\tau(\gamma)$ and consider whether an alternative incentive compatible tariff function would yield a higher value for $\mathbb{E}J(\tau,\gamma)$, where the expectation is taken over γ. We restrict attention to tariff functions for which $\tau \geq 0 = \tau^E(\underline{\gamma})$. We thus focus on non-negative tariffs and hence regard r as strictly increasing.

We begin with our *first candidate solution*. Assume that there exists a region $[\gamma_0,\gamma_A]$ and values γ_1 and γ_2 such that $\underline{\gamma} \leq \gamma_0 < \gamma_1 < \gamma_2 < \gamma_A \leq \overline{\gamma}$ and

$$\tau(\gamma) = \begin{cases} \tau_1 \leq \tau_f(\gamma_0) & \text{, for } \gamma \in [\gamma_0,\gamma_1) \\ \tau_2 = \tau_f(\gamma_2) & \text{, for } \gamma \in (\gamma_1,\gamma_2] \\ \tau_f(\gamma) & \text{, for } \gamma \in [\gamma_2,\gamma_A]. \end{cases}$$

Incentive compatibility requires that type γ_1 is indifferent. This implies that $\tau_1 < \tau_f(\gamma_1) < \tau_2$. In fact, in this quadratic setting, $\tau_f(\gamma_1) = (\tau_1+\tau_2)/2$ is required. Over the interval $[\gamma_0,\gamma_A]$, this scheme starts with a low step and then jumps to a high step. The tariff function then "steps into flexibility," by joining $\tau_f(\gamma)$ for $\gamma \in [\gamma_2,\gamma_A]$. No assumptions are made about tariffs for $\gamma \notin [\gamma_0,\gamma_A]$.

We consider the following variation: Fix γ_0, γ_A and τ_1, and lower γ_1 slightly. Notice that this variation induces slight reductions in γ_2 and τ_2.

To analyze the joint-welfare implications of this variation, we define $\tau_2(\gamma_1,\tau_1)$ for given γ_1 and τ_1 by the requirement of incentive compatibility: $w(\tau_1,\gamma_1) = w(\tau_2,\gamma_1)$. And also, define $\gamma_2(\gamma_1,\tau_1)$ by $\tau_2(\gamma_1,\tau_1) = \tau_f(\gamma_2)$. This implies

$$\frac{\partial\tau_2(\gamma_1,\tau_1)}{\partial\gamma_1} = \frac{r(\tau_1)-r(\tau_2)}{\frac{\partial w(\tau_2,\gamma_1)}{\partial\tau}} > 0, \quad \text{and} \quad \frac{\partial\gamma_2(\gamma_1,\tau_1)}{\partial\gamma_1} = \frac{\frac{\partial\tau_2(\gamma_1,\tau_1)}{\partial\gamma_1}}{\frac{\partial\tau_f(\gamma_2)}{\partial\gamma}} > 0.$$

Now consider $\mathbb{E}J$. The tariffs for types below γ_0 and above γ_A can be left unaltered, since we vary γ_1 and γ_2 while maintaining τ_1 and $\tau_A \equiv \tau_f(\gamma_A)$. Regarding expected joint welfare as a function of γ_1 and τ_1 and differentiating with respect to γ_1, we have

$$\frac{\partial \mathbb{E}J(\gamma_1,\tau_1)}{\partial\gamma_1} = \frac{\partial}{\partial\gamma_1}\left\{\int_{\gamma_0}^{\gamma_1} J(\tau_1,\gamma)dF + \int_{\gamma_1}^{\gamma_2} J(\tau_2,\gamma)dF + \int_{\gamma_2}^{\gamma_A} J(\tau_f(\gamma),\gamma)dF\right\},$$

where we use γ_2 as shorthand for $\gamma_2(\gamma_1,\tau_1)$ and τ_2 as shorthand for $\tau_2(\gamma_1,\tau_1) = \tau_f(\gamma_2)$. Thus,

$$\frac{\partial \mathbb{E}J(\gamma_1,\tau_1)}{\partial\gamma_1} = [J(\tau_1,\gamma_1) - J(\tau_2,\gamma_1)]f(\gamma_1) + \frac{\partial\gamma_2(\gamma_1,\tau_1)}{\partial\gamma_1}[J(\tau_2,\gamma_2) - J(\tau_f(\gamma_2),\gamma_2)]f(\gamma_2) + \int_{\gamma_1}^{\gamma_2} \frac{\partial J(\tau_2,\gamma)}{\partial\tau}\frac{\partial\tau_2(\gamma_1,\tau_1)}{\partial\gamma_1}dF. \quad (18)$$

Since $\tau_2 = \tau_2(\gamma_1,\tau_1) = \tau_f(\gamma_2)$, the second term is eliminated. The first term is positive, and the third term is negative. Finally, since $w(\tau_1,\gamma_1) = w(\tau_2,\gamma_1)$, we may simplify the first term in (18) and write

$$\frac{\partial \mathbb{E}J(\gamma_1,\tau_1)}{\partial\gamma_1} = [w_*(\tau_1) - w_*(\tau_2)]f(\gamma_1) + \frac{\partial\tau_2(\gamma_1,\tau_1)}{\partial\gamma_1}\int_{\gamma_1}^{\gamma_2} \frac{\partial J(\tau_2,\gamma)}{\partial\tau}dF. \quad (19)$$

Proceeding mechanically, we find that $[w_*(\tau_1) - w_*(\tau_2)]f(\gamma_1) = \frac{3}{49}(\tau_1-\tau_2)[3(\tau_1+\tau_2)-1]f(\gamma_1)$. We know $\tau_1 + \tau_2 = 2\tau_f(\gamma_1)$, and so $\tau_1 - \tau_2 = 2(\tau_f(\gamma_1) - \tau_2)$. We also know $\tau_2 = \tau_f(\gamma_2)$, and thus $\tau_1 - \tau_2 = 2(\tau_f(\gamma_1) - \tau_f(\gamma_2))$. Thus,

$$[w_*(\tau_1) - w_*(\tau_2)]f(\gamma_1) = \frac{6}{49}(\tau_f(\gamma_1) - \tau_f(\gamma_2))[6\tau_f(\gamma_1) - 1]f(\gamma_1).$$

Using $\tau_2 = 2\tau_f(\gamma_1) - \tau_1$, we also have that $\frac{\partial\tau_2(\gamma_1,\tau_1)}{\partial\gamma_1} = 2\frac{\partial\tau_f(\gamma_1)}{\partial\gamma} = \frac{63}{(17-2\gamma_1)^2}$. Given $\tau_2 = \tau_f(\gamma_2) = \frac{8\gamma_2-5}{4(17-2\gamma_2)}$, we also find

$$\int_{\gamma_1}^{\gamma_2} \frac{\partial J(\tau_2,\gamma)}{\partial\tau}dF = \frac{1}{49}\frac{1}{2(17-2\gamma_2)}\int_{\gamma_1}^{\gamma_2}(252\gamma - 168\gamma_2 - 147)dF$$

Using these findings, we may rewrite (19) as an expression whose right-hand side depends only on γ_1 and γ_2, where $\gamma_2(\gamma_1,\tau_1)$ is an implicit function.

At this point, we employ the uniform distribution $F(\gamma) = \frac{\gamma-\underline{\gamma}}{\overline{\gamma}-\underline{\gamma}}$. Under this specification, we can rewrite (19) as

$$\frac{\partial \mathbb{E}J(\gamma_1,\tau_1)}{\partial\gamma_1} = \frac{-27(\gamma_2-\gamma_1)^2}{(17-2\gamma_1)^2(17-2\gamma_2)(\overline{\gamma}-\underline{\gamma})} < 0. \quad (20)$$

We conclude from (20) that a strict gain in joint welfare would be achieved, if we were to undertake a variation entailing a slight reduction in γ_1. Consequently, it is not optimal to step into flexibility, at least when the distribution is uniform.

As noted in the text, we can use this work to consider a related variation in which a small step is injected into a region of flexibility. Let us take an interval $[\gamma_0,\gamma_A]$ and assume that tariffs are flexible over that interval ($\tau(\gamma) = \tau_f(\gamma)$ for all $\gamma \in [\gamma_0,\gamma_A]$). Consider a variation in which we introduce a small step in the interior of the interval. We could evaluate this variation using the analysis above if we think of starting with the flexible tariff function, which corresponds to a limit case of $\gamma_0 = \gamma_1$, $\tau_1 = \tau_f(\gamma_0)$ and $\gamma_2 = \gamma_1$. Now we imagine introducing a slight step by increasing γ_1 to a value slightly above γ_0. Holding fixed γ_0, τ_1 and γ_A, the new values for γ_1, γ_2 and τ_2 will be slightly higher, and at the new values we would have $\tau_1 = \tau_f(\gamma_0) < \tau_f(\gamma_1) < \tau_2 = \tau_f(\gamma_2)$. To see the effect of this change, we evaluate (20) at the flexible starting point where $\gamma_2 = \gamma_1$. Clearly, there is no first-order effect (i.e., $\frac{\partial \mathbb{E}J(\gamma_1,\tau_1)}{\partial \gamma_1} = 0$ when $\gamma_2 = \gamma_1$). In fact, there is no second-order effect either (i.e., $\frac{\partial^2 \mathbb{E}J(\gamma_1,\tau_1)}{(\partial \gamma_1)^2} = 0$ when $\gamma_2 = \gamma_1$). Calculations confirm, however, that there is a negative third-order effect (i.e., $\frac{\partial^3 \mathbb{E}J(\gamma_1,\tau_1)}{(\partial \gamma_1)^3} < 0$ when $\gamma_2 = \gamma_1$), provided that $\frac{\partial \gamma_2(\gamma_1,\tau_1)}{\partial \gamma_1} \neq 1$. This provision indeed holds in the linear-quadratic model, since we may show that $\frac{\partial \gamma_2(\gamma_1,\tau_1)}{\partial \gamma_1} = 2$ as γ_2 approaches γ_1. Thus, due to third-order effects, expected joint welfare is strictly reduced, when a tiny step is introduced into a region of flexibility.

We come now to our *second candidate solution*. Assume that there exists a region $[\gamma_0, \gamma_2]$ and a value γ_1 such that $\underline{\gamma} \leq \gamma_0 < \gamma_1 < \gamma_2 \leq \overline{\gamma}$ and

$$\tau(\gamma) = \begin{cases} \tau_f(\gamma) & \text{, for } \gamma \leq \gamma_0 \\ \tau_1 = \tau_f(\gamma_0) & \text{, for } \gamma \in [\gamma_0, \gamma_1) \\ \tau_2 & \text{, for } \gamma \in (\gamma_1, \gamma_2]. \end{cases}$$

Incentive compatibility requires that type γ_1 is indifferent, which ensures that $\tau_1 < \tau_f(\gamma_1) < \tau_2$. In fact, in the quadratic setting, we must have $\tau_f(\gamma_1) = (\tau_1 + \tau_2)/2$. Over the interval $[\underline{\gamma}, \gamma_2]$, this tariff function starts flexible but eventually "steps out of flexibility."

We will consider a variation in which we leave τ_2, γ_2 and thus the play of types above γ_2 (if any) unaltered. In particular, we consider the following variation: Fix τ_2 and γ_2 and raise γ_1 slightly. Notice that this variation induces slight increases in τ_1 and γ_0.

To analyze the joint-welfare implications of this variation, we must characterize the induced changes in τ_1 and γ_0. We thus define $\tau_1(\gamma_1, \tau_2)$ by the incentive compatibility requirement that $w(\tau_1, \gamma_1) = w(\tau_2, \gamma_1)$. And, we define $\gamma_0(\gamma_1, \tau_2)$ by $\tau_1(\gamma_1, \tau_2) = \tau_f(\gamma_0)$. This implies

$$\frac{\partial \tau_1(\gamma_1, \tau_2)}{\partial \gamma_1} = \frac{r(\tau_2) - r(\tau_1)}{\frac{\partial w(\tau_1,\gamma_1)}{\partial \tau}} > 0, \quad \text{and} \quad \frac{\partial \gamma_0(\gamma_1, \tau_2)}{\partial \gamma_1} = \frac{\frac{\partial \tau_1(\gamma_1,\tau_2)}{\partial \gamma_1}}{\frac{\partial \tau_f(\gamma_0)}{\partial \gamma}} > 0.$$

Now consider $\mathbb{E}J$. Our variation will not affect the behavior of types above γ_2. Thus, regarding expected joint welfare as a function of γ_1 and τ_2 and differentiating with respect to γ_1, we have

$$\frac{\partial \mathbb{E}J(\gamma_1, \tau_2)}{\partial \gamma_1} = \frac{\partial}{\partial \gamma_1} \left\{ \int_{\underline{\gamma}}^{\gamma_0} J(\tau_f(\gamma), \gamma) dF + \int_{\gamma_0}^{\gamma_1} J(\tau_1, \gamma) dF + \int_{\gamma_1}^{\gamma_2} J(\tau_2, \gamma) dF \right\},$$

where we use γ_0 as shorthand for $\gamma_0(\gamma_1, \tau_2)$ and τ_1 as shorthand for $\tau_1(\gamma_1, \tau_2)$. Thus,

$$\frac{\partial \mathbb{E}J(\gamma_1, \tau_2)}{\partial \gamma_1} = \frac{\partial \gamma_0(\gamma_1, \tau_2)}{\partial \gamma_1} [J(\tau_f(\gamma_0), \gamma_0) - J(\tau_1, \gamma_0)] f(\gamma_0) + [J(\tau_1, \gamma_1) - J(\tau_2, \gamma_1)] f(\gamma_1) + \int_{\gamma_0}^{\gamma_1} \frac{\partial J(\tau_1, \gamma)}{\partial \tau} \frac{\partial \tau_1(\gamma_1, \tau_2)}{\partial \gamma_1} dF. \quad (21)$$

Since $\tau_1 = \tau_1(\gamma_1, \tau_2) = \tau_f(\gamma_0)$, the first term is eliminated. The second term is positive, and the third term may be negative (if, for example, $\tau_1 > \tau_e(\gamma_1)$). Finally, since $w(\tau_1, \gamma_1) = w(\tau_2, \gamma_1)$ by incentive compatibility, we may simplify the second term in (21) and write

$$\frac{\partial \mathbb{E}J(\gamma_1, \tau_2)}{\partial \gamma_1} = [w_*(\tau_1) - w_*(\tau_2)]f(\gamma_1) + \frac{\partial \tau_1(\gamma_1, \tau_2)}{\partial \gamma_1} \int_{\gamma_0}^{\gamma_1} \frac{\partial J(\tau_1, \gamma)}{\partial \tau} dF. \tag{22}$$

Proceeding mechanically, we find that $[w_*(\tau_1) - w_*(\tau_2)]f(\gamma_1) = \frac{3}{49}(\tau_1 - \tau_2)[3(\tau_1 + \tau_2) - 1]f(\gamma_1)$. We know $\tau_1 + \tau_2 = 2\tau_f(\gamma_1)$, and so $\tau_2 - \tau_1 = 2(\tau_f(\gamma_1) - \tau_1) = 2(\tau_f(\gamma_1) - \tau_f(\gamma_0))$. Thus, $[w_*(\tau_1) - w_*(\tau_2)]f(\gamma_1) = \frac{6}{49}(\tau_f(\gamma_0) - \tau_f(\gamma_1))[6\tau_f(\gamma_1) - 1]f(\gamma_1)$. Using $\tau_1 = 2\tau_f(\gamma_1) - \tau_2$, we have that $\frac{\partial \tau_1(\gamma_1, \tau_2)}{\partial \gamma_1} = 2\frac{\partial \tau_f(\gamma_1)}{\partial \gamma} = \frac{63}{(17-2\gamma_1)^2}$. Given $\tau_1 = \tau_f(\gamma_0) = \frac{8\gamma_0 - 5}{4(17-2\gamma_0)}$,we also find

$$\int_{\gamma_0}^{\gamma_1} \frac{\partial J(\tau_1, \gamma)}{\partial \tau} dF = \frac{1}{49} \frac{1}{2(17 - 2\gamma_0)} \int_{\gamma_0}^{\gamma_1} (252\gamma - 168\gamma_0 - 147) dF.$$

Using these findings, we may rewrite (22) as an expression whose right-hand side depends only on γ_0 and γ_1, where $\gamma_0(\gamma_1, \tau_2)$ is an implicit function.

At this point, we suppose F is uniform, so that $F(\gamma) = \frac{\gamma - \underline{\gamma}}{\bar{\gamma} - \underline{\gamma}}$. Under this specification, we can rewrite (22) as

$$\frac{\partial \mathbb{E}J(\gamma_1, \tau_1)}{\partial \gamma_1} = \frac{27(\gamma_1 - \gamma_0)^2}{(17 - 2\gamma_1)^2(17 - 2\gamma_0)(\bar{\gamma} - \underline{\gamma})} > 0. \tag{23}$$

We conclude from (23) that a strict gain in joint welfare would be achieved, if we were to undertake a variation entailing a slight increase in γ_1. Consequently, it is not optimal to step out of flexibility, at least when the distribution is uniform.

D Proof of Lemma 4

Let $d(x) \equiv \mathbb{E}[\gamma|\gamma > x] - x = \int_x^{\bar{\gamma}} \frac{1-F(\gamma)}{1-F(x)} d\gamma$. The following lemma is useful.

Lemma 9. *If $f(x)$ is non-decreasing, then $g(x) \equiv \frac{d(x)}{1-F(x)}$ is such that $g(x) \leq \frac{1}{2f(x)}$*

Proof. Note that

$$g'(x) = \frac{d'(x)}{1 - F(x)} + \frac{d(x)}{1 - F(x)} \frac{f(x)}{1 - F(x)} = \frac{g(x)f(x) - 1}{1 - F(x)} + \frac{g(x)f(x)}{1 - F(x)} = \frac{2g(x)f(x) - 1}{1 - F(x)}$$

where we used that $d'(x) = -1 + d(x)\frac{f(x)}{1-F(x)}$. We also know that $\lim_{x \to \bar{\gamma}} g(x) = \frac{1}{2f(\bar{\gamma})}$ (which follows from applying L'Hopital's rule on $d(x)/(1 - F(x))$). From the ODE it follows then that if $g(x_0) > \frac{1}{2f(x_0)}$ for some x_0 then $g'(x_0) > 0$ and given that $f(x)$ is non-decreasing, this implies that $g(x) > \frac{1}{2f(x_0)} \geq \frac{1}{2f(\bar{\gamma})}$ for all $x > x_0$, which is a contradiction of the limit condition. □

Now we are ready to prove Lemma 4. Assumption 2(b) can be written as:

$$G(\gamma^c) = \mathbb{E}[\gamma|\gamma > \gamma^c] - \mathbb{E}[\gamma|\gamma > \gamma^p] - \kappa(\gamma^c - \gamma^p) \leq 0$$

for all $\gamma \geq \gamma^p$ by using Lemma 2. Note that $G(\gamma^p) = 0$ and

$$G'(\gamma^c) = \frac{d}{d\gamma^c}\left(\mathbb{E}[\gamma|\gamma > \gamma^c]\right) - \kappa = d'(\gamma^c) + 1 - \kappa = \frac{d(\gamma^c)f(\gamma^c)}{1 - F(\gamma^c)} - \kappa \leq \frac{1}{2} - \kappa$$

where the last inequality follows from f non-decreasing and Lemma 9. Letting $\kappa \geq \frac{1}{2}$ implies that $G'(\gamma^c) \leq 0$ for all $\gamma^c > \gamma^p$ which proves that $G(\gamma^c) \leq 0$ for all $\gamma^c > \gamma^p$.

Assumption 2(a) is that $H(\gamma) = \kappa F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$ is non-decreasing. Taking the derivative:

$$\begin{aligned} H'(\gamma) =& \kappa f(\gamma) + \frac{v''(\pi_f(\gamma))}{b''(\pi_f(\gamma))} f(\gamma) - v'(\pi_f(\gamma))f'(\gamma) \\ =& \left\{\kappa + \frac{v''(\pi_f(\gamma)) + b''(\pi_f(\gamma))}{b''(\pi_f(\gamma))} - 1\right\} f(\gamma) - v'(\pi_f(\gamma))f'(\gamma) \end{aligned}$$

where we used that $\pi_f'(\gamma) = -1/b''(\pi_f(\gamma))$. It follows that $H'(\gamma) \geq (2\kappa - 1)f(\gamma) - v'(\pi_f(\gamma))f'(\gamma)$. Using the definition of κ, it then follows that for $\kappa \geq 1/2$ and $f(\gamma)$ non-decreasing we have that $H'(\gamma) \geq 0$. Thus, we have proved Assumption 2.

E Proof of Lemma 8

The proof follows closely the proof for the sufficiency result of the Lemma of Optimality in Amador et al. (2006). First we show that if there exists a non-decreasing Λ_0 such that

$$\mathcal{L}(x, y|\Lambda_0) \leq \mathcal{L}(\pi^\star, 0|\Lambda_0), \text{ for all} (x, y) \in \Phi$$

then $(\pi^\star, 0)$ solves the relaxed problem. This result follows from Theorem 1, pg. 220 in Luenberger (1969), by letting $X \equiv \{\pi, \omega | \omega \geq 0 \text{ and } \pi : \Gamma \to \mathbb{R}\}$, $\Omega \equiv \Phi$, $Z \equiv \{z | z : \Gamma \to \mathbb{R} \text{ with } \sup_{\gamma\in\Gamma} |z(\gamma)| < \infty\}$ with norm $||z|| = \sup_{\gamma\in\Gamma} |z(\gamma)|$, and $P \equiv \{z | z \in Z \text{ and } z(\gamma) \geq 0 \text{ for all } \gamma \in \Gamma\}$. The value of f will be the objective function, and the value of $G((\pi, w))$ defines a mapping from X to a function of γ given by the left-hand side of inequality (13). Given that the proposed allocation satisfies (13) with equality, it follows that $G((\pi^\star, 0))(\gamma) = 0$ for all $\gamma \in \Gamma$. Then our set up satisfies the hypothesis of the Theorem, implying that $(\pi^\star, 0)$ maximizes the objective function subject to $G((x, y))(\gamma) \leq G((\pi^\star, 0)) = 0$ for all $\gamma \in \Gamma$. To be able to use the rest of the results in Amador et al. (2006) we need to extend the function $v(.)$ and $b(.)$ to the entire positive side of the real line. To do this, we let $\hat{v}$ be the linear extension of v starting from $\bar{\pi}$:

$$\hat{v}(\pi) = \begin{cases} v(\pi) & \text{, if } \pi \in [0, \bar{\pi}] \\ v(\bar{\pi}) + v'(\bar{\pi})(\pi - \bar{\pi}) & \text{, if } \pi > \bar{\pi} \end{cases}$$

which is well-defined if $v'(\bar{\pi})$ is finite. We define $\hat{b}(\pi)$ in a similar fashion.[32] Then we let $\hat{\Phi} = \{(\pi, \underline{\omega}) | \underline{\omega} \geq 0, \pi : \Gamma \to \mathbb{R}_+ \text{ and } \pi \text{ non-decreasing}\}$. Note that $\hat{\Phi}$ is a convex cone and that both $\hat{b}$ and $\hat{v} + (1-\kappa)\hat{b}$ remain continuous, differentiable and concave on $\pi \in \mathbb{R}_+$. This implies that the results of Lemma 7 extend to $(\pi, \omega) \in \hat{\Phi}$, and the extended Lagrangian, which we denote by $\hat{\mathcal{L}}$, is concave. We can then use Lemma A.1 from Amador et al. (2006) and show that the Gateaux differential of the extended Lagrangian exists because the Lagrangian functional when evaluated at the proposed multiplier can be written as the sum of terms that can be expressed as integrals with concave differentiable integrands and that the extended Lagrangian is defined over a convex cone $\hat{\Phi}$. Finally using Lemma A.2 from Amador et al. (2006) (which is itself an extension of Lemma 1 in Luenberger, 1969, page 227), we obtain that a sufficient condition for optimality is that:

$$\partial\hat{\mathcal{L}}(\pi^\star, 0; \pi^\star, 0 | \Lambda_0) = 0$$
$$\partial\hat{\mathcal{L}}(\pi^\star, 0; x, y | \Lambda_0) \leq 0$$

hold for all $(x, y) \in \hat{\Phi}$. Now, note that if $(x, y) \in \hat{\Phi}$, then for all sufficiently small $\alpha > 0$ we have that $(\alpha x, \alpha y) \in \Phi$ and $\partial\hat{\mathcal{L}}(\pi^\star, 0; x, y|\Lambda_0) = \frac{1}{\alpha}\partial\hat{\mathcal{L}}(\pi^\star, 0; \alpha x, \alpha y|\Lambda_0)$. So it is sufficient to check the above two first order conditions for all $(x, y) \in \Phi$. But given that both $(\pi^\star, 0) \in \Phi$ and that $(\pi^\star + \alpha x, \alpha y) \in \Phi$ for all α small enough (as $\pi^\star(\gamma) < \bar{\pi}$ for all $\gamma \in \Gamma$ given our assumption of an interior flexible allocation), we have then, by the definition of the Gateaux differential, that $\partial\hat{\mathcal{L}}(\pi^\star, 0; x, y|\Lambda_0) = \partial\mathcal{L}(\pi^\star, 0; x, y|\Lambda_0)$ for all $(x, y) \in \Phi$, which completes the proof.

F Proof of Proposition 4

The following Lemma will be used:

Lemma 10. *In the linear-quadratic case, if $\kappa F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$ is non-decreasing for all $\gamma \in \Gamma$, then Assumption 2 is satisfied.*

Proof. Let $X(\gamma) = (1 - F(\gamma))G(\gamma)$. Then we can show that:

$$X'(\gamma) = -\kappa + \kappa F(\gamma) - \left[v'(\pi_f(\gamma^p)) + (1-\kappa)(\gamma - \gamma^p)\right] f(\gamma)$$

In the linear-quadratic case, we have that $v'(\pi_f(\gamma)) = v'(\pi_f(\gamma^p)) + (1-\kappa)(\gamma - \gamma^p)$, and thus

$$X'(\gamma) = -\kappa + \kappa F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$$

which is non-decreasing by the hypothesis of the Lemma. This implies then that $X(\gamma)$ is a convex function of γ. Note that $X(\bar{\gamma}) = 0$ and $X'(\bar{\gamma}) = -v'(\pi_f(\bar{\gamma}))f(\bar{\gamma}) > 0$. It then follows that $X(\gamma)$ has at most another 0 for $\gamma < \bar{\gamma}$, which corresponds to γ^p. This also implies that $X(\gamma) < 0$ for all $\gamma \in (\gamma^p, \bar{\gamma})$ and thus $G(\gamma) < 0$ as well, which proves Assumption 2(b). Assumption 2(a) follows directly from the hypothesis of the Lemma. □

[32] In case that $\lim_{\pi\to\bar{\pi}} b'(\pi) = -\infty$, then one can define the linear extension of b starting from some $\pi_0 \in (\pi_f(\bar{\gamma}), \bar{\pi})$ such that $b'(\pi_0)$ is finite. Concavity of b implies $\hat{b}(\pi) \geq b(\pi)$ for $\pi \in [\pi_0, \bar{\pi}]$, and hence the Lagrangian using this extension of b will lie weakly above the original Lagrangian for $\pi \in [\pi_0, \bar{\pi}]$. In case that $\lim_{\pi\to\bar{\pi}} v'(\pi) = -\infty$, then there exists a $\pi_0 \leq (\pi_f(\bar{\gamma}), \bar{\pi})$ which is sufficiently close to $\bar{\pi}$ such that the linear extension of v from π_0 to $+\infty$ generates a $\hat{v}$ such that $\hat{v}(\pi) \geq v(\pi)$ for all $\pi \in [\pi_0, \bar{\pi}]$. This follows from showing that there always exists a π_0 sufficiently close to $\bar{\pi}$ such that $v'(\pi_0) \geq v'(\pi)$ for $\pi \in [\pi_0, \bar{\pi}]$. Again this extension implies a relaxation of the Lagrangian. Once these extensions are done, the rest of the proof proceeds in similar fashion.

To prove Proposition 4 we just need to show that $\kappa F(\gamma) - v'(\pi_f(\gamma))\gamma) f(\gamma)$ is non-decreasing in $\gamma \in \Gamma$ and invoke Lemma 10. Assuming differentiability of γ, and using that in our example $\kappa = 2/3$ and that $v''/b'' = -1/3$, we get that $\kappa F(\gamma) - v'(\pi_f(\gamma)) f(\gamma)$ is non-decreasing if

$$\frac{2}{3} f(\gamma) + \frac{v''(\pi_f(\gamma))}{b''(\pi_f(\gamma))} f(\gamma) - v'(\pi_f(\gamma)) f'(\gamma) \geq 0,$$

or equivalently $\frac{1}{3} f(\gamma) - v'(\pi_f(\gamma)) f'(\gamma) \geq 0$, the condition stated in Proposition 4.

G An Example with Logarithmic Utility

In what follows we develop an endowment example and show how the result above allow us to characterize the optimal trade agreement. Assume that $u(c) = \log(c)$ and that $Q(p) = 1$ and $Q_\star(p) = A$ where $A > 1$. Then we can write that $B = -p - p_\star z - \log(p)$, $V = p_\star z - \log(p_\star)$ and $\Pi = p$, and where $p = (1+z)^{-1}$ and $p_\star = (A-z)^{-1}$.

Note that free trade is $z = \frac{1}{2}(A-1)$. Writing everything in terms of π delivers:

$$b(\pi) = -\pi + \frac{\pi - 1}{(A+1)\pi - 1} - \log(\pi), \quad \text{and} \quad v(\pi) = \frac{1-\pi}{(A+1)\pi - 1} - \log\left(\frac{\pi}{(A+1)\pi - 1}\right)$$

and where $z = \frac{1}{\pi} - 1$.

The free trade allocation corresponds to: $\pi^{ft} = \frac{2}{1+A}$. The amount of zero trade is given by $\bar{\pi} = 1$. We will restrict attention to a set of admissible $\pi \in [\pi^{ft}, 1]$, which is equivalent to restricting tariffs to be non-negative. Note that $v'(\pi) = \frac{\pi - 1}{\pi((A+1)\pi-1)^2} \leq 0$ and note as well that $b''(\pi) = \frac{1}{\pi^2} - \frac{2A(1+A)}{((A+1)\pi-1)^3}$, which is non-positive for all $\pi \in [\pi^{ft}, 1]$ if $1 \leq A \leq 1 + \sqrt{3}$. And thus Assumption 1 is satisfied for $1 \leq A \leq 1 + \sqrt{3}$.

Similarly one can show that $v''(\pi) + b''(\pi) = \frac{2 + (A+1)\pi((A+1)\pi - 4)}{\pi^2((A+1)\pi-1)^2}$. Using the above, the value of κ can be found to be:

$$\kappa = \begin{cases} \frac{2(A+1)}{7A-1} & ; \text{for } 1 \leq A \leq \frac{4+\sqrt{41}}{5} \\ \frac{-1-2A+A^2}{-2-2A+A^2} & ; \text{for } \frac{4+\sqrt{41}}{5} \leq A \leq 1+\sqrt{3} \end{cases}$$

Note that $\kappa \geq 0$ for all $A \in [1, 1+\sqrt{2}]$. Also for A close to 1, $\kappa \sim 2/3$, which implies that Assumption 2 will be satisfied for distributions with non-decreasing densities when A is sufficiently close to 1.

References

Alonso, Ricardo and Matouschek, Niko (2008). "Optimal Delegation." *Review of Economic Studies*, 75(1), 259–293.
URL http://ideas.repec.org/a/bla/restud/v75y2008i1p259-293.html

Amador, Manuel, Werning, Iván, and Angeletos, George-Marios (2006). "Commitment

vs. Flexibility." *Econometrica*, 74(2), 365–396.
URL http://ideas.repec.org/a/ecm/emetrp/v74y2006i2p365-396.html

Athey, Susan, Atkeson, Andrew, and Kehoe, Patrick J. (2005). "The Optimal Degree of Discretion in Monetary Policy." *Econometrica*, 73(5), 1431–1475.
URL http://ideas.repec.org/a/ecm/emetrp/v73y2005i5p1431-1475.html

Athey, Susan and Bagwell, Kyle (2008). "Collusion With Persistent Cost Shocks." *Econometrica*, 76(3), 493–540.
URL http://ideas.repec.org/a/ecm/emetrp/v76y2008i3p493-540.html

Athey, Susan, Bagwell, Kyle, and Sanchirico, Chris (2004). "Collusion and Price Rigidity." *Review of Economic Studies*, 71(2), 317–349.
URL http://ideas.repec.org/a/bla/restud/v71y2004i2p317-349.html

Bagwell, Kyle (2009). "Self-Enforcing Trade Agreements and Private Information." NBER Working Papers 14812, National Bureau of Economic Research, Inc.
URL http://ideas.repec.org/p/nbr/nberwo/14812.html

Bagwell, Kyle and Lee, Gea Myoung (2010). "Advertising Collusion in Retail Markets." Working paper, Stanford University.
URL http://www.stanford.edu/~kbagwell/papers/Bagwell%20Lee%20AdCollusion%20Dec23%2009.pdf

Bagwell, Kyle and Staiger, Robert W. (1999). "An Economic Theory of GATT." *American Economic Review*, 89(1), 215–248.
URL http://ideas.repec.org/a/aea/aecrev/v89y1999i1p215-248.html

Bagwell, Kyle and Staiger, Robert W. (2001). "Reciprocity, non-discrimination and preferential agreements in the multilateral trading system." *European Journal of Political Economy*, 17(2), 281–325.
URL http://ideas.repec.org/a/eee/poleco/v17y2001i2p281-325.html

Bagwell, Kyle and Staiger, Robert W. (2005). "Enforcement, Private Political Pressure, and the General Agreement on Tariffs and Trade/World Trade Organization Escape Clause." *Journal of Legal Studies*, 34, 471–513.
URL http://ideas.repec.org/a/ucp/jlstud/v34y2005p471-513.html

Bagwell, Kyle and Sykes, Alan O. (2004). "Chile price band system and safeguard measures relating to certain agricultural products." *World Trade Review*, 3(03), 507–528.
URL http://ideas.repec.org/a/cup/wotrrv/v3y2004i03p507-528_00.html

Baldwin, Richard (1987). "Politically realistic objective functions and trade policy PROFs and tariffs." *Economics Letters*, 24(3), 287–290.
URL http://ideas.repec.org/a/eee/ecolet/v24y1987i3p287-290.html

Bchir, Mohamed Hedi, Jean, Sébastien, and Laborde, David (2006). "Binding Overhang and Tariff-Cutting Formulas." *Review of World Economics (Weltwirtschaftliches Archiv)*,

142(2), 207–232.
URL http://ideas.repec.org/a/spr/weltar/v142y2006i2p207-232.html

Bouet, Antoine and Laborde, David (2008). "The cost of non-doha." Briefing Note November, International Food Policy Research Institute (IFPRI).
URL http://www.ifpri.org/

Feenstra, Robert C and Lewis, Tracy R (1991). "Negotiated Trade Restrictions with Private Political Pressure." *The Quarterly Journal of Economics*, 106(4), 1287–307.
URL http://ideas.repec.org/a/tpr/qjecon/v106y1991i4p1287-307.html

Fudenberg, Drew, Levine, David I, and Maskin, Eric (1994). "The Folk Theorem with Imperfect Public Information." *Econometrica*, 62(5), 997–1039.
URL http://ideas.repec.org/a/ecm/emetrp/v62y1994i5p997-1039.html

Grossman, Gene M and Helpman, Elhanan (1995). "Trade Wars and Trade Talks." *Journal of Political Economy*, 103(4), 675–708.
URL http://ideas.repec.org/a/ucp/jpolec/v103y1995i4p675-708.html

Holmström, Bengt (1984). "On the Theory of Delegation." In Marcel Boyer and Richard E. Kihlstrom, editors, "Bayesian Models in Economic Theory," volume 5 of *Studies in Bayesian Econometrics*, pages 115–41.
URL http://ideas.repec.org/p/nwu/cmsems/438.html

Horn, Henrik, Maggi, Giovanni, and Staiger, Robert W. (2010). "Trade Agreements as Endogenously Incomplete Contracts." *American Economic Review*, forthcoming.
URL http://ideas.repec.org/p/nbr/nberwo/12745.html

Luenberger, David G. (1969). *Optimization by Vector Space Methods*. Wiley, John and Sons, New York.

Maggi, Giovanni and Rodriguez-Clare, Andres (2007). "A Political-Economy Theory of Trade Agreements." *American Economic Review*, 97(4), 1374–1406.
URL http://ideas.repec.org/a/aea/aecrev/v97y2007i4p1374-1406.html

Martin, Alberto and Vergote, Wouter (2008). "On the role of retaliation in trade agreements." *Journal of International Economics*, 76(1), 61–77.
URL http://ideas.repec.org/a/eee/inecon/v76y2008i1p61-77.html

McAfee, R Preston and McMillan, John (1992). "Bidding Rings." *American Economic Review*, 82(3), 579–99.
URL http://ideas.repec.org/a/aea/aecrev/v82y1992i3p579-99.html

Milgrom, Paul and Segal, Ilya (2002). "Envelope Theorems for Arbitrary Choice Sets." *Econometrica*, 70(2), 583–601.
URL http://ideas.repec.org/a/ecm/emetrp/v70y2002i2p583-601.html

World Trade Organization (2009). *World Trade Report 2009: Trade Policy Commitments and Contingency Measures*. Geneva.
URL http://www.wto.org/english/res_e/booksp_e/anrep_e/world_trade_report09_e.pdf