Online Appendix for
"On the Optimality of Tariff Caps"

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August 2010

1 An intuition for Assumption 2b

The following appendix describes the intuition for Assumption 2b. It is shown that Assumption 2b is sufficient to rule out improvements to the cap allocation in the pooling region where a new point above π^p is offered together with some money burning so as to make a certain target type (γ_0) indifferent. This deviation must not be an improvement if all types above γ_0 decide to choose the new point as implied by incentive compatibility.

Let $\pi^p = \pi^f(\gamma^p)$. And let us define the following two objects:

$$w_u(\pi|\gamma_0) = b(\pi) + \gamma_0\pi - b(\pi^p) - \gamma_0\pi^p \tag{1}$$

$$w_s(\pi|\gamma_0) = v(\pi) + b(\pi) + E[\gamma|\gamma_0]\pi - v(\pi^p) - b(\pi^p) - E[\gamma|\gamma_0]\pi^p \tag{2}$$

where we used the short cut $E[\gamma|\gamma_0]$ to denote $E[\gamma|\gamma > \gamma_0]$.

The function $w_u(\pi|\gamma_0)$ traces the indifference curve of government's type γ_0 with respect to the point $\pi = \pi^p, w = 0$, which is the pooling point in the cap allocation.

The function $w_s(\pi|\gamma_0)$ traces the indifference curve for the "planner" if it were to pool all types above γ_0 with respect to the point $\pi = \pi^p, w = 0$.

The following claim shows that the planer's indifference curve as defined above must lie always above the government's indifference curve (in the $\pi, -w$ space) for the cap to be optimal.

Claim 1: Pooling the region $[\gamma^p, \bar{\gamma}]$ is optimal only if $w_u(\pi|\gamma_0) \geq w_s(\pi|\gamma_0)$ for all $\gamma_0 \in [\gamma^p, \bar{\gamma}]$, and all $\pi > \pi^p$ with $w_u(\pi|\gamma_0) \geq 0$.

Proof: Suppose not, and that for some $\pi_1 < \pi^p$ and $\gamma_1 \in [\gamma^p, \bar{\gamma}]$ we have that $0 \leq w_u(\pi_1|\gamma_1) < w_s(\pi_1|\gamma_1)$. Then suppose that the planner offers the point $(\pi_1, w_u(\pi_1|\gamma_1))$ in addition to the cap allocation. This is feasible given that $w_u(\pi_1|\gamma_1) \geq 0$. Then, from single crossing and using that $\pi_1 > \pi^p$, it follows that all types above γ_1 will choose the new point $(\pi_1, w_u(\pi_1|\gamma_1))$ while all other types remain in their original allocation points.

The effect of this in the planner's utility, Δ_s, is:

$$\begin{aligned}\frac{\Delta_s}{1-F(\gamma_1)} &= b(\pi_1) + v(\pi_1) + E[\gamma|\gamma_1]\pi_1 - w_u(\pi_1|\gamma_1) - (b(\pi^p) + v(\pi^p) + E[\gamma|\gamma_1]\pi^p) \\ &= w_s(\pi_1|\gamma_1) - w_u(\pi_1|\gamma_1) > 0\end{aligned}$$

where the last equality follows from the definition of w_s, w_u. Hence, the new allocation is incentive compatible and improves upon the tariff cap allocation and we get a contradiction. □

Now note that $w_u(\pi|\gamma_0) \geq w_s(\pi|\gamma_0)$ for $\pi \geq \pi^p$ is equivalent to:

$$\hat{G}(\pi, \gamma_0) \equiv \gamma_0(\pi - \pi^p) + v(\pi^p) - v(\pi) - E[\gamma|\gamma_0](\pi - \pi^p) \geq 0 \tag{A}$$

for $\pi \geq \pi^p$. Note that the restriction that $w_u(\pi|\gamma_0) \geq 0$ is equivalent to requiring that:

$$\gamma_0(\pi - \pi^p) \geq -(b(\pi) - b(\pi^p)) \tag{3}$$

Using this in equation (A), we get:

$$\begin{aligned}&\hat{G}(\pi, \gamma_0) \geq \\ &\quad -\left(\frac{v(\pi) + (1-\kappa)b(\pi) - (v(\pi^p) + (1-\kappa)b(\pi^p))}{\pi - \pi^p} - \kappa\gamma_0 + E(\gamma|\gamma_0)\right)(\pi - \pi^p)\end{aligned} \tag{4}$$

Now, from the definition of κ we have that $-v(\pi) - (1-\kappa)b(\pi)$ is convex. Using that $\pi > \pi^p$, this implies that:

$$-\left(\frac{v(\pi) + (1-\kappa)b(\pi) - (v(\pi^p) + (1-\kappa)b(\pi^p))}{\pi - \pi^p}\right) \geq -v'(\pi^p) - (1-\kappa)b'(\pi^p)$$

which follows because the average slope of a convex function is always higher than the slope at its lowest point. Plugging this back into (4) we get that:

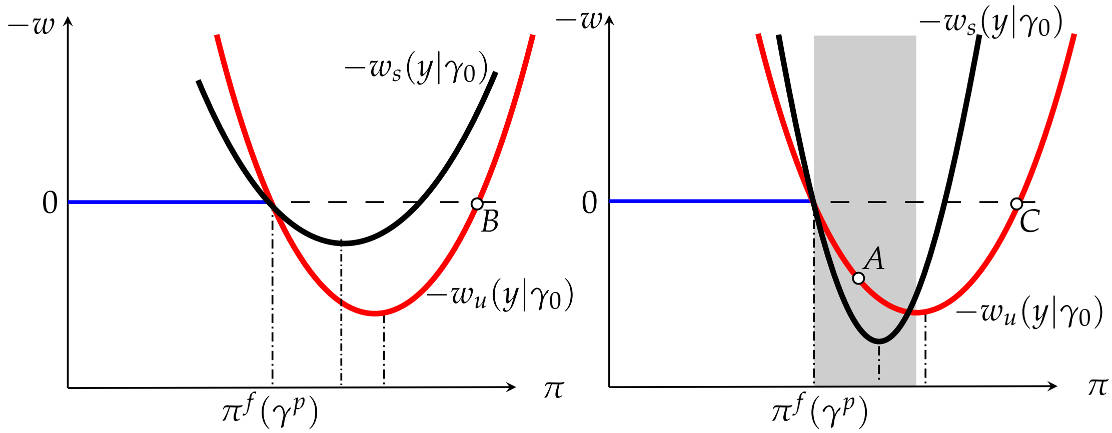
$$\begin{aligned}\hat{G}(\pi,\gamma_0) \geq& \Big(-v'(\pi^p) - (1-\kappa)b'(\pi^p) + \kappa\gamma_0 - E(\gamma|\gamma_0)\Big)(\pi - \pi^p)\\ =& \Big(-v'(\pi^p) + (1-\kappa)\gamma^p + \kappa\gamma_0 - E(\gamma|\gamma_0)\Big)(\pi - \pi^p)\\ \hat{G}(\pi,\gamma_0) \geq& -\Big[v'(\pi^p) - \gamma_0 + E(\gamma|\gamma_0) + (1-\kappa)(\gamma_0 - \gamma^p)\Big](\pi - \pi^p) \qquad \text{(B)}\end{aligned}$$

where we used in the first equality that $b'(\pi^p) = -\gamma^p$. Then we have the following claim.

Claim 2: Assumption 2b guarantees that $w_u(\pi|\gamma_0) \geq w_s(\pi|\gamma_0)$ for all $\gamma_0 \in (\gamma^p, \bar{\gamma}]$, $\pi > \pi^p$ and $w_u(\pi|\gamma_0) \geq 0$.

Proof: Note that for all $\gamma_0 > \gamma^p$ and $\pi > \pi^p$, such that $w_u(\pi|\gamma_0) \geq 0$, Assumption 2b guarantees that $\hat{G}(\pi,\gamma_0) \geq 0$ (as Assumption 2b imposes that the term in square brackets in equation (B) is non-positive). And thus, $w_u(\pi|\gamma_0) \geq w_s(\pi|\gamma_0)$ follows by the definition of $\hat{G}$. □

The following picture illustrates two possible cases.



(a) (b)

Panel (a) shows the case where $w_u(\pi) \geq w_s(\pi)$ as long as $w_u \geq 0$. Panel (b) shows the case where for some some $\pi > \pi^p$ and some $\gamma_0 > \gamma^p$ we have that $w_u(\pi) < w_s(\pi)$ for some $w_u(\pi) > 0$. In panel (b) a point such as A would represent an improvement to the cap allocation: if it were offered as a menu in addition to the cap, only agents above γ_0 would choose it, and the social planner actually prefers them to do so (this can be seen by noticing that it lies above the planer's indifference curve). No such point can be found in panel (a). Assumption 2b rules out situations such as the one in panel (b).

The existence of these improvements that destroy the optimality of the cap can be shown to rely on <u>a failure of the single crossing property</u> between the preferences of the home country and the preferences of a planner that optimally weights both home and foreign welfare. For related details, see the discussion regarding Proposition 10 of Amador, Werning and Angeletos (06).