# Online Appendix for "Trade Policy under Monopolistic Competition with Firm Selection"* 

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This Online Appendix has two parts. First, we examine and compare the entry-externality effect in the Melitz-Ottaviano (MO) and CES models, respectively, as considered by Bagwell and Lee $(2018,2020)$. Second, we provide a numerical assessment of the assumption that $U(\chi, \chi)$ is quasi-concave. A brief summary of this assessment appears in Bagwell and Lee (2020).

## 1 Entry-externality Effects under MO and CES

### 1.1 Characterizing the Entry-externality Effects

In the MO model and in the benchmark closed-economy setting, additional entry generates a positive externality if and only if

$$
\begin{equation*}
\alpha-2 \cdot c_{D}^{m}<0 \tag{1}
\end{equation*}
$$

where $\alpha$ is a preference parameter and $c_{D}^{m}$ refers to the critical cut-off cost level in market equilibrium, a function of parameters other than $\alpha$. As in Bagwell and Lee (2020), we assume that $\alpha>c_{D .}^{m}{ }^{1}$

To interpret the sign of (1), we establish now that (1) holds if and only if additional entry raises the aggregate profit of the economy starting at market equilibrium,

$$
\begin{equation*}
\left.\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}\right|_{N_{E}=N_{E}^{m}}>0 . \tag{2}
\end{equation*}
$$

[^0]The necessary and sufficient relationship between (1) and (2) can be shown by using the following equations from Bagwell and Lee (2020),

$$
\begin{gather*}
\bar{\pi}=\frac{\left(c_{M}\right)^{-k}\left(c_{D}\right)^{k+2}}{2 \gamma(k+1)(k+2)}  \tag{3}\\
N_{E}=\frac{2(1+k) \gamma\left(c_{M}\right)^{k}}{\eta} \frac{\left(\alpha-c_{D}\right)}{\left(c_{D}\right)^{k+1}}  \tag{4}\\
\frac{d c_{D}}{d N_{E}}=\left(\frac{d N_{E}}{d c_{D}}\right)^{-1}=-\left(\frac{2(1+k) \gamma\left(c_{M}\right)^{k}\left(\alpha(1+k)-k c_{D}\right)}{\eta\left(c_{D}\right)^{k+2}}\right)^{-1}<0, \tag{5}
\end{gather*}
$$

where $c_{D}$ refers to the critical cut-off cost level that is associated with a given selection for the level of entry, $N_{E}$. In these expressions, $c_{M}$ is the scale parameter and $k$ is the shape parameter of Pareto distribution $G(c)=\left(c / c_{M}\right)^{k}$ for $c \in\left[0, c_{M}\right]$, and $\alpha, \gamma, \eta$ are preference parameters as described in the main paper. We observe from (4) that $\alpha>c_{D}^{m}$ is equivalent to $N_{E}^{m}>0$.

Using (3), we derive

$$
\begin{equation*}
\frac{d \bar{\pi}}{d N_{E}}=\frac{\left(c_{M}\right)^{-k}\left(c_{D}\right)^{k+2}}{2 \gamma(k+1)(k+2)} \frac{(k+2)}{c_{D}} \frac{d c_{D}}{d N_{E}}=\bar{\pi} \frac{(k+2)}{c_{D}} \frac{d c_{D}}{d N_{E}} . \tag{6}
\end{equation*}
$$

Using (3), (4), (5), and (6), we evaluate (2) as follows:

$$
\begin{aligned}
\left.\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}\right|_{\hat{N}_{E}=\hat{N}_{E}^{m}} & =\bar{\pi}+\left.N_{E} \frac{d \bar{\pi}}{d N_{E}}\right|_{N_{E}=N_{E}^{m}} \\
& =\left.\bar{\pi}\left(1+N_{E} \frac{(k+2)}{c_{D}} \frac{d c_{D}}{d N_{E}}\right)\right|_{N_{E}=N_{E}^{m}} \\
& =\left.\bar{\pi}\left(\frac{-\alpha+2 \cdot c_{D}}{\left(\alpha(1+k)-k c_{D}\right)}\right)\right|_{N_{E}=N_{E}^{m}},
\end{aligned}
$$

where $c_{D}=c_{D}^{m}$ when $N_{E}=N_{E}^{m}$ and where $\alpha(1+k)-k c_{D}>0$ thus follows from $\alpha>c_{D}^{m}$. This shows the necessary and sufficient relationship stated above.

We may now summarize our finding for the MO model as follows: starting at the market equilibrium, additional entry generates a positive externality if and only if it raises the aggregate profit, $\left.\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}\right|_{N_{E}=N_{E}^{m}}>0$.

We now follow Bagwell and Lee (2018) and examine the model with CES preferences. ${ }^{2}$ In the CES model, firm-level productivity $\varphi \in[1, \infty)$ is distributed according to a Pareto distribution with shape parameter $k$, so that $G(\varphi)=1-\varphi^{-k}$. Under CES preferences, we

[^1]claim that $\left.\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}\right|_{N_{E}=N_{E}^{m}}>0$ holds and this inequality offers a sufficient condition for $E X T>0$ at $N_{E}=N_{E}^{m}$. To establish this claim, we proceed in three steps. First, we observe that, if $\frac{d C S}{d N_{E}}>\bar{\pi}$, then
\[

$$
\begin{equation*}
E X T=\frac{d C S}{d N_{E}}+N_{E} \frac{d \bar{\pi}}{d N_{E}}>\bar{\pi}+N_{E} \frac{d \bar{\pi}}{d N_{E}}=\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}, \tag{7}
\end{equation*}
$$

\]

and so $\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}$ would then offer a lower bound for $E X T$. Second, we show that $\frac{d C S}{d N_{E}}>\bar{\pi}$ in fact holds for $N_{E}>0$ and thus in particular for $N_{E}=N_{E}^{m}$. Third, we confirm that $\left.\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}\right|_{N_{E}=N_{E}^{m}}>0$.

The first step is immediate from (7). To confirm the second step, we use Bagwell and Lee's (2018) finding that

$$
\frac{d C S}{d N_{E}}=\frac{(\sigma-1)(1-\theta)}{\sigma(1-\theta)-1} \cdot \frac{(P)^{\frac{-\theta}{1-\theta}}}{N_{E}} \cdot \epsilon_{\varphi^{*}, N_{E}}
$$

where

$$
\begin{aligned}
(P)^{\frac{-\theta}{1-\theta}} & =N_{E} \cdot \frac{\sigma}{\sigma-1} \cdot k \cdot \bar{\pi} \\
\bar{\pi} & =\left(\varphi^{*}\right)^{-k} \frac{(\sigma-1) f_{D}}{1+k-\sigma} \\
\Upsilon\left(N_{E}\right)^{\frac{1}{\sigma-1}} & =\left(\varphi^{*}\right)^{\frac{k+1-\sigma}{\sigma-1}+\frac{(1-\theta)(\sigma-1)}{\sigma(1-\theta)-1}},
\end{aligned}
$$

where $\Upsilon>0$ is a constant and where $\epsilon_{\varphi^{*}, N_{E}} \equiv \frac{d \varphi^{*}}{d N_{E}} \frac{N_{E}}{\varphi^{*}}$ can be computed from the last of these expressions. We may now confirm that

$$
\begin{aligned}
\frac{d C S}{d N_{E}}-\bar{\pi} & =\left(\frac{(1-\theta) k \sigma}{\sigma(1-\theta)-1} \cdot \epsilon_{\varphi^{*}, N_{E}}-1\right) \bar{\pi} \\
& =\left(\frac{(1+k-\sigma)+(\sigma(1-\theta)-1)+\theta}{k(\sigma(1-\theta)-1)+\theta(\sigma-1)}\right) \bar{\pi}>0
\end{aligned}
$$

which shows that $\frac{d C S}{d N_{E}}>\bar{\pi}$ holds when $N_{E}>0$ and thus $\bar{\pi}>0$. Since $N_{E}^{m}>0$, it thus follows that (7) holds at $N_{E}=N_{E}^{m}$.

For the third step, we show that $\left.\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}\right|_{N_{E}=N_{E}^{m}}>0$ holds regardless of parameters.

In particular, for $N_{E}>0$, we find that

$$
\begin{aligned}
\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi} & =\bar{\pi}+N_{E} \frac{d \bar{\pi}}{d N_{E}}=\bar{\pi}-N_{E} \frac{k}{\varphi^{*}} \bar{\pi} \frac{d \varphi^{*}}{d N_{E}} \\
& =\left(1-k \cdot \epsilon_{\varphi^{*}, N_{E}}\right) \bar{\pi}=\left(1-\frac{\frac{k}{\sigma-1}}{\frac{k+1-\sigma}{\sigma-1}+\frac{(1-\theta)(\sigma-1)}{\sigma(1-\theta)-1}}\right) \bar{\pi}>0
\end{aligned}
$$

where the above inequality holds by

$$
\frac{k+1-\sigma}{\sigma-1}+\frac{(1-\theta)(\sigma-1)}{\sigma(1-\theta)-1}=\frac{k}{\sigma-1}+\frac{\theta}{\sigma(1-\theta)-1}>\frac{k}{\sigma-1} .
$$

It thus follows that $\left.\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}\right|_{N_{E}=N_{E}^{m}}>0$.
Summing up, under CES preferences, we have shown that $\left.\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}\right|_{N_{E}=N_{E}^{m}}>0$ holds regardless of parameter values, and that this inequality in turn is a sufficient condition for $E X T>0$ at $N_{E}=N_{E}^{m}$.

### 1.2 Conditional Expected Profit

Since the sign of $\left.\frac{d}{d N_{E}} N_{E} \cdot \bar{\pi}\right|_{N_{E}=N_{E}^{m}}$ seems to play an important role in both models, we explore further why the two models show different implications. We rewrite aggregate profit as $N \cdot \bar{\pi}_{c}$ where $N$ refers to the number of operating firms and $\bar{\pi}_{c}$ refers to expected profit of a firm conditional on its survival. ${ }^{3}$ In both models, we can separate two channels through which additional entry affects aggregate profit:

$$
\begin{equation*}
\frac{d}{d N_{E}} N \cdot \bar{\pi}_{c}=\frac{d N}{d N_{E}} \bar{\pi}_{c}+N \frac{d \bar{\pi}_{c}}{d N_{E}} . \tag{8}
\end{equation*}
$$

In the MO model, we find that two opposing forces exist at $N_{E}=N_{E}^{m}:$ i) using (3) and (5), additional entry raises the number of operating firms, $\frac{d N}{d N_{E}} \bar{\pi}_{c}>0$; and ii) using (4) and (5), the surviving firms have lower profit due to the higher competition, $N \frac{d \bar{\pi}_{c}}{d N_{E}}<0$. However, in the CES model, the second channel is absent since the conditional expected profit is constant, $\bar{\pi}_{c}=(\sigma-1) f_{D} /(1+k-\sigma) .{ }^{4}$ This feature of constant conditional expected profit provides some additional insight into why the CES model shows EXT $>0$ regardless of parameter values.

[^2]
### 1.3 Role of Endogenous Mark-up

We now argue that constant expected profit conditional on survival can be explained by the role of constant markup. To make this argument, we follow Mrazova et al (2017) and consider the CREMR (Constant Revenue Elasticity of Marginal Revenue) family of demand functions defined as

$$
p^{c r}(q)=\frac{\beta}{q}(q-\psi)^{\frac{\sigma-1}{\sigma}}
$$

where $\beta>0, \sigma>1$ and $q>\psi \sigma$. As Mrazova et al (2017) observe, this family includes CES demand as a special case: when $\psi=0$, the elasticity of demand is constant and equal to $\sigma$. We focus here on $\psi \geq 0$.

The associated profit-maximization problem is

$$
\pi^{c r}(\varphi)=\max \left\{\max _{q}\left[\left(p^{c r}(q)-\frac{1}{\varphi}\right) q-f_{D}\right], 0\right\}
$$

where $f_{D}>0$ and the firm's productivity is $\varphi$. We assume that $\varphi$ follows a Pareto distribution with shape parameter $k$ (i.e. $G(\varphi)=1-\varphi^{-k}$ ) where $1+k-\sigma>0$. We denote the markup of a firm with productivity $\varphi$ as

$$
\mu(\varphi) \equiv \frac{p^{c r}\left(q^{c r}(\varphi)\right)-\frac{1}{\varphi}}{p^{c r}\left(q^{c r}(\varphi)\right)}=\frac{1}{\sigma}-\frac{\psi \varphi^{-\sigma}}{B}
$$

where $B \equiv \beta^{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}$ and $q^{c r}(\varphi)=\left(\frac{\sigma-1}{\sigma} \beta \cdot \varphi\right)^{\sigma}+\psi$ refers to the equilibrium output level of a firm with productivity $\varphi$. If $\psi>0$, then we have that a more efficient firm charges a higher markup

$$
\begin{equation*}
\mu^{\prime}(\varphi)=\psi \frac{\sigma}{B} \varphi^{-\sigma-1}>0 \tag{9}
\end{equation*}
$$

as in the MO setup. By contrast, if $\psi=0$, then the markup is constant with respect to productivity as in the CES model.

We calculate the conditional expected profit with CREMR preferences as follows

$$
\begin{align*}
\bar{\pi}_{c}^{c r} & =\int_{\varphi^{*}}^{\infty} \pi^{c r}(\varphi) d G\left(\varphi \mid \varphi>\varphi^{*}\right)=B \int_{\varphi^{*}}^{\infty} \mu(\varphi) \varphi^{\sigma-1} d G\left(\varphi \mid \varphi>\varphi^{*}\right)-f_{D} \\
& =\frac{f_{D}(\sigma-1)}{1-\sigma+k}+\frac{k \cdot B}{1-\sigma+k}\left(\varphi^{*}\right)^{k} \int_{\varphi^{*}}^{\infty} \mu^{\prime}(\varphi) \cdot \varphi^{\sigma-k-1} d \varphi \tag{10}
\end{align*}
$$

where the third equality holds by integration by parts and the ZCP condition

$$
\pi^{c r}\left(\varphi^{*}\right)=0 \Rightarrow \mu\left(\varphi^{*}\right) \cdot\left(\varphi^{*}\right)^{\sigma-1}=\frac{f_{D}}{B}
$$

The first term in (10) coincides with the conditional expected profit under CES preferences. The second term in (10) shows the role of endogenous markups, a missing channel under CES preferences.

If $\psi=0$ (i.e. $\mu^{\prime}(\varphi)=0$ ), then it is clear from (10) that $\bar{\pi}_{c}^{c r}$ is constant with respect to the critical productivity cutoff level, $\varphi^{*}$, as in the CES model. By contrast, if $\psi>0$ (i.e. $\left.\mu^{\prime}(\varphi)>0\right)$, then $\bar{\pi}_{c}^{c r}$ decreases with $\varphi^{*}$. To establish this latter point, we substitute (9) into (10) and find that

$$
\bar{\pi}_{c}^{c r}=\frac{f_{D}(\sigma-1)}{1-\sigma+k}+\frac{\psi \cdot k \cdot \sigma}{(1-\sigma+k)(k+1)} \frac{1}{\varphi^{*}}
$$

from which it directly follows that, for $\psi>0$,

$$
\frac{d \bar{\pi}_{c}^{c r}}{d \varphi^{*}}=-\frac{\psi \cdot k \cdot \sigma}{(1-\sigma+k)(k+1)} \frac{1}{\left(\varphi^{*}\right)^{2}}<0 .
$$

If we were to embed this analysis into a model of monopolistic competition and assume that more entry generates fiercer competition, $\frac{d \varphi^{*}}{d N_{E}}>0$, as is standard, then for $\psi>0$ the expected profit conditional on survival would decrease with additional entry, $\frac{d \bar{\tau}_{c} c^{r}}{d N_{E}}<0$, as in the MO model. For $\psi=0$, by contrast, the expected profit conditional on survival would be constant with respect to additional entry, $\frac{d \bar{\pi}_{c}^{c r}}{d N_{E}}=0$, as in the CES setup. From this perspective, one component of the business-stealing effect in (8) is eliminated under CES preferences alone among all preferences in the CREMR family. This feature relates to our discussion in the preceding subsection and provides some reinforcing insight into why the CES model delivers $E X T>0$ regardless of parameter values.

## 2 Numerical Assessment: Quasi-concavity of $U(\chi, \chi)$

In this section, we provide numerical evidence for quasi-concavity of $U(\chi, \chi)$, as assumed in Propositions 10-12. However, it is not trivial to numerically test quasi-concavity. ${ }^{5}$ Hence, we first provide a proposition to guide our numerical analysis. For notational convenience, we will write $U(\chi, \chi)$ as $U(\chi)$.
Definition $U$ is quasi-concave on $\chi \in\left[\chi_{L}, \chi_{U}\right] \Longleftrightarrow$ For any $\chi_{1}, \chi_{2} \in\left[\chi_{L}, \chi_{H}\right], U\left(\chi_{1}\right) \leq$ $U\left(\chi_{2}\right)$ implies
$U\left(t \cdot \chi_{1}+(1-t) \chi_{2}\right) \geq U\left(\chi_{1}\right)$ for $t \in[0,1]$.

[^3]Proposition $1 U$ is quasi-concave over $\chi \in\left[\chi_{L}, \chi_{U}\right]$ if the following conditions are satisfied:

1. There exists unique $\chi^{*} \in\left(\chi_{L}, \chi_{H}\right)$ such that $U^{\prime}\left(\chi^{*}\right)=0$,
2. $U^{\prime}\left(\chi_{L}\right)>0$,
3. $U^{\prime}\left(\chi_{U}\right)<0$,
4. $U^{\prime}(\chi)$ is continuous over $\chi \in\left[\chi_{L}, \chi_{U}\right]$.

Proof. Note that the above four conditions imply:

$$
\begin{align*}
U^{\prime}(\chi) & >0 \text { for } \chi<\chi^{*}  \tag{11}\\
U^{\prime}(\chi) & <0 \text { for } \chi>\chi^{*}
\end{align*}
$$

Otherwise, $\chi^{*}$ is not unique. Now we check the definition of quasi-concave using (11). We pick some $\chi_{1}, \chi_{2} \in\left[\chi_{L}, \chi_{U}\right]$. Without loss of generality, we assume $U\left(\chi_{1}\right)<U\left(\chi_{2}\right)$. Under the given assumptions, we consider the four cases: 1. $\chi_{1}, \chi_{2}>\chi^{*}, 2$. $\chi_{1}, \chi_{2}<\chi^{*}, 3$. $\chi_{1}<\chi^{*}<\chi_{2}, 4 . \chi_{2}<\chi^{*}<\chi_{1}$.

In case $1, \chi_{2}<\chi_{1}$ by (11). For any $\chi$ such that $\chi_{2}<\chi<\chi_{1}, U(\chi)>U\left(\chi_{1}\right)$ holds by (11).

In case $2, \chi_{2}>\chi_{1}$ by (11). For any $\chi$ such that $\chi_{1}<\chi<\chi_{2}, U(\chi)>U\left(\chi_{1}\right)$ holds by (11).

In case 3, we consider two sub-cases.
i) For $\chi \in\left[\chi_{1}, \chi^{*}\right], U(\chi)$ is increasing with $\chi$ by (11). Therefore, $U\left(\chi_{1}\right)<U(\chi)$.
ii) For $\chi \in\left[\chi^{*}, \chi_{2}\right], U(\chi)$ is decreasing with $\chi$ by (11). Therefore, $U\left(\chi_{2}\right)<U(\chi)$. By the given assumptions, $U\left(\chi_{1}\right)<U\left(\chi_{2}\right)<U(\chi)$.

In case 4 , we can show $U\left(\chi_{1}\right)<U(\chi)$ for any $\chi \in\left[\chi_{2}, \chi_{1}\right]$ by the same logic as in case 3.

We numerically confirm quasi-concavity of $U(\chi)$ by checking the four conditions in Proposition 1. We start with the benchmark case in the draft as $\alpha=2, c_{M}=1, k=1.1, f_{e}=0.1$, $\tau=1.1$, and $\gamma=\eta=1$. Then we vary the values of individual parameters $k, \tau, \alpha$, and $\gamma$ one by one. ${ }^{6}$ For the range of $\chi$, the lower bound is determined as $0.95\left(=\tau^{-1 / 2}\right)$ following footnote 16 in the main text, and its upper bound is determined as $10 .{ }^{7}$

[^4]For example, Figure 3 illustrates $U^{\prime}(\chi)$ for $\chi \in[0.95,10]$ with the benchmark parameters and shows that the four conditions in Proposition 1 are satisfied. First, there exists unique $\chi^{*}=1.03$ such that $U^{\prime}\left(\chi^{*}\right)=0$. Second, $U^{\prime}(0.95)>0$. Third, $U^{\prime}(10)<0$. Lastly, $U^{\prime}(\chi)$ is continuous on $\chi \in[0.95,10]$. We automatize this graphical analysis for the remaining cases using Mathematica. ${ }^{8}$ We build a vector of $\chi$ ranging from 0.95 to 10 with the length of 100 (i.e. size of each grid is approximately 0.09 ). Then, we numerically check whether this $\chi$-vector satisfies these four conditions for different parameter sets. With the $\chi$-vector, we numerically check whether $U^{\prime}(0.95)>0$ and $U^{\prime}(10)<0$ hold, and we also confirm the sign of $U^{\prime}(\chi)$ is reversed for only one time to conclude quasi-concavity. ${ }^{9}$ If one of the four sufficient conditions fail, we say that quasi-concavity "may fail."

In the main text, $\alpha>c_{D}^{F T}$ is assumed to guarantee $N_{E}^{l}>0$, and many of our main findings depend on the sign of $\alpha-2 c_{D}^{F T}$. Hence, our numerical analysis only considers parameter sets satisfying $\alpha>c_{D}^{F T}$ and studies whether the quasi-concavity depends on the sign of $\alpha-2 c_{D}^{F T}$. Here is the summary of our numerical findings:

1. For variations of $\tau$ and $k$, the quasi-concavity of $U(\chi)$ seems very robust. We do not find any such parameter variations for which quasi-concavity fails, regardless the sign of $\alpha-2 c_{D}^{F T}$.
2. For variations of demand parameters $\alpha$ and $\gamma$, the relative size of $\alpha$ to $c_{D}^{F T}$ plays a role. When $\alpha / c_{D}^{F T}$ is closed to 1 , quasi-concavity of $U(\chi)$ may fail. But when $\alpha / c_{D}^{F T}$ is close to or bigger than 2, quasi-concavity holds. This implies that the quasi-concavity may fail if the marginal utility of differentiated good is too low ( $\alpha$ is too low) or the degree of differentiation within the differentiated sector is too high ( $\gamma$ is too high). But quasi-concavity is robust in neighborhood of a parameter set satisfying $\alpha=2 c_{D}^{F T}$.

## Details of the numerical exercises

## $k$ variation

We vary the value of $k$ while other parameters are fixed at their benchmark values. The value of $k$ varies from 1.01 to $10 .{ }^{10}$ The number of grids is 60 , and thus the size of each grid is approximately 0.15 . Since $\alpha>c_{D}^{F T}$ holds for all $k$ in our sample and $\alpha=2 c_{D}^{F T}$ is satisfied at $k=1.59$, we cover both cases of $\alpha \geq 2 c_{D}^{F T}$ and $\alpha<2 c_{D}^{F T}$. For each value of $k$, we build a

[^5]vector of $U^{\prime}(\chi)$ using the $\chi$-vector we described above and find that the four conditions in Proposition 1 are satisfied.
$\tau$ variation
We vary the value of $\tau$ while other parameters are fixed at their benchmark values. The value of $\tau$ varies from 1 to 10 . The number of grids is 60 , and thus the size of each grid is 0.15. Since $\alpha>c_{D}^{F T}$ holds for all $\tau$ in our sample and $\alpha=2 c_{D}^{F T}$ is satisfied at $\tau=2.97$, we cover both cases of $\alpha \geq 2 c_{D}^{F T}$ and $\alpha<2 c_{D}^{F T}$. For each value of $\tau$, we build a vector of $U^{\prime}(\chi)$ using the $\chi$-vector we described above and find that the four conditions in Proposition 1 are satisfied. ${ }^{11}$
$\alpha$ variation
We vary the value of $\alpha$ while other parameters are fixed at their benchmark values. We vary the value of $\alpha$ from $0.88\left(=c_{D}^{F T}\right)$ to $2.65\left(=3 c_{D}^{F T}\right)$. Since $\alpha \geq 2 c_{D}^{F T}$ is satisfied for $\alpha \geq 1.77$, we cover both cases of $\alpha \geq 2 c_{D}^{F T}$ and $\alpha<2 c_{D}^{F T}$. The number of grids between these two values is 60 and thus the size of grid is approximately 0.03 . For robustness, we also check the cases of $\alpha=4$ and $\alpha=10$. For each value of $\alpha$, we build a vector of $U^{\prime}(\chi)$ using the $\chi$-vector we described above and check the four conditions in Proposition 1. We find that quasi-concavity fails if the value of $\alpha<1.53$ but holds otherwise. Our numerical results thus suggest that quasi-concavity holds in the neighborhood of a parameter set satisfying $\alpha \geq 2 c_{D}^{F T}$ but may fail if the marginal utility from the differentiated good is too low ( $\alpha$ is too low).

## $\gamma$ variation

We vary the value of $\gamma$ while other parameters are fixed at their benchmark values. We vary the value of $\gamma$ from 0.01 to 12.52 (satisfying $\alpha=c_{D}^{F T}$ ). Since $\alpha \geq 2 c_{D}^{F T}$ is satisfied for $\gamma<1.46$, we cover both cases of $\alpha \geq 2 c_{D}^{F T}$ and $\alpha<2 c_{D}^{F T}$. The number of grids between these two values is 60 and thus the size of grid is approximately 0.21 . For each value of $\gamma$, we build a vector of $U^{\prime}(\chi)$ using the $\chi$-vector we defined above and check the four conditions in Proposition 1. We find that quasi-concavity fails if the value of $\gamma \geq 2.51$ but holds otherwise. Our numerical results thus suggest that quasi-concavity holds in the neighborhood of a parameter set satisfying $\alpha \geq 2 c_{D}^{F T}$ but may fail if the degree of differentiation within the differentiated good is too high ( $\gamma$ is too high).

[^6]
## 3 References

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## 4 Figures


(a) $U(\chi)$ at benchmark parameter values.

(b) $U^{\prime \prime}(\chi)$ at benchmark parameter values.

Figure 1: Global maximum and quasi-concavity.


Figure 2: $\chi^{N}$ as $\alpha$ varies from its benchmark value $(\alpha=2)$.


Figure 3: $U^{\prime}(\chi)$ at benchmark parameter values.


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    ${ }^{1}$ When we discuss here the MO model, the notation is understood in the context of that model as examined by Bagwell and Lee (2020).

[^1]:    ${ }^{2}$ When we discuss here the CES model, the notation is understood in the context of that model as examined by Bagwell and Lee (2018).

[^2]:    ${ }^{3}$ We note that $\bar{\pi}_{c}=\bar{\pi} /\left(1-G\left(\varphi^{*}\right)\right)$ in the CES model and $\bar{\pi}_{c}=\bar{\pi} / G\left(c_{D}\right)$ in the MO model, where the notation in each case is understood in the context of the model to which it is applied.
    ${ }^{4}$ The derivation of $\bar{\pi}_{c}$ follows from the expression given above for $\bar{\pi}$ and from the fact that $1-G\left(\varphi^{*}\right)=$ $\left(\varphi^{*}\right)^{-k}$ for the Pareto distribution.

[^3]:    ${ }^{5}$ We cannot simply check the second order conditions since $U(\chi, \chi)$ does not satisfy (global) concavity for some parameter sets. In Figure 1, we replace $U(\chi, \chi)$ with $U(\chi)$ for notational convenience, and illustrate $U(\chi)$ and $U^{\prime \prime}(\chi)$ with the benchmark parameters in the main text as $\alpha=2, c_{M}=1, k=1.1, f_{e}=0.1$, $\tau=1.1$, and $\gamma=\eta=1$. Sub-figure (a) shows that global maximum of $U(\chi)$ is well defined at $\chi=1.03$, but sub-figure (b) shows that concavity fails for $\chi>1.6$.

[^4]:    ${ }^{6}$ Note that variations of $c_{M}$ and $f_{e}$ have equivalent effect as variations of $\alpha$ since they affect the equilibrium through $c_{D}^{l}$. Variations of $\eta$ are not provided either since their impact on the equilibrium may be determined by the relative size of other demand parameters, $\alpha$ and $\gamma$.
    ${ }^{7}$ We conservatively pick this upper bound of $\chi \cdot \chi^{N}$ is bounded above by 3 as $\alpha$ increases from the benchmark case. See Figure 2.

[^5]:    ${ }^{8}$ Mathematica codes are available upon request.
    ${ }^{9}$ There are two potential limitations in our numerical exercise. The last condition of continuity is hard to automatize. Our numerical method does not exclude a case where there exists a tiny interval (within a grid) where the sign of $U^{\prime}(\chi)$ is reversed twice. After drawing a number of figures, we assume that $U(\chi)$ is continuous and our grids are fine enough.
    ${ }^{10}$ We restrict $k>1$ in order to avoid zeros in denominators.

[^6]:    ${ }^{11}$ When we vary the value of $\tau$, the lower bound of $\tau^{-1 / 2}$ should also change. For convenience, we set the lower bound of $\chi$ to 0.8 for every case, instead of 0.95 .

