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Advertising Competition in Retail Markets*

Kyle Bagwell and Gea M. Lee

Abstract

We consider non-price advertising by retail firms that are privately informed as to their respective production costs. We construct an advertising equilibrium in which informed consumers use an advertising search rule whereby they buy from the highest-advertising firm. Consumers are rational in using the advertising search rule since the lowest-cost firm advertises the most and also selects the lowest price. Even though the advertising equilibrium facilitates productive efficiency, we establish conditions under which firms enjoy higher expected profit when advertising is banned. Consumer welfare falls in this case, however. Under free entry, social surplus is higher when advertising is allowed. In addition, we consider a benchmark model of price competition; we provide comparative-statics results with respect to the number of informed consumers, the number of firms and the distribution of costs; and we consider the possibility of sequential search.

KEYWORDS: advertising, regulation, private information, retail markets

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1 Introduction

The typical retail firm sells a large number of products. When a retail firm advertises using traditional media, such as radio, TV, billboards and magazines, the firm may thus be unable to communicate substantial "hard" information to its potential consumers. In particular, a retail firm that advertises using such media may be able to communicate only limited information regarding the prices of the large range of products that it carries. Retail advertising in such media is often thus best described as "non-price" advertising.

The prevalence of non-price advertising in retail markets raises a number of interesting questions. First, is non-price retail advertising consistent with equilibrium behavior? Non-price advertising is costly, so firms use it only if consumers respond. Since non-price advertising may have little or no hard information, however, it is not clear why rational consumers would respond. Second, would banning non-price retail advertising raise or reduce consumer welfare? An answer to this question is necessary in order to evaluate the appropriate public policy toward non-price advertising. Finally, do retail firms have an incentive to encourage regulatory restrictions on non-price advertising? An affirmative answer to this question, for example, would support the view that restrictions on retail advertising may be a form of "regulatory capture."

These questions are of particular interest in light of striking empirical relationships between advertising and prices that have been identified in some retail industries. The classic study is by Benham (1972). Examining transaction prices in the retail eyeglass industry in the U.S. in the 1960s, he reports that prices were higher in states that prohibited all advertising than in states that had no restrictions on advertising; moreover, prices were only slightly higher in states that allowed just non-price advertising than in states that also allowed price advertising. Evidently, the ability to advertise, even if only in a non-price form, is sometimes associated with lower prices. Cady (1976) documents similar relationships in the U.S. retail market for prescription drugs in 1970.[1] At a broad level, this work raises the possibility that retail firms might gain if they were able to limit advertising. An agreement to limit advertising is directly achieved in the presence of a state law that prohibits advertising;

[1] Related findings are reported in other studies which focus on different retail markets, including the markets for gasoline, routine legal services and liquor. See Bagwell (2007, Section 3.2.4) for a survey. Rizzo and Zeckhauser (1992) consider the market for physicians and report evidence that advertising is associated with higher prices (and quality). As they note, however, this market has several novel features: quality is difficult to assess, consumers are highly sensitive to perceived quality, and consumers are relatively insensitive to price (given third-party reimbursement).

alternatively, retail firms might achieve such an agreement by forming a professional association that imposes advertising restrictions on its members.[2]

Bagwell and Ramey (1994a) develop a complete-information model of retail competition with which to interpret Benham's findings. In their model, some consumers can identify the highest-advertising firm, while other consumers do not observe advertising levels. The former (latter) consumers are referred to as informed (uninformed) consumers. Each consumer possesses a downward-sloping demand function and lacks direct information about firms' prices: a consumer observes a firm's price only after choosing to visit that firm. Bagwell and Ramey compare two equilibria. In a random equilibrium, consumers ignore advertising and choose firms at random. Firms do not advertise, and they enjoy symmetric market shares. By contrast, in an advertising equilibrium, the informed consumers go to the firm that advertises the most. Firms then use a symmetric mixed strategy, in which higher advertising choices are paired with greater investments in cost reduction and thus lower prices. Informed consumers are then rational in visiting the highest-advertising retailer, since this retailer also offers the lowest price. For a given number of firms, expected profit is higher in the random equilibrium, because advertising expenses are thereby avoided. Bagwell and Ramey then include an initial entry stage so as to endogenize the number of firms. Comparing free-entry advertising and random equilibria, they show that, in the advertising equilibrium, the market is more concentrated, prices are lower, and social welfare is higher. If the random equilibrium is associated with a setting in which advertising is banned, these findings are broadly consistent with the empirical patterns that Benham reports.

In this paper, we modify the Bagwell-Ramey model by assuming that each firm has private information about its exogenous costs of production. Specifically, we consider a model with a continuum of possible cost types, where cost types are iid across firms. We then characterize an advertising equilibrium, in which firms use pure strategies and lower-cost firms advertise more than do higher-cost firms. In the incomplete-information model that we analyze here, informed consumers are rational in using the advertising search rule, since the lowest-cost firm advertises the most and also selects the lowest price. We also compare the advertising equilibrium with the random equilibrium in which no firm advertises, both when the number of firms is fixed and when the number of firms is endogenous. We thereby consider the short- and long-run implications of advertising competition for consumer surplus and

[2]The FTC has argued that anti-competitive effects may be associated with price and non-price advertising restrictions imposed by a professional association. See *California Dental Association v. Federal Trade Commission* (1999). A further possibility is that firms interact repeatedly and achieve a self-enforcing agreement to limit advertising. We consider this latter possibility in our companion paper (Bagwell and Lee, 2010).

firm profit. In addition, we analyze a benchmark model of price competition; we provide comparative-statics results with respect to the number of informed consumers, the number of firms and the distribution of costs; and we consider the possibility of sequential search.

At a broad level, we can interpret our model as a "purified" version of the Bagwell-Ramey model. In fact, our analysis explores two notions of purification. Our first notion reflects the traditional view of purification under which the mixed-strategy equilibrium of a complete-information game can be interpreted in terms of the pure-strategy equilibrium of a nearby incomplete-information game. For the special case in which the support of possible costs is small, we report that the distribution of advertising levels in the pure-strategy advertising equilibrium of the incomplete-information game is approximately the same as the distribution of advertising levels in the mixed-strategy advertising equilibrium of the associated complete-information game.[3] Correspondingly, we show that the main predictions of Bagwell and Ramey directly extend to the private-information setting, if the support of possible costs is sufficiently small. Under our second notion of purification, we consider whether the main predictions are robust across equilibria for the complete- and incomplete-information games, even when these games are not nearby to one another. For the general case in which the support of possible cost types may be large, we establish conditions under which the main predictions derived in the complete-information game emerge as well in the incomplete-information game. This second notion of purification sometimes requires additional structure on the distribution and demand functions.

We develop our results while allowing for the general case of a large support of possible cost types. As mentioned, we establish that an advertising equilibrium exists, in which lower-cost firms advertise more and price lower than do higher-cost firms. We then establish four additional results. First, for any given number of firms, expected consumer surplus is higher in the advertising equilibrium than in the random equilibrium. Intuitively, in our model, the distribution of posted prices is not affected by firms' ability to engage in non-price advertising; thus, uninformed consumers are indifferent between the advertising and random equilibria.[4] At the same time, non-price advertising enables informed consumers to locate the firm with the lowest cost and thus

[3]Bagwell and Ramey (1992, 1994a) report a similar finding when the cost of advertising is private information and varies slightly across firms. We also note that the associated complete-information game that we consider here is closely related to but distinct from that considered by Bagwell and Ramey. An important difference is that we do not allow scale economies achieved through endogenous investments in cost reduction.

[4]The prediction of an invariant distribution for posted prices is special to our model. It holds since we assume that costs are exogenous and exhibit constant returns to scale.

the lowest price; hence, the average *transaction* price is lower when non-price advertising is allowed. This first result is consistent with Benham's observation and holds independently of any assumption about the distribution of types or the elasticity of demand.

Second, for any given number of firms, if the distribution of types is log-concave and demand is sufficiently inelastic, then firms earn higher expected profit in the random than in the advertising equilibrium. When the number of firms is fixed, therefore, firms and informed consumers rank the advertising and random equilibria in opposite fashion. Intuitively, in comparison to the advertising equilibrium, the random equilibrium offers firms an advantage as well as a disadvantage. The advantage of the random equilibrium is that firms avoid the expenses that are associated with advertising competition. The disadvantage of the random equilibrium is that it assigns the same market share to all firms, whereas the advertising equilibrium achieves greater productive efficiency by assigning higher expected market shares to lower-cost firms. Building on techniques used by Athey et al. (2004) in their study of price collusion, we show that the random equilibrium's advantage overwhelms its disadvantage, if the distribution of types is log-concave and demand is sufficiently inelastic.[5] Our second result thus gives conditions under which firms have an incentive, at least in the short run, to seek regulatory restrictions on advertising.

The third result follows directly from the second: when the number of firms is endogenously determined by a free-entry condition, if the distribution of types is log-concave and demand is sufficiently inelastic, more firms enter when the random equilibrium is anticipated. Thus, advertising competition leads to a more concentrated market structure.

Finally, our fourth result concerns social welfare when the number of firms is endogenously determined. Under free entry, firms are indifferent between the advertising and random equilibria, since in each setting the number of firms adjusts until each firm earns zero profit. Whether social surplus is higher in the advertising or random equilibrium thus hinges entirely on the consumer surplus that is expected in each equilibrium. In fact, our first result may be easily extended to cover settings in which different numbers of firms enter in the advertising and random equilibria, respectively. Without making any assumption on the distribution of types or the elasticity of demand, we show that social surplus is weakly higher in the advertising than in the random

[5] Athey et al. (2004) establish related conditions under which optimal collusion for sellers in a first-price procurement auction entails pooling at the buyer's reservation value. See also McAfee and McMillan (1992) for a related theory of identical bidding among collusive bidders in a first-price auction. Our model of advertising, by contrast, is analogous to an all-pay auction.

equilibrium when the number of firms is endogenous; moreover, social surplus is strictly higher in the advertising equilibrium if at least two firms enter in that equilibrium.

As noted, Bagwell and Ramey (1994a) also construct an advertising equilibrium, find that firms earn greater short-run profit in the random equilibrium, and find that the advertising equilibrium leads to a more concentrated market structure and greater social welfare in the long run. Our second, third and fourth results above thus establish a sense in which Bagwell and Ramey's main predictions extend to the private-information setting. We emphasize, however, that the second and third results now employ additional assumptions on the distribution of types and the elasticity of demand. These additional assumptions arise because of the productive-efficiency advantage that the advertising equilibrium offers in our incomplete-information setting.

We also compare the advertising equilibrium with another benchmark. In particular, we follow Varian (1980) and suppose that informed consumers observe prices and buy from the lowest-priced firm while uninformed consumers pick a firm at random. Following Spulber (1995) and Bagwell and Wolinsky (2002), we modify Varian's model and allow that firms are privately informed about their production costs.[6] Let us refer to the (symmetric) equilibrium of this game as the pricing equilibrium. For any fixed number of firms, we show that firms earn higher expected profit in the pricing equilibrium than in the advertising equilibrium. This is perhaps surprising, since competition in advertising is sometimes argued to be less aggressive than competition in prices. As we discuss, the key intuition is that price competition induces greater in-store demand from consumers and thus elevates the size of expected information rents for firms.

With an analysis of the benchmark model in place, we are able to offer a more complete comparison across different advertising regulatory regimes. Provided that the market always has at least two firms, our results indicate that the average transaction price is lowest in the pricing equilibrium, somewhat higher in the advertising equilibrium, and higher yet in the random equilibrium. Likewise, when the number of firms is endogenous, social welfare is highest in the pricing equilibrium, somewhat lower in the advertising equilibrium, and lower yet in the random equilibrium. If we associate the pricing equilibrium with a setting in which price advertising is allowed, the advertising equilibrium with a setting in which only non-price advertising is allowed, and the random equilibrium with a setting in which all advertising is banned, then our results are broadly consistent with Benham's findings.

[6] Bagwell and Wolinsky follow Varian and assume that each consumer possesses an inelastic demand function. We generalize this analysis slightly and allow for downward-sloping demand functions. Spulber considers a related model in which all consumers are informed.

We next examine the comparative-statics properties of the advertising equilibrium. When the number of informed consumers is increased, advertising increases for all types other than the highest type. Intuitively, firms advertise more heavily when the "prize" from advertising the most is increased.[7] Interestingly, the effect on advertising of an increase in the number of firms depends on a firm's cost type: lower-cost firms compete more aggressively and increase their advertising, but higher-cost firms perceive a reduced chance of winning the informed consumers and advertise less. An implication is that the support of observed advertising levels may be larger in markets with a greater number of firms. We also find that, for all types other than the lowest type, if the number of firms is sufficiently large, the equilibrium level of advertising is negligible. Finally, building on Hopkins and Korneinko's (2007) analysis of all-pay auctions, we show that, if the cost distribution shifts to make lower-cost types more likely in the sense of the monotone likelihood ratio order, then lower-cost firms advertise more while higher-cost firms become discouraged and advertise less.[8]

We next modify the game to allow for sequential search. If demand is sufficiently inelastic or if the cost of sequential search is sufficiently high, then our results are maintained without modification. If these conditions do not hold, however, then higher-cost firms must "limit price" (i.e., price below their monopoly prices), in order to deter sequential search.[9] An advertising equilibrium then continues to exist, if the support of possible cost types is not too large and the number of informed consumers is not too great. In this equilibrium, informed consumers use observed advertising behavior to locate the lowest price, and limit pricing by higher-cost firms ensures that uninformed consumers do not gain from actually undertaking sequential search. We argue as well that the possibility of sequential search may even strengthen our results, by raising the relative profitability of the random equilibrium.

Other authors have also considered the effects of advertising regulations on the conduct of firms that are privately informed as to their respective costs of production. For example, Peters (1984) and LeBlanc (1998) consider the effects of a prohibition on price advertising in models with a fixed number of firms where each firm is privately informed about its production cost. By contrast, here we emphasize the effects of a prohibition on non-price advertising,

[7] Bagwell and Ramey (1994a) report a related finding for their complete-information model.

[8] This finding contrasts interestingly with the standard monotone comparative statics result for first-price auctions. See, for example, Athey (2002) and Lebrun (1998).

[9] Our analysis here builds on Reinganum (1979) and Bagwell and Ramey (1996). Reinganum examines sequential search in a model with privately informed firms that are not allowed to advertise. Bagwell and Ramey examine sequential search when advertising is allowed but private information is absent.

and we utilize a free-entry condition to endogenize the number of firms. Also, Bagwell and Ramey (1994b) consider a duopoly model in which one firm has private information as to whether its costs are high or low. For settings in which non-price advertising is legal, they show that non-price advertising may be used to signal low costs and thus low prices. In the current paper, by contrast, we adopt a continuum-type model, assume that all firms are privately informed as to their costs, endogenize the number of firms using a free-entry condition, and report (non-monotone) comparative statics results.

The paper is organized as follows. Section 2 defines the advertising game and contains our main findings. In Section 3, we present a benchmark model of price competition. Our comparative-statics results are found in Section 4. We consider the possibility of sequential search in Section 5. Section 6 concludes. Remaining proofs are in the Appendix.

2 The Advertising Game

In this section, we define an advertising game in which a fixed number of firms compete through advertising for market share. Firms are privately informed as to their respective costs, and each firm's advertising choice may signal its costs, and thus its price, to those consumers who are informed of advertising activities. We establish the existence of an advertising equilibrium, in which informed consumers visit the firm with the highest level of advertising. We compare the expected profit earned by firms in the advertising equilibrium with that which they earn in a random equilibrium, wherein all consumers pick firms at random. We also compare expected consumer surplus in the advertising and random equilibria. We then endogenize the number of firms and compare market concentration and social welfare across the two equilibria.

2.1 The Model

We assume $N \geq 2$ ex ante identical firms compete for sales in a homogeneous-good market. Each firm i is privately informed of its unit cost level θ_i. Cost levels are iid across firms, and cost type θ_i is drawn from the support $[\underline{\theta}, \overline{\theta}]$ according to the twice-continuously differentiable distribution function, $F(\theta)$, where $\overline{\theta} > \underline{\theta} \geq 0$. The density $f(\theta) \equiv F'(\theta)$ is positive on $[\underline{\theta}, \overline{\theta}]$. As discussed in further detail below, after firms learn their respective cost types, they simultaneously choose their prices and levels of advertising. Following Bagwell and Ramey (1994a), we assume that advertising is a dissipative expense that does not directly affect consumer demand.

The firms face a unit mass of consumers, where each consumer possesses a twice-continuously differentiable demand function $D(p)$ that satisfies $D(p) >$

$0 > D'(p)$ over the relevant range of prices p. We assume that price information cannot be directly communicated in this market; thus, consumers cannot observe prices prior to selecting a firm to visit and from which to purchase. Some consumers, however, do observe advertising activity prior to picking a firm. In particular, a fraction I of consumers are informed, in the sense that they observe firms' advertising expenses.[10] Given this information, informed consumers form beliefs as to firms' cost types and determine a visitation (search) strategy. For example, informed consumers may use an *advertising search rule*, whereby a consumer goes to the firm that advertises the most.[11] The remaining fraction $U = 1 - I$ are uninformed. Uninformed consumers do not observe advertising expenditures and thus may adopt a *random search rule*, whereby a consumer randomly chooses which firm to visit.

The interaction between firms and consumers is represented by the following *advertising game*: (i) firms learn their own cost types, (ii) firms make simultaneous choices of advertising and price, and (iii) given any advertising information, each consumer chooses a firm to visit, observes that firm's price and makes desired purchases given this price. Note that a consumer is assumed to visit only one firm.[12] As we explain below, this assumption simplifies our analysis, by ensuring that each firm chooses the monopoly price that is associated with its cost type for any sales that it makes.

We are interested in Perfect Bayesian Equilibria for the advertising game. Before formally defining our equilibrium concept, however, we impose two requirements. First, we focus on equilibria in which consumers do not condition their visitation decisions on firms' "names." Thus, uninformed consumers must use the random search rule; furthermore, for any given vector of advertising levels, informed consumers must treat symmetrically any two firms which advertise at the same level. Informed consumers hence satisfy this restriction if they use the random or advertising search rules.[13] Second, we focus on equilibria in which firms use symmetric advertising and pricing strategies. We note that the random search rule is indeed an optimal search strategy for uninformed consumers, when firms use symmetric pricing strategies.

Given symmetry, we can define a pure advertising strategy for firm i as a function $A(\theta_i)$ that maps from the set of cost types $[\underline{\theta}, \overline{\theta}]$ to the set of possible

[10] We assume that informed consumers observe advertising levels for simplicity. All of our results hold under the assumption that informed consumers observe only the identity of the highest-advertising firm(s).

[11] If several firms tie for the highest advertising level, then the informed consumers divide up evenly over those firms.

[12] We extend the analysis to allow for sequential search in Section 5.

[13] Under the random search rule, consumers randomly pick a firm from the set of *all* firms. Similarly, under the advertising search rule, informed consumers randomize over *all* firms that advertise at the highest level (if more than one such firm exists).

advertising expenditures $\mathbb{R}_+ \equiv [0, \infty)$. Consider now firms other than firm i. Let $\mathbf{A}(\boldsymbol{\theta}_{-i})$ denote the vector of these firms' selections when their cost types are given by the $(N-1)$-tuple $\boldsymbol{\theta}_{-i}$. Given the search rule used by informed consumers, the market share for firm i is determined by the vector of advertising levels selected by firm i and its rivals. The market share for firm i thus maps from $\mathbb{R}_+^N$ to $[0,1]$ and in equilibrium may be represented as $m(A(\theta_i), \mathbf{A}(\boldsymbol{\theta}_{-i}))$.[14] Note that, under our first requirement above, firm i's market share is not indexed by i and thus does not depend on firm i's name. If firm i has cost type θ_i, its interim-stage market share under these strategies is $E_{\boldsymbol{\theta}_{-i}}[m(A(\theta_i), \mathbf{A}(\boldsymbol{\theta}_{-i}))]$.

We next describe firm i's expected profit. Firm i's net revenue is $r(p, \theta_i) \equiv (p - \theta_i)D(p)$ (excluding advertising expenses) when it has cost type θ_i, sets the price p and captures the entire unit mass of consumers. We assume that $r(p, \theta_i)$ is strictly concave in p with a unique maximizer $p(\theta_i) = \arg\max_p r(p, \theta_i)$. It follows that the monopoly price $p(\theta_i)$ strictly increases in θ_i whereas $r(p(\theta_i), \theta_i)$ strictly decreases in θ_i. We further assume that the price at the top has a positive margin: $p(\overline{\theta}) > \overline{\theta}$. Given our requirement that all consumers, and particularly uninformed consumers, treat all firms symmetrically, we know that all firms receive positive expected market share. Therefore, in the equilibria upon which we focus, each firm must select the monopoly price given its cost type. Embedding the monopoly price into the revenue function, we may define the interim-stage net revenue for firm i by $R(A(\theta_i), \theta_i; A) \equiv r(p(\theta_i), \theta_i) E_{\boldsymbol{\theta}_{-i}}[m(A(\theta_i), \mathbf{A}(\boldsymbol{\theta}_{-i}))]$. Thus, $R(A(\theta_i), \theta_i; A)$ is the interim-stage net revenue for firm i when firm i has cost type θ_i, advertises at level $A(\theta_i)$, and anticipates that other firms also use the advertising function A to determine their respective advertising levels upon observing their cost types. Firm i's expected revenue is $E_{\theta_i} R(A_i(\theta_i), \theta_i; A)$, and firm i's expected profit is thus $E_{\theta_i}[R(A(\theta_i), \theta_i; A) - A(\theta_i)]$.

Given our embedded requirements, we may now define an *equilibrium* as an advertising strategy A, a belief function and search rules for consumers that collectively satisfy three remaining conditions. First, given the market share function, m, that is induced by consumers' search rules, the advertising strategy A is such that, for all i and θ_i, $A(\theta_i) \in \arg\max_{a_i} [R(a_i, \theta_i; A) - a_i]$.[15] Second, given an observed advertising level a_i by firm i, informed consumers

[14] For example, if all consumers use the random search rule, then $m(A(\theta_i), \mathbf{A}(\boldsymbol{\theta}_{-i})) = 1/N$. If instead the uninformed consumers use the random search rule while the informed consumers use the advertising search rule, then $m(A(\theta_i), \mathbf{A}(\boldsymbol{\theta}_{-i})) = I + U/N$ if $A(\theta_i) > A(\theta_j)$ for all $j \neq i$, while $m(A(\theta_i), \mathbf{A}(\boldsymbol{\theta}_{-i})) = U/N$ if $A(\theta_i) < A(\theta_j)$ for some $j \neq i$. For this latter set of consumer search strategies, if firm i ties with $k-1$ other firms for the highest advertising level, then $m(A(\theta_i), \mathbf{A}(\boldsymbol{\theta}_{-i})) = I/k + U/N$.

[15] Notice that $A(\theta_i)$ must be an optimal choice for firm i with cost type θ_i in comparison to advertising deviations that are "on-schedule" (i.e., a_i such that $a_i = A(\theta) \neq A(\theta_i)$

use Bayes' Rule whenever possible (i.e., whenever $a_i = A(\theta_i)$ for some $\theta_i \in [\underline{\theta}, \overline{\theta}]$) in forming their beliefs as to firm i's cost type θ_i and thus price $p(\theta_i)$. Third, for any observed vector of advertising levels $[a_1, ..., a_N] \in \mathbb{R}_+^N$, given their beliefs, informed consumers' search rule directs them to the firm or firms with the lowest expected price.

We may now simplify our notation for equilibrium variables somewhat further. We may define firm i's interim-stage market share as $M(A(\theta_i); A) \equiv E_{\boldsymbol{\theta}_{-i}}\left[m(A(\theta_i), \mathbf{A}(\boldsymbol{\theta}_{-i}))\right]$. Similarly, we can define firm i's interim-stage profit and net revenue as follows:

$$\begin{aligned} \Pi(A(\theta_i), \theta_i; A) &\equiv r(p(\theta_i), \theta_i) M(A(\theta_i); A) - A(\theta_i). \\ &\equiv R(A(\theta_i), \theta_i; A) - A(\theta_i). \end{aligned}$$

We note that the interim-stage profit function satisfies a single-crossing property: higher types are less willing to engage in higher advertising to increase expected market share.[16] For here and later use, we now write interim-stage profit in direct-form notation, ignoring subscript i: if a firm of type θ picks an advertising level $A(\widehat{\theta})$ when its rivals employ the strategy A, then we define $\Pi(\widehat{\theta}, \theta; A) \equiv \Pi(A(\widehat{\theta}), \theta; A)$, $M(\widehat{\theta}; A) \equiv M(A(\widehat{\theta}); A)$ and $R(\widehat{\theta}, \theta; A) \equiv R(A(\widehat{\theta}), \theta; A)$.

We are primarily interested in two kinds of equilibria. In an *advertising equilibrium*, informed consumers use the advertising search rule. Since $p(\theta)$ is strictly increasing, such equilibria can exist only if the advertising schedule A is nonincreasing, so that higher-advertising firms have lower costs and thus offer lower prices. In a *random equilibrium*, informed consumers ignore advertising and use the random search rule. A random equilibrium thus can exist only if firms maximize expected profits and do not advertise (i.e., $A \equiv 0$).

2.2 Advertising Equilibrium

In an advertising equilibrium, informed consumers use the advertising search rule while uninformed consumers are randomly distributed across all N firms. We now report the following existence and uniqueness result.

Proposition 1. *There exists a unique advertising equilibrium, and in this equilibrium $A(\theta)$ is strictly decreasing and differentiable and satisfies $A(\overline{\theta}) = 0$.*

for some $\theta \in [\underline{\theta}, \overline{\theta}]$) as well as "off-schedule" (i.e., a_i such that $a_i \neq A(\theta)$ for any $\theta \in [\underline{\theta}, \overline{\theta}]$).

[16] When a firm increases its advertising level, it may confront a trade off between the larger advertising expense, a_i, and the consequent higher expected market share, $M(a_i; A)$. When the interim-stage profit is held constant, the slope $da_i/dM(a_i; A)$ is given by $r(p(\theta_i), \theta_i)$, which is strictly decreasing in θ_i.

Proof. We first derive the necessary features of an advertising equilibrium. The following incentive constraints are necessary: For any $\theta \in [\underline{\theta}, \overline{\theta}]$ and any $\widehat{\theta} \in [\underline{\theta}, \overline{\theta}]$,

$$\begin{aligned} r(p(\widehat{\theta}), \widehat{\theta})M(\widehat{\theta}; A) - A(\widehat{\theta}) &\geq r(p(\widehat{\theta}), \widehat{\theta})M(\theta; A) - A(\theta) \\ r(p(\theta), \theta)M(\theta; A) - A(\theta) &\geq r(p(\theta), \theta)M(\widehat{\theta}; A) - A(\widehat{\theta}). \end{aligned}$$

Adding yields $[r(p(\widehat{\theta}), \widehat{\theta}) - r(p(\theta), \theta)][M(\widehat{\theta}; A) - M(\theta; A)] \geq 0$. Since $r(p(\theta), \theta)$ is strictly decreasing in θ, it is thus necessary that $M(\theta; A)$ is nonincreasing. It follows from the incentive constraints that $A(\theta)$ is nonincreasing. Further, given the advertising search rule, it is clear that $A(\theta)$ cannot be constant over any interval of types: by increasing its advertising an infinitesimal amount, a firm with a type on this interval would experience a discrete gain in its expected market share. Thus, $A(\theta)$ must be strictly decreasing, and consequently it is necessary that $M(\theta; A) = \frac{U}{N} + [1 - F(\theta)]^{N-1}I$. It follows that $M(\overline{\theta}; A) = \frac{U}{N}$. A firm with type $\overline{\theta}$ thus cannot be deterred from selecting zero advertising, and hence $A(\overline{\theta}) = 0$ is also necessary.

We next establish that $A(\theta)$ must be differentiable, and we also derive the necessary expression for $A'(\theta)$. Consider any $\widehat{\theta} < \theta$. Rearranging the incentive constraints presented above, we find that

$$\frac{r(p(\theta), \theta)[M(\widehat{\theta}; A) - M(\theta; A)]}{\widehat{\theta} - \theta} \geq \frac{A(\widehat{\theta}) - A(\theta)}{\widehat{\theta} - \theta} \geq \frac{r(p(\widehat{\theta}), \widehat{\theta})[M(\widehat{\theta}; A) - M(\theta; A)]}{\widehat{\theta} - \theta}.$$

Similarly, consider any $\widehat{\theta} > \theta$. The incentive constraints may now be rearranged to yield

$$\frac{r(p(\theta), \theta)[M(\widehat{\theta}; A) - M(\theta; A)]}{\widehat{\theta} - \theta} \leq \frac{A(\widehat{\theta}) - A(\theta)}{\widehat{\theta} - \theta} \leq \frac{r(p(\widehat{\theta}), \widehat{\theta})[M(\widehat{\theta}; A) - M(\theta; A)]}{\widehat{\theta} - \theta}.$$

Allowing that $\widehat{\theta}$ may approach θ from the right or the left, we may now take limits as $\widehat{\theta} \to \theta$, use the differentiability of the function $M(\theta; A) = \frac{U}{N} + [1 - F(\theta)]^{N-1}I$, and conclude that

$$A'(\theta) = r(p(\theta), \theta)\frac{\partial M(\theta; A)}{\partial \theta}.$$

When combined with the boundary condition $A(\overline{\theta}) = 0$, this differential equation may be solved to yield

$$A(\theta) = -\int_{\theta}^{\overline{\theta}} r(p(x), x)[\partial M(x; A)/\partial x]dx,$$

where $\frac{\partial M(x;A)}{\partial x} = -(N-1)[1 - F(x)]^{N-2}f(x)I < 0$ for all $x < \overline{\theta}$.

We now integrate by parts and establish that $A(\theta)$ must take the following unique form:

$$A(\theta) = R(\theta, \theta; A) - R(\overline{\theta}, \overline{\theta}; A) - \int_{\theta}^{\overline{\theta}} D(p(x)) \left[\frac{U}{N} + [1 - F(x)]^{N-1} I \right] dx, \quad (1)$$

where $R(\overline{\theta}, \overline{\theta}; A) = r(p(\overline{\theta}), \overline{\theta}) \frac{U}{N}$. Rearranging, we note that interim-stage profit for type θ then must be given as

$$\Pi(\theta, \theta; A) = R(\overline{\theta}, \overline{\theta}; A) + \int_{\theta}^{\overline{\theta}} D(p(x)) \left[\frac{U}{N} + [1 - F(x)]^{N-1} I \right] dx. \quad (2)$$

Observe that interim-stage profit is positive for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

The second step in our proof is to construct an advertising equilibrium using the $A(\theta)$ function defined in (1). Observe that $\Pi_1(\theta, \theta; A) = r(p(\theta), \theta) \frac{\partial M(\theta; A)}{\partial \theta} - A'(\theta) = 0$ when this function is used. It follows that no type θ will deviate by mimicking some other type $\widehat{\theta}$, since for all $\widehat{\theta} < \theta$ we have

$$\begin{aligned} \Pi(\theta, \theta; A) - \Pi(\widehat{\theta}, \theta; A) &= \int_{\widehat{\theta}}^{\theta} \Pi_1(x, \theta; A) dx \\ &= \int_{\widehat{\theta}}^{\theta} \left[\Pi_1(x, \theta; A) - \Pi_1(x, x; A) \right] dx \\ &= \int_{\widehat{\theta}}^{\theta} \int_{x}^{\theta} \Pi_{12}(x, y; A) dy dx > 0, \end{aligned}$$

where the inequality follows from

$$\Pi_{12}(x, y; A) = D(p(y))(N-1)[1 - F(x)]^{N-2} f(x) I > 0 \text{ for all } x < \overline{\theta}.$$

A similar argument ensures that $\Pi(\theta, \theta; A) > \Pi(\widehat{\theta}, \theta; A)$ for all $\widehat{\theta} > \theta$. Next, if no type $\theta > \underline{\theta}$ gains from deviating to $A(\underline{\theta})$, then a deviation to any advertising level $a > A(\underline{\theta})$ is also unattractive. Finally, since $A'(\theta) < 0$, the advertising search rule is optimal for informed consumers. ■

Proposition 1 thus establishes the existence and uniqueness of an advertising equilibrium.[17] The advertising equilibrium acts as a fully sorting (separating)

[17] See Maskin and Riley (1984) for a related equilibrium characterization of bidding functions in the context of optimal auctions when buyers are risk averse. Our model also endogenizes the beliefs and strategies of informed consumers. For an advertising equilibrium, beliefs are uniquely defined on the equilibrium path (by Bayes' rule) and off the equilibrium path (since the advertising search rule is optimal for informed consumers when they observe an advertising level in excess of $A(\underline{\theta})$ only if they believe that the deviating firm has cost type $\underline{\theta}$).

mechanism: firms truthfully reveal their cost types along the downward-sloping advertising schedule. The informed consumers behave rationally in the advertising model: the lowest-cost firm advertises the most and offers the lowest price, and the informed consumers purchase from the highest-advertising firm. Thus, ostensibly uninformative advertising directs market share to the lowest-cost supplier and promotes productive efficiency.

In the advertising equilibrium, the expected market share allocated to a firm of type θ takes the following form: $M(\theta; A) = \frac{U}{N} + [1 - F(\theta)]^{N-1} I$. A firm is sure to get its share of uninformed consumers; further, since the advertising schedule is downward sloping, a firm wins the informed consumers with the probability that the other $N - 1$ firms draw higher types. The advertising equilibrium thus induces a market share allocation that is strictly decreasing in a firm's type. The highest type does not advertise and sells only to its uninformed consumers: $M(\overline{\theta}; A) = \frac{U}{N}$.

We now characterize the expected profit for firms in the advertising equilibrium. Using (2) and integrating by parts, we find that expected profit may be represented as:

$$E_\theta \left[\Pi(\theta, \theta; A)\right] = r(p(\overline{\theta}), \overline{\theta})\frac{U}{N} + E_\theta \left[D(p(\theta))\frac{F}{f}(\theta)\left[\frac{U}{N} + [1 - F(\theta)]^{N-1} I\right]\right]. \tag{3}$$

The first term on the RHS is the "profit at the top." As noted, the fully sorting scheme allocates a market share of only $\frac{U}{N}$ to the highest type, $\overline{\theta}$. The second term represents the expected information rents. It is not immediately clear whether a strictly decreasing market share allocation enhances the magnitude of this term. The strength of the fully sorting scheme is based on downward-sloping demand. Lower-cost firms set lower prices and thus generate greater demand from visiting consumers; hence, $D(p(\theta))$ is decreasing in θ. By directing more market share to lower-cost firms, the fully sorting scheme thus acts to expand the size of the market and increase expected information rents. The weakness of the fully sorting scheme is associated with the term $\frac{F}{f}(\theta)$. For many popular distributions, F is log-concave ($\frac{F}{f}(\theta)$ is nondecreasing in θ).[18] By allocating less market share to higher types, the fully sorting scheme works against the direct to which log-concavity of F appeals.

Log-concavity of F plays a prominent role in our analysis below. It is thus important to develop some intuition for the role played by this property.[19] Market share must be allocated so as to satisfy incentive compatibility. When greater market share is directed to type θ, this type earns greater profit

[18] This assumption is common in the contract literature and is satisfied by many distribution functions.

[19] For further discussion, see also Athey et al. (2004).

and is thus less tempted to mimic lower types. Lower types can then lower advertising expenses and earn greater profit without inducing a violation of incentive compatibility. Intuitively, the ratio $\frac{F}{f}(\theta)$ describes the contribution to expected profit that is made when type θ receives greater market share, since this ratio measures the proportion of types below θ conditional on the occurrence of type θ. When $\frac{F}{f}(\theta)$ is nondecreasing in θ, an increase in market share to type θ contributes more to expected profit when type θ is higher.[20]

The advertising equilibrium can be understood as a purification of Bagwell and Ramey (1994a). In their paper, advertising directs market share to the firm that offers the best deals, but equilibrium advertising takes the form of a mixed strategy. To see how our model constructs a purified version, consider a complete-information game, where production costs are fixed at a constant $c > 0$. Then, as we establish in the Appendix, there exists a unique symmetric mixed-strategy equilibrium in this game as in Bagwell and Ramey (1994a) and Varian (1980).[21] Consider next an incomplete-information game, where production costs rise in types θ. As we show in the Appendix, if a firm of type θ uses the advertising strategy $A(\theta)$ in the unique advertising equilibrium of the incomplete-information game, then the probability distribution induced by A is approximately the same as the distribution of advertising in the mixed-strategy equilibrium of the complete-information game, when the production costs for types θ (say, $c(\theta)$) approximate the constant c.[22] This purification result offers a useful link between the complete- and incomplete-information analyses; however, it does not establish whether the main predictions of Bagwell and Ramey carry over when, as seems plausible, production costs vary significantly with types. As we show below, when some additional structure is placed on the demand and distribution functions, the main predictions of the complete-information model can be captured in the general incomplete-information setting.

[20]We build on this intuition below, when we compare expected information rents under the advertising and random equilibria. The random equilibrium is a limiting case in which firms pool at zero advertising. The resulting market share allocation is incentive compatible.

[21]In the complete-information game considered here, all firms set the same price and informed consumers are indifferent when using the advertising search rule. By contrast, Bagwell and Ramey (1994a) allow firms to make cost-reducing investments, and this ensures that higher-advertising firms offer strictly lower prices. In the analysis of advertising equilibria considered here, the advertising search rule is strictly optimal for informed consumers provided that incomplete information is present so that production costs vary (at least a little) with types.

[22]Our analysis here builds on Bagwell and Ramey (1992, 1994a), who establish a related finding when the cost of advertising is private information and approximately constant.

2.3 Random Equilibrium

In this subsection, we analyze the random equilibrium, wherein all consumers use the random search rule and thus divide up evenly across firms. Each firm then receives an equal share, $\frac{1}{N}$, of the unit mass of consumers. Given the random search rule, firms necessarily choose zero advertising, since even informed consumers are unresponsive to advertising; furthermore, when firms pool and do not advertise, the random search rule is a best response for each consumer.[23] The random equilibrium thus exists and takes the form of a pooling equilibrium.

In the random equilibrium, the interim-stage profit for the firm of type θ is given by $r(p(\theta), \theta)\frac{1}{N}$. The random equilibrium sacrifices productive efficiency; however, all advertising expenses are avoided. Using $\frac{dr(p(\theta),\theta)}{d\theta} = -D(p(\theta))$, it is straightforward to confirm that the expected profit for a firm in the random equilibrium is

$$E_\theta \left[r(p(\theta), \theta) \frac{1}{N} \right] = r(p(\overline{\theta}), \overline{\theta}) \frac{1}{N} + E_\theta \left[D(p(\theta)) \frac{F}{f}(\theta) \frac{1}{N} \right]. \tag{4}$$

The RHS contains the profit at the top and the expected information rents, respectively.

2.4 Comparison of Advertising and Random Equilibria

We now compare the advertising and random equilibria. We begin by comparing the expected consumer surplus in these equilibria. An uninformed consumer expects the same consumer surplus whether the advertising or random equilibrium is anticipated. For both equilibria, the uninformed consumers samples from the induced distribution of monopoly prices and expects to pay $E_\theta [p(\theta)]$. By contrast, an informed consumer expects strictly higher consumer surplus in the advertising equilibrium than in the random equilibrium. The key point is that, in the advertising equilibrium, an informed consumer can infer the identity of the lowest-cost, and thus lowest-price, firm. Accordingly, while

[23] If informed consumers observe a deviation whereby some firm selects positive advertising, then random search remains optimal in the event that informed consumers believe that the deviating firm has an average type. Since such a deviation may be more attractive to a lower-cost type, the random equilibrium may fail to be a "refined" equilibrium in the static model. See Bagwell and Ramey (1994b) for an analysis of the refined equilibrium in a related model of advertising in which one firm has two possible cost types. As noted in the Introduction, the random equilibrium can also be associated with a setting in which advertising is prohibited (in which case deviant positive advertising selections are not possible). Our analysis here of random equilibria is also useful for our companion paper (Bagwell and Lee, 2010), where we consider the repeated game and the possibility of a self-enforcing agreement among firms in which a deviation from zero advertising would cause a future advertising war.

the distribution of prices is not altered across equilibria, the informed consumer in the advertising equilibrium transacts at the lowest available price. Formally, an informed consumer expects to pay $E_\theta\left[p(\theta)\right]$ in the random equilibrium and $E_\theta[p(\theta_{\min})]$, where $\theta_{\min} \equiv \min\{\theta_1, ..., \theta_N\}$, in the advertising equilibrium. Under our assumption that $N \geq 2$, we have that $E_\theta[p(\theta_{\min})] < E_\theta\left[p(\theta)\right]$.

The comparison of expected profit across equilibria is more subtle. As illustrated in (3) and (4), in both types of equilibria, expected profit consists of two terms: the profit at the top and the expected information rents. To increase the profit at the top, the random equilibrium (pooling) is strictly preferred to the advertising equilibrium (full sorting). Intuitively, the highest-cost firm is never "out-advertised" in the random equilibrium and thus sells to its share of all consumers, $\frac{1}{N}$; by contrast, in the advertising equilibrium, the highest-cost firm is always out-advertised and thus sells only to its share of uninformed consumers, $\frac{U}{N}$. To increase expected information rents, however, it is not immediately clear whether the random or advertising equilibrium is preferred. On the one hand, if $\frac{F}{f}(\theta)$ is nondecreasing, then the random equilibrium is attractive, since this equilibrium allocates more market share to higher-cost types. On the other hand, downward-sloping demand creates a force that favors the advertising equilibrium, which allocates more market share to lower-cost types, since these types price lower and thus generate larger demand $D(p(\theta))$.

For the special case in which the support of possible cost types is small, we can unambiguously rank expected profits under the advertising and random equilibria. As $\overline{\theta} - \underline{\theta}$ approaches zero, expected information rents also approach zero in both the advertising and random equilibria. Profit at the top remains strictly higher under the random equilibrium, however, since the highest-cost firm gets strictly more market share in the random than the advertising equilibrium. Thus, for $\overline{\theta} - \underline{\theta}$ sufficiently small, expected profit is strictly higher under the random equilibrium than under the advertising equilibrium. Given the purification result described above and established in the Appendix, this finding can be understood as a direct extension of Bagwell and Ramey's (1994a) analogous finding for the associated complete-information game.

Consider next the general case in which the support of possible costs may be large. To go further in ranking expected profits, we must formally analyze the expected information rents.[24] Let A denote the advertising schedule used in the advertising equilibrium, in which the market share allocation, $M(\theta; A) = \frac{U}{N} + [1 - F(\theta)]^{N-1} I$, is strictly decreasing. Similarly, let $A^p \equiv 0$ denote the advertising schedule used in the random (pooling) equilibrium, in

[24] Our analysis here builds on arguments made by Athey et al. (2004) in their analysis of price collusion.

which the market share allocation, $M(\theta; A^p) \equiv \frac{1}{N}$, is constant. We now define the distribution function

$$G(\theta; A) \equiv \frac{\int_{\underline{\theta}}^{\theta} M(x; A) f(x) dx}{\int_{\underline{\theta}}^{\overline{\theta}} M(x; A) f(x) dx}.$$

The distribution $G(\theta; A^p)$ is similarly defined. The denominator represents the (ex ante) expected market share, which equals $\frac{1}{N}$. Since $M(\theta; A)$ is strictly decreasing, $M(\theta; A^p) = \frac{1}{N}$ crosses $M(\theta; A)$ from below. This implies in turn that $G(\theta; A^p)$ first-order stochastically dominates $G(\theta; A)$: $G(\theta; A^p) \leq G(\theta; A)$. Thus, if $D(p(\theta))\frac{F}{f}(\theta)$ is nondecreasing, then

$$\int_{\underline{\theta}}^{\overline{\theta}} D(p(\theta))\frac{F}{f}(\theta) dG(\theta; A^p) \geq \int_{\underline{\theta}}^{\overline{\theta}} D(p(\theta))\frac{F}{f}(\theta) dG(\theta; A).$$

The inequality can be rewritten as

$$E_\theta\left[D(p(\theta))\frac{F}{f}(\theta)M(\theta; A^p)\right] \geq E_\theta\left[D(p(\theta))\frac{F}{f}(\theta)M(\theta; A)\right]. \qquad (5)$$

Referring to (3)-(5), we conclude that, if $D(p(\theta))\frac{F}{f}(\theta)$ is nondecreasing, then expected information rents are weakly higher in the random equilibrium than in the advertising equilibrium. In fact, given our assumption that $f(\underline{\theta}) > 0$, we can go further and establish that, if $D(p(\theta))\frac{F}{f}(\theta)$ is nondecreasing, then the random equilibrium must generate strictly higher information rents than the advertising equilibrium. This follows since $D(p(\theta))\frac{F}{f}(\theta)$ is strictly increasing at $\underline{\theta}$, given $f(\underline{\theta}) > 0$.

We may now summarize our findings regarding the comparison of expected profit across equilibria. Relative to the advertising equilibrium, the random equilibrium has strictly higher profit at the top and, if $D(p(\theta))\frac{F}{f}(\theta)$ is nondecreasing, strictly higher expected information rents. As suggested above, $D(p(\theta))\frac{F}{f}(\theta)$ is nondecreasing if the log-concavity of F is significant in comparison to the extent to which demand slopes down. Further insight is possible by considering the limiting case in which $D(p(\theta))$ is perfectly inelastic, so that $D(p(\theta))$ is constant for all prices up to a reservation value. In this case, if $F(\theta)$ is log-concave, then $D(p(\theta))\frac{F}{f}(\theta)$ is nondecreasing.

We may now state the following conclusion:

Proposition 2. *(i) Informed consumers enjoy strictly higher expected consumer surplus in the advertising equilibrium than in the random equilibrium, and uninformed consumers enjoy exactly the same expected consumer surplus in both equilibria. (ii) If F is log-concave and demand is sufficiently inelas-*

tic, or if the support of possible cost types is sufficiently small, then firms make a strictly higher expected profit in the random equilibrium than in the advertising equilibrium.

Proposition 2 (i) captures the implications of advertising restrictions for consumer welfare. While the ability to advertise does not affect the distribution of posted prices, it does lead to a strict reduction in the average transaction price. This finding is consistent with Benham's observations. We discuss this point further in the next subsection, when we allow for endogenous entry.

Proposition 2 (ii) indicates that important circumstances exist under which firms gain when the use of advertising is restricted. As our discussion of the random equilibrium confirms, advertising would not be used if informed consumers were to ignore it. If informed consumers were responsive to advertising, however, then firms might nevertheless achieve a restriction on the use of advertising if advertising were legally prohibited. For a fixed industry structure, Proposition 2 (ii) thus suggests that retail firms might benefit from a prohibition on non-price retail advertising.[25]

Proposition 2 (ii) establishes that firms gain by restricting the use of advertising if F is log-concave and demand is sufficiently inelastic or if the support of possible cost types is sufficiently small.[26] It is important to note, though, that this conclusion may hold even when the assumptions are weakened. Consider the constant-elasticity demand function, $D(p) = p^{-\epsilon}$, and suppose that demand is elastic (i.e., $\epsilon > 1$). Assume further that F is log-concave in the specific sense that types are distributed uniformly over $[\underline{\theta}, \overline{\theta}]$ where $\underline{\theta} > 0$. For this example, calculations reveal that $\frac{d}{d\theta}[D(p(\theta))\frac{F}{f}(\theta)] > 0$ for all θ if $\overline{\theta}/[\overline{\theta} - \underline{\theta}] > \epsilon$. Firms thus earn a strictly higher expected profit by pooling at zero advertising than by following the advertising equilibrium, provided that the elasticity of demand, ϵ, does not exceed a critical level where this level is higher when the support of possible cost types is smaller.

[25] A further possibility is that firms are able to eliminate the use of advertising through a self-enforcing collusive agreement and that firms prefer such a restriction to *any* other self-enforcing advertising scheme. We consider this possibility in our companion paper (Bagwell and Lee, 2010).

[26] Under these conditions, firms prefer the random equilibrium to the advertising equilibrium, because separation through advertising is too costly as a means of informing consumers of firms' cost types. Information transmission itself would be valuable to firms if it could be achieved at lower cost. For example, firms would earn even higher expected profit if they could costlessly and verifiably tell informed consumers the identity of the lowest-cost firm, since firms would then enjoy productive efficiency without incurring any advertising expenditures. See Ziv (1993) for an analysis of a similar issue in the context of oligopoly information sharing.

2.5 Free-Entry Equilibrium

We now relax the assumption that the number of firms is fixed. To this end, following Bagwell and Ramey (1994a), we include now an initial stage for the game in which firms simultaneously decide whether to enter, where entry entails a positive setup (or opportunity) cost. After a firm chooses to enter, it privately learns its cost type. The number of entering firms is publicly observed, and the game then proceeds as above.

It is clear from (3) and (4) that expected profit is strictly decreasing in the number of firms, N, whether firms anticipate the advertising or random equilibrium. Thus, in each case, an equilibrium number of firms is implied such that the profit from entry (inclusive of the fixed cost) would be negative were one more firm to enter. Let N^s denote the equilibrium number of entering firms when the advertising (full sorting) equilibrium is anticipated, and let N^p denote the equilibrium number of entering firms when the random (pooling) equilibrium is expected. It is also clear from Proposition 2 that, if F is log-concave and demand is sufficiently inelastic, or if $\overline{\theta} - \underline{\theta}$ is sufficiently small, then $N^p \geq N^s$. Under these conditions, at least as many firms enter when the random equilibrium is expected as when the advertising equilibrium is anticipated.

The model also leads to welfare comparisons. Assume that $\min(N^s, N^p) \geq 1$.[27] When the number of firms is endogenized, if we ignore integer constraints, then firms earn zero expected profit whether the random or advertising equilibrium is anticipated. Uninformed consumers are also indifferent. Under either equilibrium, an uninformed consumer picks a firm at random and thus faces an expected price of $E_\theta p(\theta)$. Finally, consider the informed consumers. When the random equilibrium occurs, an informed consumer also faces an expected price of $E_\theta p(\theta)$; however, when the advertising equilibrium occurs, an informed consumer is guided by advertising activity to the lowest market price and thus faces the expected *minimum* price in the market. Provided that $N^s \geq 2$, an informed consumer thus strictly prefers the advertising equilibrium. When the number of firms is endogenous, it follows that expected welfare is higher when the advertising equilibrium is anticipated than when the random equilibrium is expected. This conclusion does not require any assumption as to the elasticity of demand or the log-concavity of the distribution function.

We may now summarize with the following proposition:

Proposition 3. *Assume that* $\min(N^s, N^p) \geq 1$. *(i) If* F *is log-concave and demand is sufficiently inelastic, or if the support of possible cost types is sufficiently small, then* $N^p \geq N^s$ *(concentration is at least as high in the*

[27] When $N = 1$, a single firm enters the market and chooses $A = 0$, and all consumers visit it.

advertising equilibrium as in the random equilibrium). (ii) Social surplus is as high in the advertising equilibrium as in the random equilibrium; further, if $N^s \geq 2$, then social surplus is strictly higher in the advertising equilibrium than in the random equilibrium.

Allowing that the support of possible costs may be large, we thus establish a sense in which Bagwell and Ramey's main findings extend to the private-information setting. When legal or other considerations lead to the absence of advertising, if the distribution of types is log-concave and demand is sufficiently inelastic, then the market is less concentrated than it would be were advertising competition to occur. Furthermore, the average transaction price is lower, and social welfare is thus higher, when entry is endogenized and firms compete in advertising. Note, however, that some findings such as Proposition 2 (ii) and Proposition 3 (i) are not straightforward, given downward-sloping demand. For a given number of firms, pooling at zero advertising acts to increase the profit at the top but sorting through advertising acts to increase expected information rents when demand is substantially larger for lower prices. This conflict suggests that market concentration could be lower in the advertising equilibrium than in the random equilibrium, when demand is sufficiently elastic. Thus, the established positive association between advertising and market concentration employs additional assumptions on the distribution of types and the elasticity of demand in the general private-information setting.

It is interesting to compare these findings with empirical patterns emphasized in the earlier literature on advertising. Benham (1972) provides evidence for retail markets that prices are lower and market concentration is higher, when non-price retail advertising is allowed. Our findings offer theoretical support for these associations. In another set of studies, Bain (1956), Comanor and Wilson (1974) and others find a positive relationship between manufacturer advertising and profitability. These authors suggest that the relationship may reflect the role of advertising in deterring entry. Consistent with interpretations offered by Demsetz (1973) and Nelson (1974), our work suggests that advertising and profitability may be positively related, since they are both implications of superior efficiency. In the advertising equilibrium, lower-cost firms advertise more, have larger sales and earn greater expected profit.[28]

[28]While (interim) expected profit is decreasing in cost, ex post profit need not decrease in cost. For U sufficiently small and $\theta < \overline{\theta}$, ex post profit increases in cost among those firms that do not have the lowest cost in the market. This is because such "losing" firms make few sales when U is small and incur lower advertising expenses when costs are higher. Thus, in some circumstances, higher ex post profit may be paired with lower advertising for a subset of firms. We thank a referee for this observation. Note that, under our assumption that demand is not perfectly inelastic, sales decrease with cost even in an ex post sense, because higher-cost firms select higher prices.

3 Comparison with Pricing Equilibrium

In this section, we compare the advertising equilibrium with the analogous *pricing equilibrium* that emerges in a benchmark model in which $N \geq 2$ ex ante identical firms compete in prices. We follow Varian (1980) and suppose that informed consumers observe prices and buy from the lowest-priced firm while uninformed consumers pick a firm at random. Following Spulber (1995) and Bagwell and Wolinsky (2002), we modify Varian's model and allow that firms are privately informed as to their costs. We characterize the pricing equilibrium of this benchmark game and compare the associated expected profit with that achieved in the advertising equilibrium of our advertising game.

In the benchmark game, if a pricing strategy is denoted by ρ, then the interim-stage profit in direct form is given by

$$\Pi^B(\widehat{\theta}, \theta; \rho) = [\rho(\widehat{\theta}) - \theta] D(\rho(\widehat{\theta})) M^B(\widehat{\theta}; \rho),$$

where we use the superscript B to denote the benchmark (Bertrand) game. When a firm selects the price $\rho(\widehat{\theta})$ and other firms use the pricing strategy ρ, then the firm's expected market share is denoted as $M^B(\widehat{\theta}; \rho)$. The profit-if-win is defined by $[\rho(\widehat{\theta}) - \theta] D(\rho(\widehat{\theta})) \equiv r(\rho(\widehat{\theta}), \theta)$. As in Spulber (1995), a unique and symmetric equilibrium can be established. A new feature in our benchmark model is that uninformed consumers exist. The pricing equilibrium ρ satisfies:

$$\rho'(\theta) = -\frac{r(\rho(\theta), \theta)[\partial M^B(\theta; \rho)/\partial\theta]}{r_\rho(\rho(\theta), \theta) M^B(\theta; \rho)} \text{ and } \rho(\overline{\theta}) = p(\overline{\theta}), \tag{6}$$

where $M^B(\theta; \rho) = \frac{U}{N} + [1 - F(\theta)]^{N-1} I$. Straightforward arguments ensure that the equilibrium price is lower than the monopoly price except the price at the top, so that $r_\rho > 0$. As (6) confirms, the equilibrium pricing schedule is strictly increasing; thus, firms are fully sorted by their types in the pricing equilibrium. Notice that the highest-cost firm selects its monopoly price, $p(\overline{\theta})$, and sells only to uninformed consumers.

In the pricing equilibrium, interim-stage profit can be written as

$$\Pi^B(\theta, \theta; \rho) = \Pi^B(\overline{\theta}, \overline{\theta}; \rho) + \int_\theta^{\overline{\theta}} D(\rho(x)) \left[\frac{U}{N} + [1 - F(x)]^{N-1} I\right] dx, \tag{7}$$

where the profit at the top is $\Pi^B(\overline{\theta}, \overline{\theta}; \rho) = r(p(\overline{\theta}), \overline{\theta}) \frac{U}{N}$. Integrating by parts, we find that expected profit is given as:

$$E_\theta\left[\Pi^B(\theta, \theta; \rho)\right] = r(p(\overline{\theta}), \overline{\theta}) \frac{U}{N} + E_\theta\left[D(\rho(\theta)) \frac{F}{f}(\theta) \left[\frac{U}{N} + [1 - F(\theta)]^{N-1} I\right]\right]. \tag{8}$$

Comparing (8) with (3), we see that the profit at the top is the same in the advertising equilibrium as in the pricing equilibrium. In each case, the highest-cost firm monopolizes only uninformed consumers. The expected information rents are higher in the pricing equilibrium, however, since demand is greater when prices are set below monopoly levels. We thus have the following conclusion: for any fixed number of firms, a firm's expected profit is strictly higher in the pricing equilibrium than in the advertising equilibrium.[29] Evidently, when firms possess private information about their costs, competition in (non-price) advertising is more aggressive than (Bertrand) competition in prices. Intuitively, price competition induces greater in-store demand from consumers and thus elevates the size of expected information rents for firms. When the number of firms is fixed, both consumers and firms agree that the pricing equilibrium is preferred to the advertising equilibrium. When the number of firms is endogenized by the free-entry condition, more firms enter in the former equilibrium than in the latter equilibrium. Once market structure is endogenized, firms are indifferent between pricing and advertising competition, but consumers strictly prefer the former to the latter (provided that at least two firms enter in the pricing equilibrium).

We may thus summarize the findings of this section as follows:

Proposition 4. *There exists a unique and symmetric pricing equilibrium, and in this equilibrium the pricing function $\rho(\theta)$ satisfies $\rho(\theta) > \theta$ and is strictly increasing and differentiable. Expected profit and consumer surplus are both strictly higher in the pricing equilibrium than in the advertising equilibrium. When the number of firms is endogenized, at least as many firms enter in the pricing equilibrium as in the advertising equilibrium; furthermore, if at least two firms enter in the pricing equilibrium, then social surplus is strictly higher in the pricing equilibrium than in the advertising equilibrium.*

With these findings at hand, we may now offer a further interpretation of Benham's findings. Let us associate the advertising equilibrium with a setting in which only non-price advertising is allowed, the pricing equilibrium with a setting in which price advertising is allowed, and the random equilibrium with a setting in which advertising is banned.[30] Provided that the market always

[29] In a different context, Bagwell and Ramey (1988) present a somewhat related finding. Working with a two-type signaling model, they show that a low-cost incumbent earns greater profit when it separates using price as a signal than when it separates using wasteful advertising (money-burning) as a signal.

[30] Our association of the pricing equilibrium with a setting in which price advertising is allowed implicitly assumes that price advertising is not costly and that firms do not choose the intensity of price advertising. It would be interesting to explore as well a model of price advertising that relaxes these assumptions. We leave this to future research.

has at least two firms, our results in this section indicate that the average transaction price is lowest when price advertising is allowed, somewhat higher when only non-price advertising is allowed, and higher yet when all advertising is banned. Likewise, when the number of firms is endogenous, social welfare is highest when price advertising is allowed, somewhat lower when only non-price advertising is allowed, and lower yet when all advertising is banned. Finally, when demand is sufficiently inelastic and the distribution of types is log-concave, the market is less concentrated when advertising is banned than when non-price or price advertising is allowed.[31] These findings are broadly consistent with Benham's findings.

4 Comparative Statics

We now return to the advertising model and conduct comparative-statics analysis. We consider how the advertising equilibrium responds to changes in parameters I and N, and to shifts of the distribution function of types.

To analyze comparative statics associated with distribution functions, we consider distribution functions F and G that have the same support $[\underline{\theta}, \overline{\theta}]$. As above, the distribution functions are twice-continuously differentiable and have positive densities f and g. We then compare two advertising equilibrium strategies, $A_F(\theta)$ and $A_G(\theta)$, that correspond to the distribution functions, F and G, respectively. We compare the distributions F and G by using the monotone likelihood ratio (MLR) order. The distribution function F dominates G in terms of the MLR order if $\frac{f(\theta)}{g(\theta)}$ is strictly increasing for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Using the MLR order, we can show how firms choose their advertising when lower-cost (higher-advertising) types are more likely under G than under F.

Our comparative-statics results are contained in the following proposition:

Proposition 5. *(i) Equilibrium advertising $A(\theta)$ is strictly increasing in I for all $\theta \in [\underline{\theta}, \overline{\theta})$, where $A(\overline{\theta}) = 0$. (ii) If N rises, then there exists $\widehat{\theta} \in (\underline{\theta}, F^{-1}(1 - e^{-\frac{1}{N-1}}))$ such that equilibrium advertising strictly increases for $\theta \in [\underline{\theta}, \widehat{\theta})$, strictly decreases for $\theta \in (\widehat{\theta}, \overline{\theta})$, and is unchanged for $\theta \in \{\widehat{\theta}, \overline{\theta}\}$. (iii) For all $\theta > \underline{\theta}$ and $\varepsilon > 0$, there exists N' such that, for all $N > N'$, $A(\theta) < \varepsilon$. (iv) If distribution function F dominates G in terms of the MLR order, then*

[31] For a fixed number of firms, if demand is perfectly inelastic, the expected information rents in the pricing equilibrium are the same as in the advertising equilibrium. Thus, when demand is sufficiently inelastic, market concentration is approximately the same in these two equilibria. Further, if $\overline{\theta} - \underline{\theta}$ is sufficiently small, then the market is less concentrated when advertising is banned than when non-price or price advertising is allowed. This is because the random equilibrium generates the largest market share for a firm with cost type $\overline{\theta}$.

there exists $\widetilde{\theta} \in (\underline{\theta}, \overline{\theta})$ *such that* $A_F(\theta) < A_G(\theta)$ *for* $\theta \in [\underline{\theta}, \widetilde{\theta})$, $A_F(\theta) > A_G(\theta)$ *for* $\theta \in (\widetilde{\theta}, \overline{\theta})$, *and* $A_F(\theta) = A_G(\theta)$ *for* $\theta \in \{\widetilde{\theta}, \overline{\theta}\}$.

The proofs of parts (ii) and (iv) are in the Appendix.

Using the derivation of $A(\theta)$ in the proof of Proposition 1, we can immediately confirm that part (i) holds.[32] Intuitively, firms compete more intensely by raising advertising when the gain from capturing informed consumers rises. It is less clear, however, whether advertising increases when N rises. On the one hand, an increase in the number of firms may lead to greater competition for the informed consumers and thus an increase in advertising. On the other hand, an increase in the number of firms may also cause firms to become discouraged about the prospect of winning the informed consumers and thus result in a decrease in advertising. In part (ii), we confirm that these competing considerations weigh differently across firms with different cost types: when the number of firms increases, lower-cost firms compete more aggressively and raise advertising, while higher-cost firms perceive a reduced chance of winning the informed consumers and lower advertising. An interesting implication is that the support of equilibrium advertising levels (i.e., $[A(\overline{\theta}) = 0, A(\underline{\theta})]$) is larger in markets with more firms. Observe, however, that as the number of firms goes to infinity, the cutoff type $\widehat{\theta}$ converges to $\underline{\theta}$; thus, for markets with a sufficiently large number of firms, further entry is almost sure to lower the advertising of any given firm. In fact, we can easily confirm that part (iii) holds and thus that, for any type other than the lowest type, the equilibrium level of advertising must be near zero when the number of firms is sufficiently large.[33]

Finally, as part (iv) establishes, competing considerations arise as well when the distribution of costs changes so that lower-cost realizations become more likely in the sense of the MLR order. Following such a shift, lower-cost firms compete more aggressively for informed consumers and thus increase their advertising; however, higher-cost firms become discouraged about their chances of winning the informed consumers and thus lower their advertising levels. Our work here builds on Hopkins and Kornienko (2007), who report a similar finding for a family of all-pay auctions.

5 Sequential Search

In the advertising model considered above, we assume that consumers are unable to engage in sequential search. We now examine equilibrium behavior

[32] Formally, this follows since $\partial M(x; A)/\partial x$ is strictly decreasing in I for all $x < \overline{\theta}$.

[33] As shown in the proof of Proposition 1, $A(\overline{\theta}) = 0$ and $A'(\theta) = r(p(\theta), \theta)\partial M(\theta; A)/\partial\theta$. Part (iii) thus follows, since, for all $\theta > \underline{\theta}$, $\partial M(\theta; A)/\partial\theta$ goes to zero as N goes to infinity.

when this assumption is relaxed. Thus, we allow that after a consumer visits a firm and observes that firm's price, the consumer may elect to incur a search cost and visit another firm.

Consider then a modified advertising game, in which consumers can undertake costly sequential search and firms choose advertising levels and prices. A Symmetric Perfect Bayesian Equilibrium may be informally defined in terms of the following requirements: (i) each firm selects its advertising level and price to maximize its expected profit, given its type and the strategies of other players; (ii) each consumer selects an initial firm to visit and any subsequent firm to visit in a way that maximizes the consumer's expected welfare at each point, given the information that the consumer then has and the consumer's beliefs about prices at firms not yet visited; (iii) where possible, consumers' beliefs are formed in a manner consistent with Bayes' rule, given the equilibrium strategies of firms;[34] and (iv) firms use symmetric price and advertising strategies. An advertising equilibrium is a Symmetric Perfect Bayesian Equilibrium in which informed consumers pick an initial firm using the advertising search rule while uninformed consumers pick an initial firm at random. A random equilibrium is a Symmetric Perfect Bayesian Equilibrium in which all consumers ignore advertising and select an initial firm at random.

We begin by observing that the sequential-search option is irrelevant if the cost of sequential search is sufficiently large relative to the expected dispersion of prices in the market. Suppose that firms follow the advertising equilibrium of the original advertising game as characterized in Proposition 1. An uninformed consumer is then most tempted to search again in the event that the consumer encounters the highest possible monopoly price, $p(\overline{\theta})$. Let $U(p)$ denote consumer surplus at the price p, and let the cost of sequential search be denoted as $d > 0$.[35] Even a consumer that encounters $p(\overline{\theta})$ won't gain from sequential search, if $U(p(\overline{\theta})) \geq E_\theta U(p(\theta)) - d$. Thus, if $p(\overline{\theta}) - E_\theta p(\theta)$ is small relative to the cost of sequential search, then an uninformed consumer never gains from sequential search. This condition is sure to hold in the limiting case of perfectly inelastic demand, since then the monopoly price is independent of production costs. Likewise, for the constant-elasticity demand function, $D(p) = p^{-\epsilon}$, with elasticity $\epsilon > 1$, we have that $p(\overline{\theta}) - E_\theta p(\theta) = \frac{\epsilon}{\epsilon - 1}[\overline{\theta} - E\theta]$. Thus, if the extent of dispersion in production costs is small relative to the

[34] The concept of Perfect Bayesian Equilibrium also includes a no-signaling-what-you-don't-know requirement. In the present context, this means that, if a consumer initially visits firm i and contemplates undertaking the sequential search cost and visiting some other firm j, then the consumer's belief about the price that might be observed at firm j is not altered by the price observed at firm i. Of course, for an informed consumer, the belief about the price at firm j may be influenced by the advertising level selected by firm j.

[35] For simplicity, we assume that the initial search has zero cost.

size of the sequential-search cost, then uninformed consumers will not search again even after encountering the highest monopoly price.

If instead the cost of sequential search is small relative to the expected dispersion of prices, then higher-cost firms induce search if they select their monopoly prices. To capture this case, we assume henceforth that $U(p(\overline{\theta})) < E_\theta U(p(\theta)) - d$. Building on work by Reinganum (1979) and Bagwell and Ramey (1996), our goal is to establish conditions under which an advertising equilibrium exists in which firms with cost types at or above a critical level $\theta_c \in (\underline{\theta}, \overline{\theta})$ select the monopoly price for this critical type. We thus seek to construct an advertising equilibrium in which a firm with cost type $\theta > \theta_c$ prices at $p(\theta_c) < p(\theta)$, where $p(\theta_c)$ is determined so that the costs and benefits of sequential search are equal. Higher-cost firms then "limit price" and thereby deter uninformed consumers from searching again.

In our proposed advertising equilibrium, a firm of cost type θ thus selects the price $p^*(\theta) \equiv \min\{p(\theta), p(\theta_c)\}$ and earns the corresponding net revenue $r(p^*(\theta), \theta)$. We now impose a new assumption that $p(\underline{\theta}) > \overline{\theta}$. This assumption is sure to hold if the dispersion in cost types is not too great or if demand is sufficiently inelastic, and it ensures that $p(\theta_c) > \overline{\theta}$ so that $r(p^*(\theta), \theta)$ remains strictly positive even for the highest type. Observe also that $r(p^*(\theta), \theta)$ is strictly decreasing with $\frac{dr(p^*(\theta),\theta)}{d\theta} = -D(p^*(\theta)) < 0$. With these properties in place, we can confirm that the arguments used in the proof of Proposition 1 continue to hold when firms use the pricing function $p^*(\theta)$. Thus, the level of advertising again strictly declines as costs increase, and no firm of any type gains from undertaking an "on-schedule" deviation and mimicking the advertising level of some other type. Informed consumers are again rational in visiting the firm with the highest advertising level, since this firm selects the lowest price in the market.[36] Two issues remain. First, we must establish that a critical value $\theta_c \in (\underline{\theta}, \overline{\theta})$ indeed exists such that an uninformed consumer is indifferent to sequential search upon observing $p(\theta_c)$. Second, we must establish that no firm with cost type $\theta > \theta_c$ would gain from undertaking an "off-schedule" deviation to a higher price.

Consider the first issue. Under our assumption that $U(p(\overline{\theta})) < E_\theta U(p(\theta)) - d$, it is straightforward to establish that there exists a unique value $\theta_c \in (\underline{\theta}, \overline{\theta})$ such that

$$U(p(\theta_c)) = [1 - F(\theta_c)]U(p(\theta_c)) + \int_{\underline{\theta}}^{\theta_c} U(p(\theta))dF(\theta) - d. \tag{9}$$

The LHS of (9) represents the consumer welfare from remaining with a firm that selects $p(\theta_c)$, while the RHS represents the expected welfare from incurring

[36] Note, though, that informed consumers are indifferent about using the advertising search rule in the event that all firms draw cost types at or above θ_c.

the sequential-search cost d and finding the same price or a lower price. The critical value $\theta_c \in (\underline{\theta}, \overline{\theta})$ is then determined so as to make the consumer indifferent between the two options. Notice that θ_c is independent of the fraction of informed consumers, I, and is strictly increasing in the sequential-search cost, d. As d gets close to zero, θ_c gets close to $\underline{\theta}$ and thus almost all types select the limit price.

To understand the second issue, consider a firm with cost type $\theta > \theta_c$. This firm retains its uninformed consumers if it sets the limit price, $p(\theta_c)$, and loses its uninformed consumers if it sets any higher price. Under our assumption that $p(\underline{\theta}) > \overline{\theta}$, we know that the firm earns strictly positive net revenue on its uninformed consumers at the price $p(\theta_c)$. Thus, as regards its uninformed consumers, the firm earns strictly more by selecting the price $p(\theta_c)$ than it would make by undertaking an off-schedule deviation to any higher price. But this firm must also consider informed consumers. With probability $[1 - F(\theta)]^{N-1}$, this firm advertises more than all other firms and receives the informed consumers. In this event, as in the model analyzed by Bagwell and Ramey (1996), the informed consumers observe all advertising choices and thus know that all other firms have higher costs and thus select the price $p(\theta_c)$. The informed consumers will then tolerate a price hike without searching again, provided that the hike is not too large. The maximal price hike that informed consumers will tolerate is $h(d)$ where $h(d)$ is defined by $U(p(\theta_c) + h(d)) \equiv U(p(\theta_c)) - d$. It follows that the optimal off-schedule deviation for a firm of type $\theta > \theta_c$ is the price $p(\theta, \theta_c, d) \equiv \min\{p(\theta), p(\theta_c) + h(d)\}$, where θ_c is determined as a function of d by (9).

We may now conclude that a firm with cost type $\theta > \theta_c$ does not gain from an off-schedule deviation to a higher price if

$$\Omega(\theta, \theta_c, d) \equiv [(1 - F(\theta))^{N-1} I][r(p(\theta, \theta_c, d), \theta) - r(p(\theta_c), \theta)] - \frac{U}{N} r(p(\theta_c), \theta) \leq 0. \tag{10}$$

The first term on the RHS of (10) captures the possible benefit of a price hike in terms of more profitable sales to informed consumers whereas the second term reflects the certain cost of a price hike in terms of lost sales to uninformed consumers. Notice that $\Omega(\theta_c, \theta_c, d) < 0$, since $p(\theta_c, \theta_c, d) = p(\theta_c)$. Likewise, $\Omega(\overline{\theta}, \theta_c, d) < 0$ follows, since the highest-cost firm wins the informed consumers with probability zero and earns strictly positive net revenue at the price $p(\theta_c)$ under our assumption that $p(\underline{\theta}) > \overline{\theta}$. Outside of these boundary cases, we cannot immediately sign $\Omega(\theta, \theta_c, d)$. We can, however, state the following sufficient condition: There exists $I^* \in (0, 1)$ such that if $I < I^*$ then for all $\theta \in (\theta_c, \overline{\theta})$, $\Omega(\theta, \theta_c, d) < 0$. In other words, if the fraction of informed consumers is not too great, then no type of firm will undertake an off-schedule deviation by raising price.

We may now summarize our findings as follows.

Proposition 6. *Consider the advertising game, modified to allow for sequential search. Assume that the search cost satisfies* $U(p(\overline{\theta})) < E_\theta U(p(\theta)) - d$ *and that* $p(\underline{\theta}) > \overline{\theta}$. *There exists* $I^* \in (0,1)$ *such that if* $I < I^*$ *then an advertising equilibrium exists. In this equilibrium, (i) firms use an advertising strategy* $A(\theta)$ *that is strictly decreasing and differentiable and satisfies* $A(\overline{\theta}) = 0$*; (ii) firms use the pricing strategy* $p^*(\theta)$*, where* $\theta_c \in (\underline{\theta}, \overline{\theta})$ *satisfies (9); and (iii) consumers do not engage in sequential search along the equilibrium path.*

In effect, Proposition 6 establishes conditions under which Proposition 1 extends to the setting in which sequential search is possible and not prohibitively expensive.[37]

We now consider the effect of sequential search on the comparison between expected profits under the random and advertising equilibria. When sequential search is possible, our assumption that $p(\underline{\theta}) > \overline{\theta}$ ensures that a random equilibrium exists, wherein firms use the modified pricing schedule, $p^*(\theta)$.[38] As this assumption implies that profit at the top is strictly positive, the random equilibrium again generates strictly greater profit at the top than does the constructed advertising equilibrium (when it exists). When sequential search is prohibited, expected information rents are strictly higher under the random than advertising equilibrium if $\frac{F}{f}(\theta)D(p(\theta))$ is nondecreasing. Likewise, when sequential search is possible, expected information rents are strictly higher under the random than advertising equilibrium if $\frac{F}{f}(\theta)D(p^*(\theta))$ is nondecreasing. Since $p^*(\theta)$ is constant in θ for $\theta > \theta_c$, log-concavity of F alone now ensures that $\frac{F}{f}(\theta)D(p^*(\theta))$ is nondecreasing when $\theta > \theta_c$. Thus, the tension between log-concavity and reduced demand is removed for higher types when sequential search is possible. In this respect, the possibility of sequential search serves to *strengthen* our basic result that firms achieve higher expected profit when they restrict the use of advertising.[39]

[37] The advertising equilibrium of the modified static game is also unique, if the definitions of the advertising and random search rules are extended to cover sequential search decisions. Otherwise, some uninformed consumers that encounter the price $p(\theta_c)$ may undertake sequential search out of indifference, for example.

[38] The existence of the random equilibrium does not require any additional assumption on the fraction of informed consumers, since firms do not advertise in the random equilibrium and thus all consumers are, in effect, uninformed. Thus, the random equilibrium is the counterpart of the equilibrium featured by Reinganum (1979).

[39] Note, though, that sequential search lowers profit at the top, since higher-cost firms earn lower profit when sequential search is possible. Sequential search thus diminishes the magnitude of the profit-at-the-top advantage that the random equilibrium has in comparison to the advertising equilibrium.

6 Conclusion

We investigate the advertising behavior of firms with private information as to their respective production costs. We show that an advertising equilibrium exists, in which informed consumers use an advertising search rule whereby they buy from the highest-advertising firm. The key point is that the highest-advertising firm has the lowest cost and thus selects the lowest price. In this way, "non-informative" advertising directs consumers to the lowest price in the market. We establish conditions under which firms earn greater expected profit when advertising is banned. Consumer welfare falls in this case, however. Thus, advertising can promote productive efficiency and raise consumer welfare; however, firms often have incentive to diminish advertising competition through regulatory restrictions.

We also consider three extensions of the model. First, we present a benchmark model in which privately informed firms compete in prices for informed consumers, and we argue that the associated pricing equilibrium generates greater profit and consumer welfare than does the advertising equilibrium. Second, we show that the advertising equilibrium exhibits non-monotone comparative statics: when the number of firms increases, or when the lower-cost firms become more likely in the sense of the monotone-likelihood ratio, lower-cost firms advertise more while higher-cost firms become discouraged and advertise less. Third, we modify the advertising model to allow for sequential search, and we establish conditions under which an advertising equilibrium continues to exist in the modified model.

We close by mentioning one example of a further extension that represents a promising direction for future research. In our model, advertising expenses are incurred prior to the realization of sales, and so firms implicitly rely on retained earnings or external capital markets when incurring advertising expenditures. Further, if the number of uninformed consumers is small, then it is possible that a firm may not make sufficient profit to cover its advertising expenditure in the current period. These considerations provide motivation for a model in which each firm's advertising expenditure is capped at some common level. Gavious, Moldavanu and Sela (2002) consider a related contest model. Their results suggest the possibility of an advertising equilibrium in which the advertising function is discontinuous, intermediate types advertise *more* when a cap is in place, and all but the highest types advertise at the capped level and thus pool. Interesting future work might build on this analysis, in order to consider the effects of an advertising cap on expected advertising expenditures and social welfare.

7 Appendix

This appendix has two parts. In support of the discussion at the end of Section 2.2, the first part defines a complete-information game, characterizes the associated symmetric mixed-strategy equilibrium, and shows that the distribution of advertising in this equilibrium is approximately the same as that which is induced by the pure-strategy advertising equilibrium of the incomplete-information game when production costs vary sufficiently little with respect to types. The second part completes the proof of Proposition 5.

7.1 Purification

7.1.1 Equilibrium in Complete-Information Game

Suppose that N firms sell a homogeneous good at a constant cost $c > 0$. A pure strategy for firm i is $A_i \in [0, r(p(c), c)]$ and A_{-i} denotes the $(N-1)$-tuple of advertising selected by other firms. The profit for firm i is

$$\Pi_i(A_i, A_{-i}) = \begin{cases} r(p(c), c)\frac{U}{N} - A_i & \text{if } A_i < \max_{j \neq i} A_j \\ r(p(c), c)\left[\frac{U}{N} + \frac{I}{k}\right] - A_i & \begin{array}{l}\text{if } A_i \geq \max_{j \neq i} A_j \text{ and} \\ \|\{j \mid A_j = A_i\}\| = k-1.\end{array} \end{cases}$$

The term $r(p(c), c)$ represents $[p(c) - c]D(p(c))$. A mixed strategy for firm i is a distribution function Φ over $[\underline{A}(\Phi), \overline{A}(\Phi)]$. The profit for firm i is

$$E_i(\Phi_i, \Phi_{-i}) = \int_{\underline{A}}^{\overline{A}} \cdots \int_{\underline{A}}^{\overline{A}} \Pi(A_i, A_{-i}) d\Phi_1 \cdots d\Phi_N,$$

where $\overline{A}$ and $\underline{A}$ are defined below. This complete-information game has a unique symmetric mixed-strategy equilibrium, $\Phi = \Phi_i$ for all i, which is characterized as follows:

Lemma A1. *(i) There is no pure-strategy Nash equilibrium. (ii) There is a unique symmetric mixed-strategy equilibrium:*

$$\Phi(A) = \left(\frac{A}{r(p(c), c)I}\right)^{\frac{1}{N-1}} \text{ with } \underline{A}(\Phi) = 0 \text{ and } \overline{A}(\Phi) = r(p(c), c)I. \qquad \text{(A1)}$$

Proof. To prove (i), assume that there are k firms that select the highest advertising A. First, suppose that $2 \leq k \leq N$. If $A < r(p(c), c)I$, then a firm can gain by raising A slightly by ε and winning all the informed consumers:

$$r(p(c), c)\left[\frac{U}{N} + I\right] - A - \varepsilon > r(p(c), c)\left[\frac{U}{N} + \frac{I}{k}\right] - A.$$

If $A = r(p(c), c)I$, then a firm can increase its profit by reducing A to zero and winning only the uninformed consumers:

$$r(p(c), c)\frac{U}{N} > r(p(c), c)\left[\frac{U}{N} + \frac{I}{k}\right] - A.$$

Second, suppose that $k = 1$. The highest-advertising firm can raise its profit by setting $A - \varepsilon$ which is slightly above the second-highest advertising.

To prove (ii), we begin by showing that any symmetric Nash equilibrium, Φ, must satisfy (A1). To this end, we establish four findings. First, there is no mass point in Φ. If A is a mass point of Φ, then there is a positive probability of tie at A. A firm can increase its profit, if it preserves the hypothesized equilibrium strategy, except that it replaces the selection of A with the selection of $A + \varepsilon$ for small ε. Second, $\underline{A}(\Phi) = 0$. Suppose that $\underline{A}(\Phi) > 0$. If a firm chooses $\underline{A}(\Phi)$, then it wins only the uninformed consumers with probability one, since ties occur with zero probability (because of no mass point). The firm can increase its profit when it replaces the selection of $[\underline{A}(\Phi), \underline{A}(\Phi) + \varepsilon]$ with the selection of zero advertising. Third, $\overline{A}(\Phi) = r(p(c), c)I$. This result is immediate, since the profit at the top is equal to the profit at the bottom in the mixed-strategy equilibrium:

$$r(p(c), c)\left[\frac{U}{N} + I\right] - \overline{A}(\phi) = r(p(c), c)\frac{U}{N}.$$

Fourth, Φ is strictly increasing over $(\underline{A}(\Phi), \overline{A}(\Phi))$. Suppose that there is a gap (A_1, A_2) such that $\underline{A}(\Phi) < A_1 < A_2 < \overline{A}(\Phi)$ and $\Phi(A_1) = \Phi(A_2)$. Advertisements in the interval (A_1, A_2) are then selected with zero probability. For ε small, a firm would gain by replacing the selection of advertising levels in the interval $[A_2, A_2 + \varepsilon]$ with the selection of $A_1 + \varepsilon$. This deviation has the same probability of winning but uses a lower level of advertising. Given these four findings, we may conclude that, in any symmetric Nash equilibrium, Φ, and for all $A \in [0, r(p(c), c)I]$,

$$r(p(c), c)\left[\frac{U}{N} + [\Phi(A)]^{N-1}I\right] - A = r(p(c), c)\frac{U}{N}. \tag{A2}$$

This equation yields (A1). Thus, (A1) is necessarily satisfied in a symmetric Nash equilibrium. Observe next that (A1) identifies a well-defined and unique distribution function $\Phi(A)$. Lastly, we verify that Φ is a Nash equilibrium. A firm earns the same expected profit for any $A \in [\underline{A}(\Phi), \overline{A}(\Phi)]$ when all other $N - 1$ firms adopt $\Phi(A)$. It cannot increase the profit by altering the distribution over the interval. Any advertising above $\overline{A}(\Phi)$ earns a lower expected profit than does $\overline{A}(\Phi)$, because $\overline{A}(\Phi)$ wins the informed consumers with probability one. Any advertising below $\underline{A}(\Phi)$ is infeasible. ■

7.1.2 Equilibrium in Incomplete-Information Game

We consider an incomplete-information game, where production costs rise in types θ. We argue that if each firm of type θ chooses $A(\theta)$, which is the unique advertising equilibrium in the incomplete-information game, then the probability distribution induced by A is approximately the distribution of advertising in the mixed-strategy equilibrium, when the payoff relevance of types θ gets small. In the incomplete-information game, the firm of type $\theta \in [\underline{\theta}, \overline{\theta}]$ privately observes its type and has the cost $c(\theta)$. Assume that function c is differentiable and strictly increasing in θ, with $0 < c(\underline{\theta}) < c(\overline{\theta}) < p_R$, where p_R is given by $D(p_R) = 0$. The advertising game is the same as in the text. Then, arguing as in the proof of Proposition 1, there is a unique advertising equilibrium A which satisfies:

$$A'(\theta) = -r(p(\theta), \theta)(N-1)[1-F(\theta)]^{N-2} f(\theta) I < 0 \text{ and } A(\overline{\theta}) = 0, \qquad \text{(A3)}$$

where $r(p(\theta), \theta) = [p(c(\theta)) - c(\theta)]D(p(c(\theta)))$.

Lemma A2. *Given a constant $c \in (0, p_R)$, for any $\varepsilon > 0$, there exists $\delta > 0$ such that if $|c(\theta) - c| < \delta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, then the probability distribution of advertising induced by the advertising equilibrium in the incomplete-information game is ε-close to Φ_c, where Φ_c is the distribution of advertising in the mixed-strategy equilibrium of the complete-information game with constant cost c.*

Proof. The distribution induced by $A(\theta)$ is

$$prob\,(\theta \mid A(\theta) \leq x) = prob\left(\theta \geq A^{-1}(x)\right) = 1 - F\left(A^{-1}(x)\right).$$

Let Φ_c denote the symmetric mixed-strategy equilibrium with costs c. Define the function A_c by

$$A_c(\theta) = \Phi_c^{-1}(\phi(\theta)), \text{ where } \phi(\theta) \equiv 1 - F(\theta).$$

Given that $\phi(\theta)$ is strictly decreasing in θ and Φ_c is strictly increasing, $A_c(\theta)$ is strictly decreasing in θ. The proof is established as a consequence of the following results. First, if each firm of type θ chooses $A_c(\theta)$, then the distribution of advertising becomes Φ_c. In other words, $A_c(\theta)$ induces the same distribution of advertising as Φ_c:

$$\begin{aligned} prob\,(A_c(\theta) \leq x) &= prob\left(\Phi_c^{-1}(\phi(\theta)) \leq x\right) \\ &= prob\,(\phi(\theta) \leq \Phi_c(x)) \\ &= prob\,(F(\theta) \geq 1 - \Phi_c(x)) \\ &= prob\left(\theta \geq F^{-1}(1 - \Phi_c(x))\right) \\ &= 1 - F\left(F^{-1}(1 - \Phi_c(x))\right) \\ &= \Phi_c(x). \end{aligned}$$

Second, $A_c(\theta)$ solves (A3) when $c(\theta) = c$. By the definition of $A_c(\theta)$, we have that

$$A_c'(\theta) = -f(\theta)/\Phi_c'(A_c(\theta)).$$

To find $\Phi_c'(A_c(\theta))$, we recall the mixed strategy (A2) and differentiate it with respect to A :

$$1 = (N-1)r(p(c), c) [\Phi(A)]^{N-2} \Phi'(A)I.$$

Replacing Φ with Φ_c, we obtain

$$\Phi_c'(A) = \frac{1}{(N-1)r(p(c), c) [\Phi_c(A)]^{N-2} I}.$$

Substituting, we thus find that

$$A_c'(\theta) = -(N-1)r(p(c), c) [\Phi_c(A_c(\theta))]^{N-2} f(\theta)I.$$

Note also that $A_c(\overline{\theta}) = \Phi_c^{-1}(1 - F(\overline{\theta})) = \Phi^{-1}(0) = 0$. Hence, when $c(\theta) = c$, $A_c(\theta)$ solves (A3). Third, if $|c(\theta) - c|$ is small, then $A(\theta)$ induces approximately the same distribution of advertising as does Φ_c. This result is based on the first and second result. The function $A_c(\theta)$ induces Φ_c by the first result, and $A_c(\theta)$ approximates $A(\theta)$ when $c(\theta)$ approaches c by the second result: for any $\varepsilon > 0$, there exists $\delta > 0$ such that if $|c(\theta) - c| < \delta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, then $|A(\theta) - A_c(\theta)| < \varepsilon$. As $c(\theta)$ becomes closer to a constant c, the type θ becomes less payoff-relevant. Hence, the distribution of advertising induced by $A(\theta)$, prob$(\theta \mid A(\theta) \leq x)$, approximates Φ_c when the payoff relevance of types θ gets small. ■

7.2 Proof of Proposition 5

Consider part (ii). Note first that advertising at the top is held fixed at $A(\overline{\theta}) = A'(\overline{\theta}) = 0$ for all N. Differentiating $|A'(\theta)|$ with respect to N yields:

$$\frac{\partial |A'(\theta)|}{\partial N} = |A'(\theta)| \left(\frac{1 + (N-1)\ln[1 - F(\theta)]}{N-1} \right).$$

The equation means that for a slight increase of N, $A(\theta)$ becomes flatter over the types above $F^{-1}(1 - e^{-\frac{1}{N-1}}) \in (\underline{\theta}, \overline{\theta})$ and steeper over the types below $F^{-1}(1 - e^{-\frac{1}{N-1}})$. We can next show that advertising at the bottom, $A(\underline{\theta})$, strictly increases when N rises. To see this, integrating by parts, we get

$$\begin{aligned} A(\underline{\theta}) &= \int_{\underline{\theta}}^{\overline{\theta}} r(p(x), x)(N-1)[1 - F(x)]^{N-2} f(x)Idx \\ &= r(p(\underline{\theta}), \underline{\theta})I - \int_{\underline{\theta}}^{\overline{\theta}} [1 - F(x)]^{N-1} D(p(x))Idx. \end{aligned}$$

The integral on the RHS strictly decreases with N and thus $A(\underline{\theta})$ strictly increases in N. Hence, we can now conclude that there exists a cutoff type $\widehat{\theta} < F^{-1}(1 - e^{-\frac{1}{N-1}})$ such that equilibrium advertising strictly increases with N for $\theta \in [\underline{\theta}, \widehat{\theta})$, strictly decreases with N for $\theta \in (\widehat{\theta}, \overline{\theta})$, and is constant with N when $\theta \in \{\widehat{\theta}, \overline{\theta}\}$.

Consider part (iv). For the proof, we proceed with four steps as follows. First, we establish a monotonicity in the ratio of advertising equilibrium slopes under MLR dominance. Define

$$\gamma(\theta) \equiv \frac{A'_F(\theta)}{A'_G(\theta)} = \frac{f(\theta)}{g(\theta)} \left[\frac{1 - F(\theta)}{1 - G(\theta)}\right]^{N-2}.$$

For $\theta \in [\underline{\theta}, \overline{\theta})$, the ratio $\gamma(\theta)$ of two slopes is strictly increasing in θ, since $\frac{f(\theta)}{g(\theta)}$ and $\frac{1-F(\theta)}{1-G(\theta)}$ are then positive and strictly increasing under MLR dominance. The latter term, $\frac{1-F(\theta)}{1-G(\theta)}$, is strictly increasing if $\frac{1-F(\theta)}{f(\theta)} > \frac{1-G(\theta)}{g(\theta)}$. To see that this inequality holds for $\theta \in [\underline{\theta}, \overline{\theta})$, note that MLR dominance can be re-stated as $\frac{f(y)}{f(x)} > \frac{g(y)}{g(x)}$ for all $y > x$; hence, for $x \in [\underline{\theta}, \overline{\theta})$, MLR dominance implies $\int_x^{\overline{\theta}} \frac{f(y)}{f(x)} dy > \int_x^{\overline{\theta}} \frac{g(y)}{g(x)} dy$ and thus $\frac{1-F(x)}{f(x)} > \frac{1-G(x)}{g(x)}$. Second, we establish that $A_F(\underline{\theta}) < A_G(\underline{\theta})$. Note that

$$A_F(\underline{\theta}) - A_G(\underline{\theta}) = -\int_{\underline{\theta}}^{\overline{\theta}} \left([1 - F(x)]^{N-1} - [1 - G(x)]^{N-1}\right) D(p(x)) I dx.$$

We thus have that $A_F(\underline{\theta}) < A_G(\underline{\theta})$ if $\frac{1-F(\theta)}{1-G(\theta)} > 1$ for all $\theta > \underline{\theta}$. This inequality holds, since $\frac{1-F(\theta)}{1-G(\theta)}$ achieves its minimum value of 1 at $\underline{\theta}$ and (as established above) is strictly increasing for $\theta \in [\underline{\theta}, \overline{\theta})$ under MLR dominance. Third, we show that $\gamma(\underline{\theta}) = \frac{f(\underline{\theta})}{g(\underline{\theta})} < 1 < [\frac{f(\overline{\theta})}{g(\overline{\theta})}]^{N-1} = \gamma(\overline{\theta})$. The stated properties for $\gamma(\underline{\theta})$ follow immediately from the definition of $\gamma(\theta)$ and MLR dominance, while the stated properties for $\gamma(\overline{\theta})$ follow from using L'Hopital's rule and MLR dominance. Given $A_F(\underline{\theta}) < A_G(\underline{\theta})$, $A_F(\overline{\theta}) = A_G(\overline{\theta}) = 0$ and $\gamma(\overline{\theta}) > 1$, we can conclude that there exists $\widetilde{\theta} \in (\underline{\theta}, \overline{\theta})$ at which $A_G(\theta)$ crosses $A_F(\theta)$ from above. Fourth, we establish that a second interior crossing does not exist. Assume to the contrary that there exists $\theta_2 \in (\underline{\theta}, \overline{\theta})$ at which $A_G(\theta)$ crosses $A_F(\theta)$ from below and thus $\gamma(\theta_2) > 1$. Given $A_F(\overline{\theta}) = A_G(\overline{\theta}) = 0$ and $\gamma(\overline{\theta}) > 1$, there must then exist $\theta_3 \in (\theta_2, \overline{\theta})$ at which $A_G(\theta)$ crosses $A_F(\theta)$ from above and thus $\gamma(\theta_3) < 1$. But this contradicts our first result that $\gamma(\theta)$ is strictly increasing in θ over $\theta \in [\underline{\theta}, \overline{\theta})$ under MLR dominance. ■

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